TEST #2
STA 4853/5856 **Name:**______________________________________
May 3, 2013

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are 27 questions, all multiple choice.
- Circle a **single** answer for each multiple choice question. Your choice should be made clearly.
- There is no penalty for guessing.
- The exam has **10** pages.
- Each question is worth equal credit.

The following information applies to the next 5 problems:

Suppose $Y_t = v(B)X_t + N_t$ where X_t is a **step** function representing an intervention at time **10** (so that $X_t = 1$ for $t \geq 10$ and $X_t = 0$ for $t < 10$). The following five plots depict the changes $C_t = v(B)X_t$ produced in the series Y_t by different transfer functions $v(B)$. In each case, pick the simplest type of transfer function that could produce the given changes C_t.
In each case, the possibilities are:

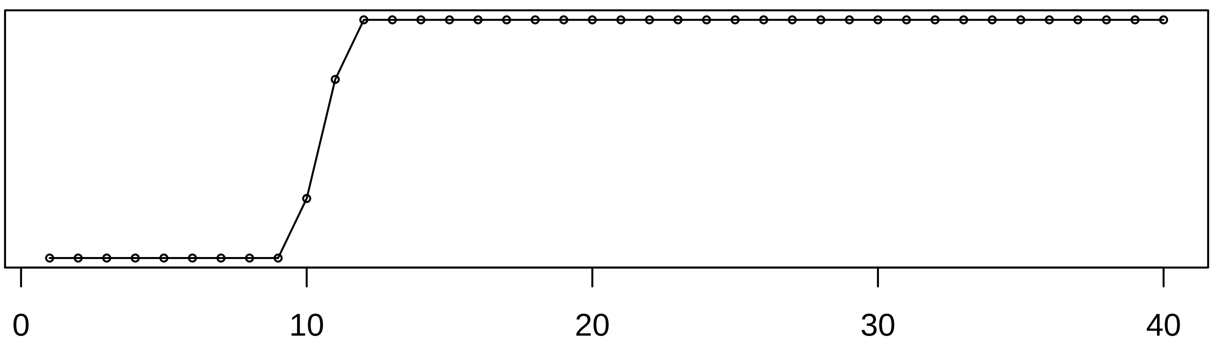
a) ω_0 **b)** $\omega_0 B^2$ **c)** $\omega_0 - \omega_1 B - \omega_2 B^2$ **d)** $B^3(\omega_0 - \omega_1 B - \omega_2 B^2)$

e) $\dfrac{\omega_0}{1 - \delta_1 B}$ **f)** $\dfrac{\omega_0}{1 - \delta_1 B - \delta_2 B^2}$ **g)** $\dfrac{\omega_0 - \omega_1 B - \omega_2 B^2}{1 - \delta_1 B}$ **h)** $\dfrac{\omega_0 - \omega_1 B - \omega_2 B^2}{1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3}$

Note: In all the plots $C_t = 0$ for $t \leq 9$, i.e., nothing changes before the intervention at time 10.

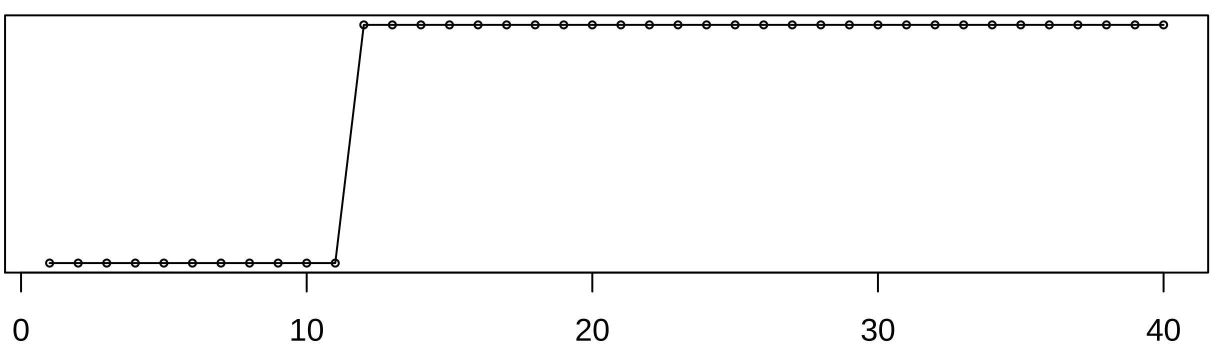
Problem 1. What is the simplest type of transfer function $v(B)$ that could produce the changes $C_t = v(B)X_t$ pictured below?

a) **b)** **c)**$\star$ **d)** **e)** **f)** **g)** **h)**



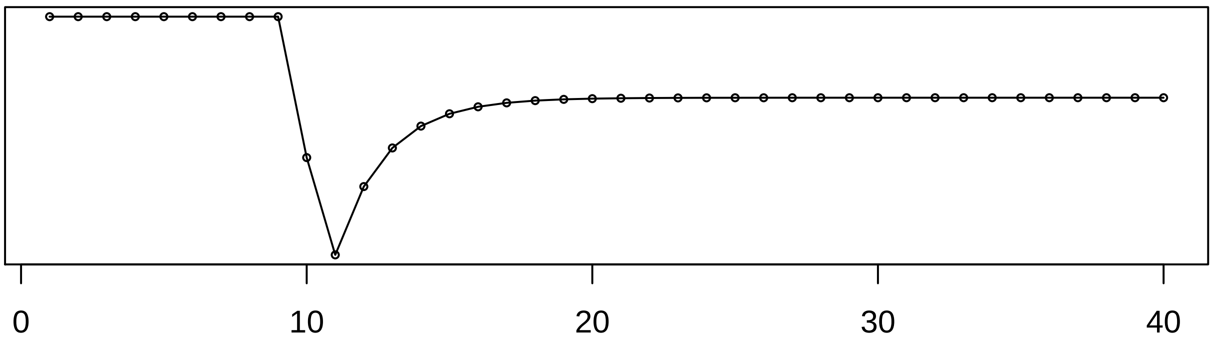
Problem 2. What is the simplest type of transfer function $v(B)$ that could produce the changes $C_t = v(B)X_t$ pictured below?

a) **b)**$\star$ **c)** **d)** **e)** **f)** **g)** **h)**



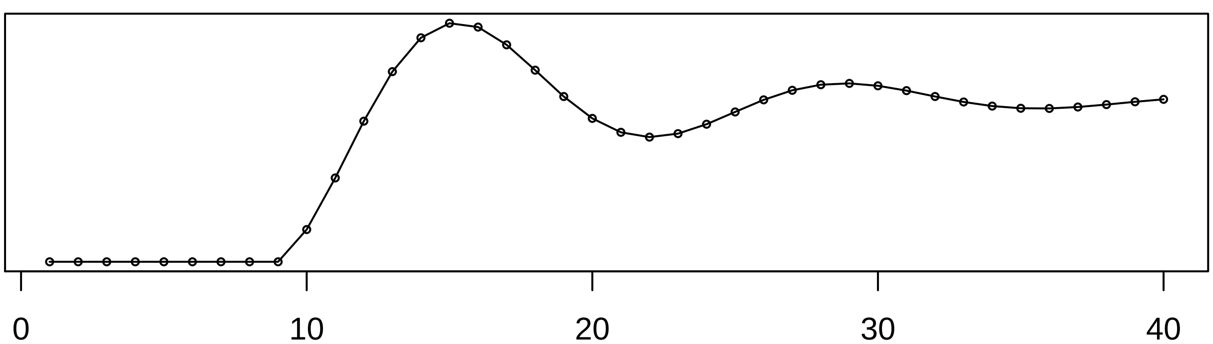
Problem 3. What is the simplest type of transfer function $v(B)$ that could produce the changes $C_t = v(B)X_t$ pictured below?

a) **b**) **c**) **d**) **e**) **f**) **g**)$\star$ **h**)



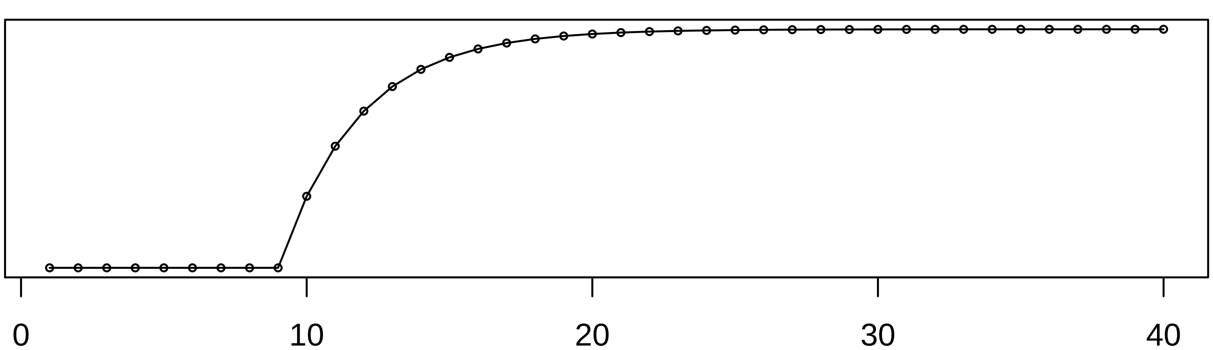
Problem 4. What is the simplest type of transfer function $v(B)$ that could produce the changes $C_t = v(B)X_t$ pictured below?

a) **b**) **c**) **d**) **e**) **f**)$\star$ **g**) **h**)



Problem 5. What is the simplest type of transfer function $v(B)$ that could produce the changes $C_t = v(B)X_t$ pictured below?

a) **b**) **c**) **d**) **e**)$\star$ **f**) **g**) **h**)



Suppose you wish to predict the random variable X based on the information $\mathcal{I}$. Let $\widehat{X}$ denote your prediction for X. Suppose you pay a penalty depending on the accuracy of your prediction $\widehat{X}$.

Problem 6. If you will have to pay a penalty of $(X - \widehat{X})^2$, then the best prediction for X is the ________ of the conditional distribution of X given $\mathcal{I}$.

a) variance **b)** standard error **c)** median **d)** mode **e)**⋆ mean

Problem 7. If you will have to pay a penalty of $|X - \widehat{X}|$, then the best prediction for X is the ________ of the conditional distribution of X given $\mathcal{I}$.

a) variance **b)** standard error **c)**⋆ median **d)** mode **e)** mean

Problem 8. If you will have to pay a penalty of \$100 unless $\widehat{X}$ is within ε of X (where ε is small), then the best prediction for X is the ________ of the conditional distribution of X given $\mathcal{I}$.

a) variance **b)** standard error **c)** median **d)**⋆ mode **e)** mean

Problem 9. If you are predicting future values of an ARIMA process with normally distributed random shocks (given values of the series up to time n), then the best prediction is ________ for all three penalties in the previous three problems.

a) different
b)⋆ the same
c) equal to the variance
d) equal to the standard error
e) never equal to the mean
f) never equal to the median
g) never equal to the mode

Problem 10. An ARMA(2,2) process will be **invertible** if . . .

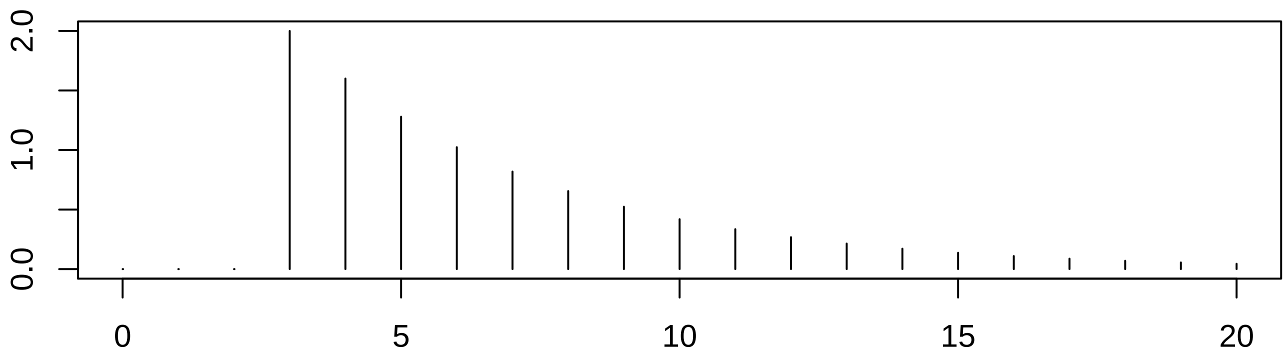
a) it can be written as an MA(∞) process
b) the long run mean is constant
c) $|\theta_1| < 1$
d) $|\phi_1| < 1$
e) $|\theta_2| < 1$
f) $|\phi_2| < 1$
g) $|\phi_2| < 1$, $\phi_2 + \phi_1 < 1$, and $\phi_2 - \phi_1 < 1$
h)⋆ $|\theta_2| < 1$, $\theta_2 + \theta_1 < 1$, and $\theta_2 - \theta_1 < 1$

Problem 11. A transfer function $v(B)$ can be expanded in a series $v(B) = v_0 + v_1 B + v_2 B^2 + v_3 B^3 + \cdots$ whose coefficients $v_0, v_1, v_2, \ldots$ are called the v-weights. The graph given below is a plot of the v-weights (plotting v_k versus k) for which of the following transfer functions?

a) $(2 - 0.8B)$ **b)** $B(2 - 0.8B)$ **c)** $B^3(2 - 0.8B)$

d) $\dfrac{2}{1 - 0.8B}$ **e)** $\dfrac{2B}{1 - 0.8B}$ **f)**$\star$ $\dfrac{2B^3}{1 - 0.8B}$

g) $\dfrac{2}{1 + 1.2B - 0.8B^2}$ **h)** $\dfrac{2B}{1 + 1.2B - 0.8B^2}$ **i)** $\dfrac{2B^3}{1 + 1.2B - 0.8B^2}$



Problem 12. Suppose you are given values of a stationary ARMA process z_t up to time n, and you compute a "long range" forecast $\hat{z}_{n+k} = \hat{z}_n(k)$ where k is large. The forecast $\hat{z}_{n+k}$ will always be approximately equal to ...

a) $\mathcal{I}_n$ **b)** z_n **c)** σ_z^2 **d)** σ_a^2 **e)** C **f)**$\star$ μ_z **g)** $1.96\sigma_a$ **h)** $1.96\sigma_z$

Problem 13. Continuing the previous problem, the variance of the forecast error will be approximately equal to ...

a) $\mathcal{I}_n$ **b)** z_n **c)**$\star$ σ_z^2 **d)** σ_a^2 **e)** C **f)** μ_z **g)** $1.96\sigma_a$ **h)** $1.96\sigma_z$

Problem 14. A transfer function model

$$Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + N_t$$

where the input X_t is an **impulse** (or pulse) function was used in lecture to model the effects of an intervention with ...

a) permanent effects on the level of Y_t.

b) permanent effects on the variance of Y_t.

c) permanent effects on the level of N_t.

d) permanent effects on the variance of N_t.

e)$\star$ short-term effects on Y_t.

f) short-term effects on X_t.

Problem 15. Suppose $\{z_t\}$ is an AR(1) process:

$$z_t = C + \phi_1 z_{t-1} + a_t$$

and you have observed all the values z_t and a_t up to time n. What is the best forecast of z_{n+1}? (The best forecast of z_{n+1} is denoted $\hat{z}_{n+1}$ or $\hat{z}_n(1)$ in the lecture notes.)

a) $C + \phi_1 C + \phi_1^2 z_n$ **b)** $C + \hat{a}_{n+1}$ **c)** $\mu_z + \phi_1 \hat{z}_n$ **d)** $\mu_z \pm 1.96\sigma_z$
e) $C + \phi_1 z_n + a_{n+1}$ **f)** μ_z **g)** C **h)**$\star$ $C + \phi_1 z_n$

Problem 16. Continuing the previous problem, if you have observed z_t and a_t up to time n, what is the best forecast of z_{n+2}? (This forecast is denoted $\hat{z}_{n+2}$ or $\hat{z}_n(2)$.)

a) $C + \phi_1 z_n$ **b)** $C + \phi_1 z_{n+1}$ **c)** $\mu_z + \phi_1 \hat{z}_{n+1}$ **d)**$\star$ $C + \phi_1 C + \phi_1^2 z_n$
e) $C + \phi_1 z_{n+1} + a_{n+2}$ **f)** μ_z **g)** $\mu_z \pm 1.96\sigma_z$ **h)** C

Problem 17. For an ARIMA(p, d, q) process $\{z_t\}$, the variance of the one-step-ahead prediction errors ...

a)$\star$ is equal to σ_a^2. **b)** is equal to σ_z^2. **c)** depends on $\phi_1, \ldots, \phi_p$, and σ_a^2.
d) depends on d. **e)** depends on $\theta_1, \ldots, \theta_q$. **f)** depends on d, $\phi_1, \ldots, \phi_p$, and σ_z^2.

Problem 18. Suppose the series $\{Y_t\}$ and $\{X_t\}$ are jointly stationary, and the series $\{X_t\}$ follows an ARMA model

$$\phi_x(B) X_t = \theta_x(B) b_t$$

where b_t denotes a white noise series. Let $f(B) = \dfrac{\phi_x(B)}{\theta_x(B)}$. Which one of the following statements is true about the filtered series $X'_t = f(B)X_t$ and $Y'_t = f(B)Y_t$?

a) the cross-correlations between X'_t and Y'_t are zero
b) Y'_t is white noise
c)$\star$ X'_t is white noise
d) X'_t is non-stationary
e) Y'_t is non-stationary
f) $Y'_t = f(B)X'_t$

Problem 19. Which one of the transfer functions given below uses the alternative parameterization which is requested by the ALTPARM option?

a) $\dfrac{\omega_0(1-\omega_1 B-\omega_2 B^2)}{\delta_0(1-\delta_1 B-\delta_2 B^2)}$

b)⋆ $\dfrac{\omega_0(1-\omega_1 B-\omega_2 B^2)}{1-\delta_1 B-\delta_2 B^2}$

c) $\dfrac{\omega_0-\omega_1 B-\omega_2 B^2}{1-\delta_1 B-\delta_2 B^2}$

d) $\dfrac{1-\omega_1 B-\omega_2 B^2}{1-\delta_1 B-\delta_2 B^2}$

e) $\dfrac{1-\omega_1 B-\omega_2 B^2}{\delta_0-\delta_1 B-\delta_2 B^2}$

Problem 20. Suppose that X_t and Y_t are non-stationary but that the differenced series $X_t^* = \nabla X_t$ and $Y_t^* = \nabla Y_t$ are jointly stationary and satisfy the transfer function model

$$Y_t^* = \frac{B^b\omega(B)}{\delta(B)}X_t^* + \frac{\theta(B)}{\delta(B)}a_t\,.$$

This model may be re-written in terms of X_t and Y_t as ...

a) $Y_t = \dfrac{B^b\omega(B)}{\delta(B)}X_t + \dfrac{(1-B)\theta(B)}{\phi(B)}\,a_t$

b) $Y_t = \dfrac{B^b\omega(B)}{\delta(B)}X_t + \dfrac{(1-B)^2\theta(B)}{\phi(B)}\,a_t$

c)⋆ $Y_t = \dfrac{B^b\omega(B)}{\delta(B)}X_t + \dfrac{\theta(B)}{(1-B)\phi(B)}\,a_t$

d) $Y_t = \dfrac{B^b\omega(B)}{\delta(B)}X_t + \dfrac{\theta(B)}{(1-B)^2\phi(B)}\,a_t$

e) $(1-B)Y_t = \dfrac{B^b\omega(B)}{\delta(B)}X_t + \dfrac{\theta(B)}{\phi(B)}\,a_t$

f) $Y_t = \dfrac{B^b\omega(B)(1-B)}{\delta(B)}X_t + \dfrac{\theta(B)}{\phi(B)}\,a_t$

g) $Y_t = \dfrac{B^b\omega(B)}{(1-B)\delta(B)}X_t + \dfrac{\theta(B)}{\phi(B)}\,a_t$

Problem 21. If $\{X_t\}$ and $\{Y_t\}$ are jointly stationary, then the cross-correlation function is defined by $\rho_{xy}(s) = \ldots$

a) $\mathrm{Corr}(X_{t-s}, Y_{t-s})$

b) $\mathrm{Corr}(X_{t+s}, Y_{t+s})$

c)⋆ $\mathrm{Corr}(X_t, Y_{t+s})$

d) $\mathrm{Corr}(X_{t+s}, Y_t)$

e) $\mathrm{Corr}(X_{t+s}, Y_{t-s})$

f) $\mathrm{Corr}(X_{t-s}, Y_{t+s})$

Problem 22. The minimum information criterion (MINIC) method can tentatively identify the order of a stationary and invertible ARMA process. For a range of values for p and q, a table supplies an estimate of the ________ divided by n (the series length) that would be obtained if an ARMA(p, q) model were fit to the data. The models with the ________ tabled values are worth considering as possible models.

The sentences above contain two blanks. Select the response below which gives two words which correctly fill these blanks.

a) P-value, smallest **b)** P-value, largest **c)**⋆ BIC, smallest **d)** BIC, largest
e) AIC, smallest **f)** AIC, largest **g)** Chi-Square, smallest **h)** Chi-Square, largest

Problem 23. Based on a series $z_1, z_2, \dots, z_n$ consisting of monthly data, you have fit an ARIMA$(1, 1, 1)$ model with no constant (i.e., $C = 0$). How would you expect the long range forecasts $\hat{z}_n(k) = \hat{z}_{n+k}$ to behave?

a)⋆ The forecasts will converge to a value which depends mainly on the last few observed values $z_n, z_{n-1}, \dots$

b) the forecasts will converge to a repetitive pattern, repeating every 12 months.

c) the forecasts will converge to a seasonal pattern (repeating every 12 months) added to a straight line with nonzero slope.

d) the forecasts will converge to a straight line with a nonzero slope equal to the mean of the differenced series ∇z_t.

e) The forecasts will converge to a value approximately equal to the overall mean of the time series.

Problem 24. The MA(1) process $z_t = a_t - 0.7a_{t-1}$ can also be written in the form

a) $z_t = a_t - 0.7a_{t-1} - 0.7^2 a_{t-2} - 0.7^3 a_{t-3} - \cdots$

b) $z_t = a_t + 0.7a_{t-1} + 0.7^2 a_{t-2} + 0.7^3 a_{t-3} - \cdots$

c)⋆ $z_t = a_t - 0.7z_{t-1} - 0.7^2 z_{t-2} - 0.7^3 z_{t-3} - \cdots$

d) $z_t = \dfrac{1}{1 - 0.7B} a_t$

e) $z_t = \dfrac{1}{1 + 0.7B} a_t$

f) $(1 - 0.7B) z_t = a_t$

g) $(1 + 0.7B) z_t = a_t$

Problem 25. Suppose we observe **all** the values z_t and a_t (the random shocks) up to time n. Call this set of information $\mathcal{I}_n$:

$$\mathcal{I}_n = \{z_n, z_{n-1}, z_{n-2}, \ldots, a_n, a_{n-1}, a_{n-2}, \ldots\}$$

What is the value of $E(z_{n-1} + z_n + a_{n-1} + a_n + a_{n+1} + a_{n+2} \,|\, \mathcal{I}_n)$? In other words, what is the best forecast of $z_{n-1} + z_n + a_{n-1} + a_n + a_{n+1} + a_{n+2}$ given the information $\mathcal{I}_n$?

a) 0

b) $z_{n-1} + z_n + a_{n+1} + a_{n+2}$

c) $a_{n-1} + a_n + a_{n+1} + a_{n+2}$

d) $z_{n-1} + z_n + a_{n-1} + a_n + z_{n+1} + z_{n+2}$

e) $z_{n-1} + z_n$

f) $z_{n-1} + z_n + z_{n+1} + z_{n+2}$

g)$\star$ $z_{n-1} + z_n + a_{n-1} + a_n$

Problem 26. Suppose you wish to identify the transfer functions in the model:

$$Y_t = C + \frac{B^{b_1}\omega_1(B)}{\delta_1(B)} X_{1,t} + \frac{B^{b_2}\omega_2(B)}{\delta_2(B)} X_{2,t} + N_t$$

One approach is to temporarily replace the transfer functions by simple linear transfer functions:

$$Y_t = C + v_1(B)X_{1,t} + v_2(B)X_{2,t} + N_t$$

where

$$\begin{aligned} v_1(B) &= v_{1,0} + v_{1,1}B + v_{1,2}B^2 + \cdots + v_{1,K}B^K \\ v_2(B) &= v_{2,0} + v_{2,1}B + v_{2,2}B^2 + \cdots + v_{2,K}B^K \,, \end{aligned}$$

and then fit (estimate) this model with a proxy model (say, ARIMA$(1,0,0)(1,0,0)_S$) for the noise N_t and use it to compute estimated values for the noise process

$$\hat{N}_t = Y_t - \hat{C} - \hat{v}_1(B)X_{1,t} - \hat{v}_2(B)X_{2,t} \,.$$

If the series $\{\hat{N}_t\}$ seems stationary, then you ...

a) use the ACF and PACF of $\{\hat{N}_t\}$ to select a model to use in pre-whitening the series $\{Y_t\}$, $\{X_{1,t}\}$, and $\{X_{2,t}\}$, and then use the CCF's of the pre-whitened series to select tentative forms for the transfer functions.

b)$\star$ use the ACF and PACF of $\{\hat{N}_t\}$ to select a tentative noise model, and the patterns in the estimated v-weights $\hat{v}_1(B)$ and $\hat{v}_2(B)$ to select tentative forms for the transfer functions.

c) obtain the CCF's of $\{X_{1,t}\}$, and $\{X_{2,t}\}$ with $\{\hat{N}_t\}$ and use them to identify tentative forms for the transfer functions.

d) use the estimated v-weights to pre-whiten the series $\{Y_t\}$, $\{X_{1,t}\}$, and $\{X_{2,t}\}$, and then use the CCF's of the pre-whitened series to select tentative forms for the transfer functions.

e) accept the proxy model as the final noise model and use it to estimate the final forms for the transfer functions.

Problem 27. Two jointly stationary series X_t (the input) and Y_t (the output) have been pre-whitened according to the usual procedure. A plot of the sample cross-correlation function of the pre-whitened series is given below. Based on this plot, identify a plausible transfer function $v(B)$ to use in the model $Y_t = v(B)X_t + N_t$.

a) $\dfrac{B^2\omega_0}{1-\delta_1 B}$ **b)** $\dfrac{B^2\omega_0}{1-\delta_1 B-\delta_2 B^2}$ **c)** $\dfrac{B\omega_0}{1-\delta_1 B-\delta_2 B^2-\delta_3 B^3}$

d) $\dfrac{B\omega_0}{1-\delta_1 B}$ **e)** $B(\omega_0-\omega_1 B-\omega_2 B^2-\omega_3 B^3)$ **f)** $B^2(\omega_0-\omega_1 B-\omega_2 B^2-\omega_3 B^3)$

g) $B^2(\omega_0-\omega_1 B-\omega_2 B^2)$ **h)** $\omega_0-\omega_1 B-\omega_2 B^2$ **i)**$\star$ $B(\omega_0-\omega_1 B-\omega_2 B^2)$

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The ARIMA Procedure