**Begin your solution to each problem on a new sheet of paper.**

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**Problem 9.** (5326) Answer the following. The parts are **not** related.

**(a)** Suppose the random variable $X$ has density

$$f_X(x) = \frac{1}{\pi}\frac{1}{1+x^2}, \quad -\infty < x < \infty.$$

Find $M_X(t)$, the moment generating function (mgf) of $X$. Give a careful description of $M_X(t)$ for all real values $t$. Prove your answer.

**(b)** For each of the two functions given below, answer the following: Does a distribution exist whose mgf $M(t)$ is the given function? If yes, specify this distribution. If no, prove it.

$$\frac{1}{3}\left(e^{3t} + e^{5t} + e^{7t}\right)$$

$$\frac{1}{3}\left(2e^{3t} + 2e^{5t} - e^{7t}\right)$$

**(c)** Suppose $Y \sim \text{Geometric}(\beta)$ and $Z \mid Y \sim \text{Binomial}(Y, p)$. Find $M_Z(t)$, the mgf of $Z$. Simplify your answer. (Hint: one approach is to use iterated expectations.)

Note: The pmf of $Y$ is $f_Y(y) = (1-\beta)^{y-1}\beta$ for $y = 1, 2, 3, \ldots$ where $0 < \beta < 1$.

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**Problem 10.** (5326) Suppose that $n$ balls are placed at random into $n$ cells. (The balls are independent of each other, and the cells are equally likely.) Let $X$ be the number of empty cells remaining after the balls are placed.

**(a)** Find $P(X = 0)$.

**(b)** Find $P(X = 1)$.

**(c)** Find $EX$.

**(d)** Find $\text{Var}(X)$.

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