Mark Hamrick

Homework 1, Exercise B3(a)

Probability that one or more people are still alive after k rounds of play = ?

Probability that anyone's gun fails to fire (and therefore the person lives) $= p = 1 - \pi$

Probability that a person is still alive after one round $= p$, after 2 rounds $= p \cdot p = p^2, \cdots$, after k rounds $= p^k$ (since the events are independent), where k is a non-negative integer.

Define "success" as the event that an individual's gun fails to fire after k rounds.
Then $Pr(\text{success}) = p^k = p'$.

Denote X_i as any of the n individual players, and

$$X_i = \begin{cases} 1, & \text{success for a person with probability } p' = p^k \\ 0, & \text{fail (gun fires and person dies) with probability } 1 - p' = 1 - p^k \end{cases}$$

The pmf of X_i is:

$$f(x_i) = (p')^x (1 - p')^{1-x}\,,\; x = 0, 1$$

And since any one individual's fate is independent of another's, the X_i's are IID Bernoulli random variables.

We are interested in the total number of people alive after k rounds, so we are interested in the total number of successes in n repeated Bernoulli trials.
Therefore $X = \sum_i^n X_i$ is a Binomial random variable with probabaility p' and the pmf:

$$\begin{aligned} f(x) &= \binom{n}{x} (p')^x \, (1 - p')^{n-x} \\ &= \binom{n}{x} ((1-\pi)^k)^x (1 - (1-\pi)^k)^{n-x}\,, \quad \text{for } x = 0, 1, 2, \ldots, n. \end{aligned}$$

So the probability that one or more people are still alive after k rounds of play =

$$\begin{aligned} Pr(X \geq 1) &= 1 - Pr(X = 0) \\ &= 1 - (1 - (1-\pi)^k)^n \end{aligned}$$