TEST #1
STA 5326 **Name:**____________________
October 2, 2023

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

- The exam is closed book and closed notes. You will be supplied with scratch paper, and a copy of the Table of Common Distributions from the back of our textbook.
- During the exam, you may use ONLY what you need to write with (pens, pencils, erasers, etc). Calculators are NOT allowed.
- All other items (INCLUDING CELL PHONES) must be left at the front of the classroom during the exam. This includes backpacks, purses, books, notes, etc. You may keep small items (keys, coins, wallets, etc., but NOT CELL PHONEs) so long as they remain in your pockets at all times.
- You must show and explain your work (including your calculations) for all the problems (except for problems labeled **NWR**). **No credit is given without work or explanation!**. But don't get carried away! Give enough explanation and work so that what you have done is clearly understandable.
- Partial credit is available (except for problems labeled **NWR**). If you know part of a solution, write it down. If you know an approach to a problem, but cannot carry it out – write down this approach. If you know a useful result, write it down.
- **No work is required** for the problems marked **NWR**. For these problems, you will receive full credit just for stating the correct answer.
- Make sure that the grader can easily see how you get from one step to the next. If you needed scratch paper to work something out, make sure to transfer your work to the exam.
- **If your answer is valid only for a certain range of values, this should be stated as part of your answer. For example, if a density is zero outside of some interval, this interval should be stated explicitly.**
- You should give only one answer to each problem. **Circle your answer** if there is any chance for confusion.
- Simplify your answers when it is easy to do so. But more difficult arithmetic does **not** have to be done completely. Answers can be left as fractions or products. You do not have to evaluate large binomial coefficients, factorials or powers. Answers can be left as summations (unless there is a simple closed form such as when summing a geometric or exponential series).
- All algebra and calculus must be done completely. (Only arithmetic can be left incomplete.)
- Do **not** quote homework results. If you wish to use a result from homework in a solution, you must prove this result.
- All the work on the exam should be your own. No "cooperation" is allowed.
- The exam has **8** problems and pages. There are a total of **100** points.

Problem 1. n figure skaters perform in a random order. Each skater is given a rating by a panel of judges. Assume there are no ties among the skaters, that is, there are no skaters that are rated the same.

(a) (12%) If the i-th skater is the best so far, what is the probability this skater will be the best overall?

[**Problem 1 continued**]

(b) (12%) Suppose that prizes are given to the top three skaters. If the i-th skater is the best so far, what is the probability this skater does NOT receive a prize? (Assume $n > 3$ and $1 \leq i \leq n-3$.)

Problem 2. Suppose n people play Russian roulette. Each person has a gun which fires with probability π when the trigger is pulled. (Assume the guns are independent of each other and successive shots of the same gun are independent.) A round of play consists of every one who is still alive raising the guns to their temples and firing simultaneously. Play continues until everyone is dead.

(a) (12%) What is the probability that one or more people are still alive after k rounds of play?

[**Problem 2 continued**]

(b) (12%) The last person (or persons) to die receives a prize (flowers on the grave). What is the probability this prize goes to **exactly two persons**? Assume $n > 2$. (Note: The answer may not have a simple form, so do not worry if your answer is messy or contains summations you do not know how to do.)

Problem 3. In each of the following find the density (pdf) of Y.

(a) (12%) $Y = X^4$ and X has density $f_X(x) = 2(x - 1)$ for $1 < x < 2$.

[**Problem 3 continued**]

(b) (12%) $Y = X^4$ and X has density $f_X(x) = (2x + 7)/30$ for $-3 < x < 2$.

Problem 4. A monomial of degree d in the k variables $x_1, x_2, \ldots, x_k$ is a product of the form $x_1^{i_1} x_2^{i_2} \cdots x_k^{i_k}$ where the k exponents $i_1, i_2, \ldots, i_k$ are nonnegative integers which sum to d. (Note that zero is allowed as an exponent.)

Some monomials of degree 6 in the 3 variables x, y, z are listed here as examples:

(1) $x^6y^0z^0 = x^6$	(2) $x^0y^0z^6 = z^6$	(3) $x^4y^2z^0 = x^4y^2$
(4) $x^2y^4z^0 = x^2y^4$	(5) $x^2y^1z^3 = x^2yz^3$	(6) $x^1y^3z^2 = xy^3z^2$

All six of these examples are different monomials because they are different functions of x, y, z. Rearranging the order of factors does not change their product; xy^2z^3, y^2z^3x, and z^3xy^2 are all considered to be the same monomial because they are equal.

(a) (12%) How many different monomials of degree d in the k variables $x_1, x_2, \ldots, x_k$ are there?

(b) (3%) How many **different** polynomials are there which are sums of two **different** monomials of degree d in the k variables $x_1, x_2, \ldots, x_k$? **(NWR)**

(Example: $x^4y^2 + xy^3z^2$ is a sum of two different monomials of degree 6 in the 3 variables x, y, z. Changing the order of the two terms does not alter the sum so that both $x^4y^2 + xy^3z^2$ and $xy^3z^2 + x^4y^2$ are considered to be the same polynomial.)

Problem 5. (4%) Approximately one-third of all human twins are identical (one-egg) and two-thirds are fraternal (two-egg) twins. Identical twins are necessarily the same sex, with male and female being equally likely. Among fraternal twins, approximately one-fourth are both female, one-fourth are both male, and half are one male and one female. Finally, among all U.S. births, approximately 1 in 90 is a twin birth.

What is the probability that a U.S. birth results in fraternal twins, with one being male and the other female?

In the remaining questions, **circle** the **single** correct response. **(NWR)**

Problem 6. (3%) If A, B, and C are mutually **exclusive** events, all having positive probability, then $P(A \cup B \,|\, B \cup C) =$ ________.

a) 0

b) $P(A)$

c) $\dfrac{P(B)}{P(B)+P(C)}$

d) $\dfrac{1}{P(C)}$

e) $\dfrac{P(B)+P(A)P(C)-P(A)P(B)P(C)}{P(B)+P(C)-P(B)P(C)}$

f) $\dfrac{P(A)P(B)P(C)}{P(B)+P(C)}$

g) $\dfrac{P(A)P(B)+P(B)P(C)-P(A)P(B)P(C)}{P(B)+P(C)-P(B)P(C)}$

h) $\dfrac{P(B)}{P(B)+P(C)-P(B)P(C)}$

Problem 7. (3%) A random variable that is continuous but **not** absolutely continuous ________.

a) has a **p**df which is continuous except at finitely many points

b) has a **c**df which is continuous except at finitely many points

c) has a continuous **c**df with finitely many flat intervals

d) has a continuous **c**df and a **p**df which is NOT continuous

e) has a continuous **c**df but does NOT have a **p**df

f) has a **c**df with NO jumps but with finitely many points where the derivative does NOT exist

Problem 8. (3%) Which one of the following statements is always true?

a) $P(A \cap B^c \cap C^c) \geq P(A) - P(A \cap B) + P(A \cap B \cap C)$

b) $P((A \cap B) \cup C) \leq P(A \cap B) + P(C)$

c) $1 - P(A^c \cap B^c) \geq P(A) + P(B)$

d) $1 - P(A^c \cup B^c) = P(A)P(B)$

e) $P(A \cap B) \geq P(A)P(B)$