

Please read the following directions.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

- The exam is closed book and closed notes. You will be supplied with scratch paper, and a copy of the Table of Common Distributions from the back of our textbook.
- During the exam, you may use **ONLY** what you need to write with (pens, pencils, erasers, etc).
- All other items (**INCLUDING CELL PHONES**) must be left at the front of the classroom during the exam. This includes backpacks, purses, books, notes, etc. You may keep small items (keys, coins, wallets, etc., but **NOT CELL PHONES**) so long as they remain in your pockets at all times.
- You must show and explain your work (including your calculations) for all the problems except those on the last page. **No credit is given without work or explanation!** But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Partial credit is available (except on the last page). If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- **If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a moment generating function is undefined outside of some range, this range should be stated explicitly.**
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- Simplify your answers when it is easy to do so. But more difficult arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate large binomial coefficients, factorials or powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- All algebra and calculus must be done completely. (Only arithmetic can be left incomplete.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- All the work on the exam should be your own. No “cooperation” is allowed.
- The exam has **8** problems and **12** pages. There are a total of **100** points.

Problem 1. Suppose X has density

$$f_X(x) = \lambda e^{-\lambda x} \quad \text{for } x > 0$$

where $\lambda > 0$ is a constant. Define $Y = X^\beta$ where $\beta > 0$ is a constant.

(a) (8%) Find the density of Y .

[**Problem 1 continued**]

(b) (8%) Find the mean of Y .

Problem 2. (14%) Suppose that clicks on a Geiger counter occur according to a Poisson process with an average rate of 2.0 clicks per second. What is the probability of the event that there are **exactly 4** clicks during the time interval $(3.5, 6.5)$ and **at least 2** clicks during the time interval $(8.0, 9.5)$?

Problem 3. (14%) Use an argument involving indicator random variables to give an expression for $P(A \cap B^c \cap C^c)$. This expression should only contain terms chosen from the list

$$P(A), P(B), P(C), P(A \cap B), P(A \cap C), P(B \cap C), P(A \cap B \cap C).$$

Problem 4. Suppose X is a discrete random variable with probability mass function (pmf) given by

$$f(x) = \begin{cases} \frac{1}{2} p(1-p)^{|x|-1} & \text{for } x = \pm 1, \pm 2, \pm 3, \dots \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$, that is, $f(x) = \frac{1}{2} p(1-p)^{|x|-1}$ for all integers except 0 for which we have $f(0) = 0$.

(a) (8%) Find the moment generating function (mgf) of X .

[**Problem 4 continued**]

(b) (6%) Find the mean of X .

[**Problem 4 continued**]

(c) (6%) Find the variance of X .

Problem 5. (14%) Suppose that $EX^2 < \infty$. Show that $E(X - a)^2$ is minimized over a when $a = EX$. Make sure to show that $a = EX$ is indeed a minimum. (If you wish, you may assume that X has a density f_X , although this is not really needed.)

Problem 6. An urn contains n balls. One of them is red, the others are all green. The balls are drawn out one by one withOUT replacement. Let X be the number of draws needed to get the red ball.

(a) (10%) Find EX .

[**Problem 6 continued**]

(b) (4%) Now suppose there are two red balls in the urn, and the other $n - 2$ balls are green. The balls are drawn out one by one withOUT replacement. Let X be the number of draws needed to get the first red ball. What is EX ?

No work is required for the problems on this page. You will receive full credit for stating the correct answer. Put your answer in the blank provided.

Problem 7. (4%) Let $X \sim \text{Gamma}(\alpha, \beta = 1)$ and define

$$\psi(\alpha) = P(\alpha - 0.5 < X < \alpha + 0.5).$$

What is the value of

$$\lim_{\alpha \rightarrow \infty} \psi(\alpha)?$$

Problem 8. (4%) Let $X_1, X_2, X_3, \dots$ be iid $\text{Poisson}(\lambda = 1)$ and define $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$. What is the value of

$$\lim_{n \rightarrow \infty} P(0.5 < \bar{X}_n < 1.5)?$$