TEST #2

STA 4853 **Name:**____________________________________

May 1, 2014

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are 30 multiple choice questions.
- Circle a **single** answer for each multiple choice question. Your choice should be made clearly.
- On the multiple choice questions, always **circle the correct response**. (Sometimes the question has an empty blank or a box, but this is **NOT** where the answer goes.)
- There is no penalty for guessing.
- The exam has **10** pages.
- Each question is worth equal credit.

Problem 1. For a stationary time series with seasonality s, if the values of the ACF at lags $s+1$ and $s-1$ differ greatly, the correct model for the time series might be ...

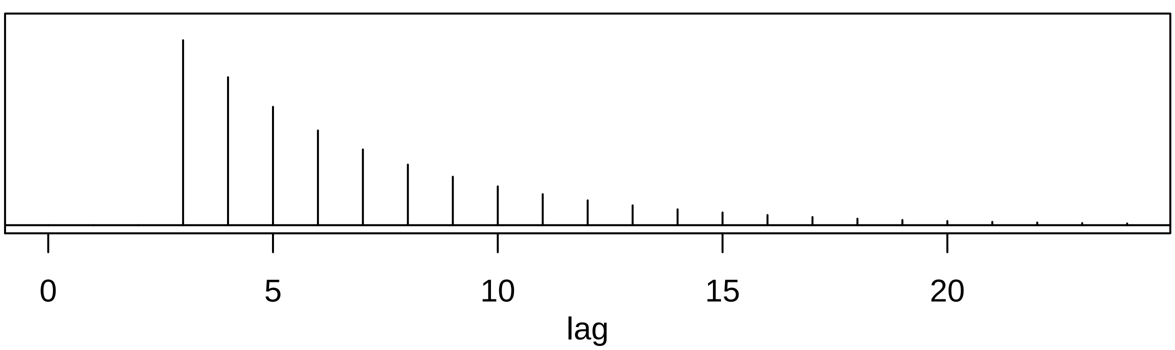
a) a non-multiplicative seasonal model
b) ARIMA$(0,0,0),(0,0,1)_s$
c) ARIMA$(0,0,0),(0,0,2)_s$
d) ARIMA$(0,0,0),(1,0,0)_s$
e) ARIMA$(0,0,0),(2,0,0)_s$
f) ARIMA$(0,0,0),(1,0,1)_s$
g) ARIMA$(0,0,1),(0,0,1)_s$
h) ARIMA$(0,0,1),(0,1,1)_s$
i) ARIMA$(0,1,1),(0,0,1)_s$

Problem 2. Immediately following this problem is a plot showing the v-weights of a transfer function $v(B)$ (i.e., a plot of lag i versus v_i). What is this transfer function?

a) $0.4 - 0.5B - 0.4B^2$ **b)** $B^3\left(0.4 + 0.5B + 0.4B^2\right)$ **c)** $\dfrac{2B^3}{1-0.8B}$

d) $0.4B + 0.5B^2 + 0.4B^3$ **e)** $\dfrac{2}{1-0.8B}$ **f)** $\dfrac{0.4B + 0.5B^2 + 0.4B^3}{1-(-0.8)B}$

g) $\dfrac{0.5}{1-1.6B-(-0.8)B^2-0.2B^3}$ **h)** $\dfrac{0.4B + 0.5B^2 + 0.4B^3}{1-1.6B-(-0.8)B^2}$ **i)** $\dfrac{2B^2}{1+0.8B^3}$



Problem 3. Suppose that

$$Y_t = C + v(B)X_t + N_t$$

and that the series $\{X_t\}$ and $\{Y_t\}$ are jointly stationary and $\{X_t\}$ is white noise. Then the v-weights are $v_k =$ ☐ for $k = 0, 1, 2, 3, \ldots$

a) $-\omega_k$ **b)** $\dfrac{\theta(B)}{\phi(B)}a_t$ **c)** $\dfrac{B^b\omega(B)}{\delta(B)}$ **d)** 0 **e)** $\dfrac{\sigma_y}{\sigma_x}\rho_{xy}(k)$ **f)** $\dfrac{\phi_x(B)}{\theta_x(B)}$ **g)** $\omega_0\delta_1^k$

Problem 4. The table immediately following this problem gives the ACF of a stationary process with seasonality $s = 8$. The ACF is exactly zero for all lags from 11 onward. What type of ARIMA process is this?

a) $(2,0,0)(1,0,0)_8$ **b)** $(1,0,0)(2,0,0)_8$ **c)** $(0,0,2)(0,0,5)_8$
d) $(0,0,2)(5,0,0)_8$ **e)** $(2,0,0)(0,0,5)_8$ **f)** $(2,0,0)(5,0,0)_8$
g) $(0,0,2)(1,0,0)_8$ **h)** $(0,0,2)(0,0,1)_8$ **i)** $(2,0,0)(0,0,1)_8$

LAG	ACF
1	-0.261194
2	-0.223881
3	0.0
4	0.0
5	0.0
6	-0.098771
7	-0.115233
8	0.441176
9	-0.115233
10	-0.098771
11	0.0
12	0.0

Problem 5. The model

$$(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^{12}) z_t = C + a_t$$

is (after expanding the product, eliminating the backshift notation, and re-arranging the terms) equivalent to a special case of a [ ] process.

a) ARMA(12,2) **b)** ARMA(2,12) **c)** ARMA(14,1) **d)** ARMA(1,14) **e)** ARMA(2,1)
f) MA(10) **g)** MA(11) **h)** MA(12) **i)** MA(13) **j)** MA(14)
k) AR(10) **l)** AR(11) **m)** AR(12) **n)** AR(13) **o)** AR(14)

Problem 6. Suppose that the series $\{z_t\}$ is a realization of a stationary AR(1) process

$$z_t = C + \phi_1 z_{t-1} + a_t \, ,$$

and that we know the values of z_t up to time n and wish to forecast k time units into the future. If k is large, the forecast $\widehat{z}_{n+k}$ will be approximately equal to [ ].

a) σ_z **b)** C **c)** $C + \phi_1 C + \phi_1^2 C$ **d)** $\phi_1^k z_n$ **e)** μ_z **f)** z_n **g)** $\dfrac{\sigma_a^2}{1 - \phi_1}$

The information below applies to the next two questions.

The v-weights of a transfer function $v(B)$ are given in the table below.

v_0	v_1	v_2	v_3	v_4	v_5	v_6
4	3	2	1	0	0	0

All the v-weights are zero from v_4 onwards.

Problem 7. An intervention occurs at time $t_{\text{event}} = 5$. The change produced by this intervention is the series $C_t = v(B)X_t$ where X_t is a **step** function at time $t_{\text{event}} = 5$. One of the rows of the following table gives the correct values for the series C_t. Circle the letter of the correct row.

a) **b)** **c)** **d)** **e)** **f)** **g)** **h)**

	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$
a)	0	0	0	0	0	4	3	2	1	0	0
b)	0	0	4	3	2	1	0	0	0	0	0
c)	0	0	0	0	0	1	2	3	4	0	0
d)	0	0	1	2	3	4	0	0	0	0	0
e)	0	0	0	0	0	1	3	6	10	10	10
f)	0	1	3	6	10	10	0	0	0	0	0
g)	0	4	7	9	10	10	0	0	0	0	0
h)	0	0	0	0	0	4	7	9	10	10	10

Problem 8. An intervention occurs at time $t_{\text{event}} = 5$. The change produced by this intervention is the series $C_t = v(B)X_t$ where X_t is an **impulse** function at time $t_{\text{event}} = 5$. One of the rows of the table above gives the correct values for the series C_t. Circle the letter of the correct row.

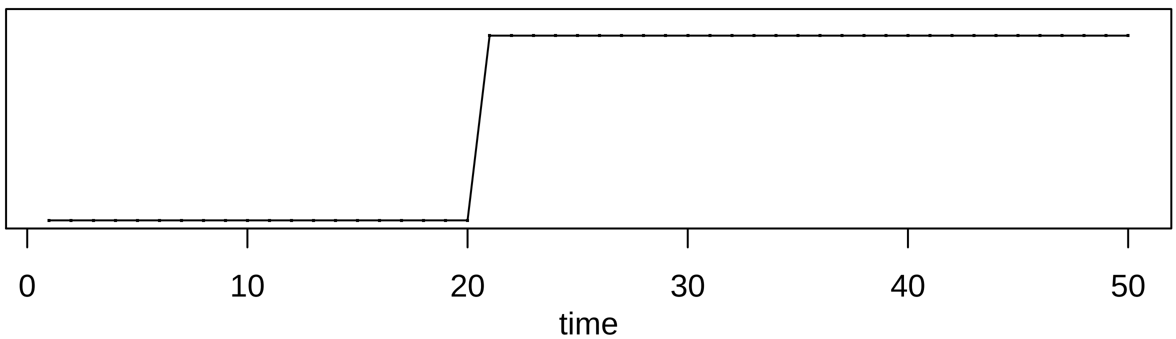
a) **b)** **c)** **d)** **e)** **f)** **g)** **h)**

Problem 9. An impulse function at time t_{event} is a series defined by ☐.

a) $X_t = 0$ if $t = t_{\text{event}}$ and $X_t = 1$ if $t \neq t_{\text{event}}$
b) $X_t = 1$ if $t = t_{\text{event}}$ and $X_t = 0$ if $t \neq t_{\text{event}}$
c) $X_t = 1$ if $t \geq t_{\text{event}}$ and $X_t = 0$ if $t < t_{\text{event}}$
d) $X_t = 1$ if $t \leq t_{\text{event}}$ and $X_t = 0$ if $t > t_{\text{event}}$
e) $X_t = t$
f) $X_t = t^2$
g) $X_t = \sin(2\pi jt/12)$
h) $X_t = \cos(2\pi jt/12)$
i) $X_t = \dfrac{\theta(B)}{\phi(B)} a_t$

Problem 10. An intervention occurs at time $t_{\text{event}} = 21$. The change produced by this intervention (the intervention effect) is the series $C_t = v(B)X_t$ given in the plot immediately after this problem. What choice of $v(B)$ and X_t (i.e., an impulse or a step function) produces this intervention effect C_t?

a) $v(B) = 3/(1 - 0.7B)$ and X_t = impulse
b) $v(B) = 3/(1 - 0.7B)$ and X_t = step
c) $v(B) = 3$ and X_t = impulse
d) $v(B) = 3$ and X_t = step
e) $v(B) = 7 - 4B$ and X_t = step
f) $v(B) = 7 - 4B$ and X_t = impulse
g) $v(B) = 3/(1 + 0.7B)$ and X_t = step
h) $v(B) = 3/(1 + 0.7B)$ and X_t = impulse



Problem 11. Suppose we wish to model a time series Y_t which has an intervention at time t_{event} by using a model of the form

$$Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{\theta(B)}{\phi(B)} a_t$$

where X_t is a step function at time t_{event}. How do we choose the transfer function (the value of b and form of $\omega(B)$ and $\delta(B)$) to use in this model?

a) By studying the ACF and PACF of X_t and Y_t
b) By studying the cross-correlations between X_t and Y_t
c) By studying the cross-correlations between X_t and Y_t after pre-whitening both series
d) By studying the pattern of the ψ-weights
e) By comparing the pattern of change in the series with known patterns
f) By studying the pattern of the π-weights

The next two problems concern this situation: Suppose $\{z_t\}$ is an ARIMA process generated by the random shocks $\{a_t\}$. We observe **all** the values z_t and a_t up to time n. Call this set of information $\mathcal{I}_n$:

$$\mathcal{I}_n = \{z_n, z_{n-1}, z_{n-2}, \ldots, a_n, a_{n-1}, a_{n-2}, \ldots\}$$

We define $\widehat{z}_t = E(z_t \,|\, \mathcal{I}_n)$ and $\widehat{a}_t = E(a_t \,|\, \mathcal{I}_n)$.

Problem 12. Which of the following statements about $\widehat{z}_t$ is always true?

a) $\widehat{z}_t = z_t$ for $t > n$
b) $\widehat{z}_t = \mu_z$ for $t \leq n$
c) $\widehat{z}_t = z_t$ for $t \leq n$
d) $\widehat{z}_t = \mu_z$ for $t > n$
e) $\widehat{z}_t = 0$ for $t \leq n$
f) $\widehat{z}_t = 0$ for $t > n$

Problem 13. Which of the following statements about $\widehat{a}_t$ is always true?

a) $\widehat{a}_t = a_t$ for $t \leq n$ and $\widehat{a}_t = 0$ for $t > n$
b) $\widehat{a}_t = 0$ for $t \leq n$ and $\widehat{a}_t = a_t$ for $t > n$
c) $\widehat{a}_t = z_t$ for $t \leq n$ and $\widehat{a}_t = a_t$ for $t > n$
d) $\widehat{a}_t = \sigma_a^2$ for $t \leq n$ and $\widehat{a}_t = 0$ for $t > n$
e) $\widehat{a}_t = 0$ for $t \leq n$ and $\widehat{a}_t = \sigma_a^2$ for $t > n$
f) $\widehat{a}_t = a_t$ for $t \leq n$ and $\widehat{a}_t = z_t$ for $t > n$
g) $\widehat{a}_t = \sigma_a^2$ for $t = n$ and $\widehat{a}_t = 0$ for $t \neq n$

Problem 14. The terms in the summation ______ are periodic functions of (integer-valued) time t with period 12, and any periodic function with period 12 can be represented as a constant plus a summation of this form for some values of α_j ad ξ_j.

a) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{\pi j t}{12}\right) + \xi_j \cos\left(\frac{\pi j t}{12}\right)\right\}$

b) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{2\pi j t}{12}\right) + \xi_j \cos\left(\frac{2\pi j t}{12}\right)\right\}$

c) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{2\pi t}{12}\right) + \xi_j \cos\left(\frac{2\pi t}{12}\right)\right\}$

d) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{2\pi j}{12}\right) + \xi_j \cos\left(\frac{2\pi j}{12}\right)\right\}$

e) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{\pi t}{12j}\right) + \xi_j \cos\left(\frac{\pi t}{12j}\right)\right\}$

f) $\displaystyle\sum_{j=1}^{6}\left\{\alpha_j \sin\left(\frac{2\pi t}{12j}\right) + \xi_j \cos\left(\frac{2\pi t}{12j}\right)\right\}$

The following two questions involve this situation:

Based on time series data $Y_1, Y_2, \ldots, Y_{100}$, SAS has fit an ARIMA model and produced forecasts for future values of Y_t. A fragment of SAS output is given below.

Obs	Forecast	Std Error	95% Confidence Limits	
101	6.5068	1.2111	4.1331	8.8804
102	6.8077	1.2429	4.3716	9.2437

We are interested in forecasts for time $t = 102$. Assuming the model is correct and the parameter estimates are accurate, based on this output we can say that Y_{102} is normally distributed with a mean of $\mu =$ ☐ and a standard deviation of $\sigma =$ ☐.

Problem 15. Circle the correct value of μ.

a) 6.5068 **b)** 1.2111 **c)** 4.1331 **d)** 8.8804
e) 6.8077 **f)** 1.2429 **g)** 4.3716 **h)** 9.2437

Problem 16. Circle the correct value of σ.

a) 6.5068 **b)** 1.2111 **c)** 4.1331 **d)** 8.8804
e) 6.8077 **f)** 1.2429 **g)** 4.3716 **h)** 9.2437

Problem 17. Suppose $Y_t = v(B)X_t + N_t$ where N_t is a stationary ARMA process,

$$v(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_h B^h$$

and X_t is a **step** function representing an intervention at time t_{event}. The long-run effect of the intervention is to permanently increase the mean level of the series Y_t by the amount ☐.

a) 0 **b)** $-\omega_0 + \omega_1 + \cdots + \omega_h$ **c)** $\omega_0 - \omega_h$ **d)** $\omega_h - \omega_0$
e) ω_0 **f)** $\omega_0 - \omega_1 - \cdots - \omega_h$ **g)** ω_h **h)** $-\omega_h$

Problem 18. If $\{Y_t\}$ and $\{X_t\}$ are jointly stationary, then $\text{Corr}(Y_8, X_6) =$ ☐.

a) $\text{Corr}(Y_6, X_4)$ **b)** $\text{Corr}(Y_6, X_8)$ **c)** $\text{Corr}(Y_4, X_6)$ **d)** $\text{Corr}(Y_9, X_6)$
e) $\text{Corr}(Y_8, X_7)$ **f)** $\text{Corr}(Y_8, X_7)$ **g)** $\text{Corr}(X_8, Y_6)$ **h)** $\text{Corr}(X_6, Y_4)$

Problem 19. For an ARMA(p, q) process, the "dual" process is the ARMA(q, p) process obtained by interchanging the roles of the ϕ_i's and the θ_i's. If an ARMA(p, q) process is stationary, the dual process is ...

a) non-stationary **b)** jointly stationary **c)** non-invertible **d)** invertible **e)** non-seasonal

Problem 20. Immediately following this question is a table giving the theoretical ACF and PACF of a seasonal ARIMA process with seasonality $s = 8$. For lags beyond those displayed in the table, the **PACF** is zero for all lags, and the **ACF** decays to zero along lags which are multiples of 8 and is zero elsewhere. What ARIMA$(p, d, q)(P, D, Q)_8$ process is this?

a) $(0, 0, 0)(2, 0, 3)_8$ **b**) $(2, 0, 0)(0, 0, 1)_8$ **c**) $(0, 0, 2)(0, 0, 1)_8$ **d**) $(0, 0, 0)(2, 0, 0)_8$
e) $(1, 0, 1)(0, 0, 1)_8$ **f**) $(0, 0, 0)(0, 0, 2)_8$ **g**) $(0, 0, 0)(1, 0, 1)_8$ **h**) $(0, 0, 0)(3, 0, 2)_8$
i) $(3, 0, 2)(1, 0, 0)_8$ **j**) $(2, 0, 3)(1, 0, 0)_8$ **k**) $(0, 1, 0)(1, 0, 1)_8$ **l**) $(0, 1, 0)(3, 0, 2)_8$

LAG	ACF	PACF
1	0.0	0.0
2	0.0	0.0
3	0.0	0.0
4	0.0	0.0
5	0.0	0.0
6	0.0	0.0
7	0.0	0.0
8	0.714286	0.714286
9	0.0	0.0
10	0.0	0.0
11	0.0	0.0
12	0.0	0.0
13	0.0	0.0
14	0.0	0.0
15	0.0	0.0
16	0.657143	0.300000
17	0.0	0.0
18	0.0	0.0
19	0.0	0.0
20	0.0	0.0
21	0.0	0.0
22	0.0	0.0
23	0.0	0.0
24	0.542857	0.0
25	0.0	0.0

Problem 21. Let $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$. A regression model with ARMA(p, q) errors which explains the response series $\{y_t\}$ using 2 input series $\{x_{1,t}\}$ and $\{x_{2,t}\}$, can be written as $\boxed{}$.

a) $\theta(B)y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \phi(B)a_t$

b) $y_t = \beta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + a_t$

c) $y_t = \beta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \theta(B)a_t$

d) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dfrac{\phi(B)}{\theta(B)} a_t$

e) $\phi(B)y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \theta(B)a_t$

f) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dfrac{\theta(B)}{\phi(B)} a_t$

Problem 22. One general approach to modeling non-stationary series which exhibit seasonal patterns or seasonal variation is to:

a) Transform the series (by taking logs or square roots or some other transformation) and then choose an appropriate ARIMA$(p, 0, q)(P, 0, Q)_s$ process to model the transformed series.

b) Make the series stationary by differencing (either ordinary or seasonal differencing or some combination), and then choose an appropriate ARIMA$(p, 0, q)(P, 0, Q)_s$ process to model the differenced series.

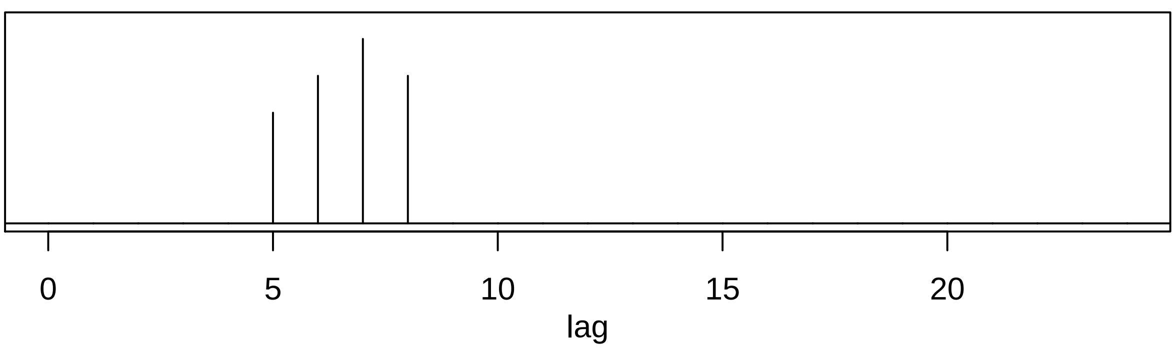
c) Integrate the series (compute cumulative sums) and then choose an appropriate ARIMA$(p, 0, q)(P, 0, Q)_s$ process to model the integrated series.

d) Pre-whiten the series and then choose an appropriate ARIMA$(p, 0, q)(P, 0, Q)_s$ process to model the pre-whitened series.

e) Mean-center the series and then choose an appropriate ARIMA$(p, 0, q)(P, 0, Q)_s$ process to model the mean-centered series.

Immediately below is a plot showing the v-weights of a transfer function of the form

$$v(B) = \frac{B^b \omega(B)}{\delta(B)} \quad \text{where } \omega(B) = \omega_0 - \omega_1 B - \ldots - \omega_h B^h \text{ and } \delta(B) = 1 - \delta_1 B - \ldots - \delta_r B^r \,.$$



In each of the next three questions select the response which correctly describes this transfer function.

Problem 23. The value of $b =$ ☐

a) 0 **b**) 1 **c**) 2 **d**) 3 **e**) 4 **f**) 5 **g**) 6 **h**) 7 **i**) 8 **j**) 9

Problem 24. The value of $r =$ ☐

a) 0 **b**) 1 **c**) 2 **d**) 3 **e**) 4 **f**) 5 **g**) 6 **h**) 7 **i**) 8 **j**) 9

Problem 25. The value of $h =$ ☐

a) 0 **b**) 1 **c**) 2 **d**) 3 **e**) 4 **f**) 5 **g**) 6 **h**) 7 **i**) 8 **j**) 9

The next question is **not** related to the previous questions.

Problem 26. If we define $v(B) = v_0 + v_1 B + v_2 B^2 + \cdots + v_h B^h$, then $v(B)X_t = \boxed{}$.

a) $\dfrac{\theta(B)}{\phi(B)} a_t$ **b)** $\dfrac{\omega(B)}{\delta(B)} a_t$ **c)** $(1-B)^h X_t$ **d)** $v_0 X_t + v_1 X_{t+1} + v_2 X_{t+2} + \cdots + v_h X_{t+h}$

e) $v_0 X_t + v_1 X_{t-1} + v_2 X_{t-2} + \cdots + v_h X_{t-h}$ **f)** $(1 - v_0 B)^h X_t$

g) $v_0 X_t - v_1 X_t - v_2 X_t - \cdots - v_h X_t$ **h)** $(1 - B^h) X_t$

Problem 27. For **non**-stationary ARIMA processes, as you forecast further and further into the future, the confidence interval **widths** for the forecasts ...

a) converge to a limiting value
b) converge to a repetitive pattern which repeats with a period of S (= the seasonality)
c) converge to a repetitive pattern added to a straight line with nonzero slope
d) converges to a straight line with a nonzero slope
e) continue to gradually increase and will eventually reach arbitrarily large values

Problem 28. An ARIMA$(1,1,0)(0,1,1)_s$ process can be written as $\boxed{}$.

a) $\dfrac{(1-B^s)(1-\Theta_1 B^s)}{(1-B)(1-\phi_1 B)} a_t$ **b)** $\dfrac{(1-B)(1-\phi_1 B)}{(1-B^s)(1-\Theta_1 B^s)} a_t$ **c)** $\dfrac{(1-B)(1-B^s)(1-\phi_1 B)}{(1-\Theta_1 B^s)} a_t$

d) $\dfrac{(1-\Theta_1 B^s)}{(1-B)(1-B^s)(1-\phi_1 B)} a_t$ **e)** $\dfrac{(1-B)(1-\Theta_1 B^s)}{(1-B^s)(1-\phi_1 B)} a_t$ **f)** $\dfrac{(1-B^s)(1-\phi_1 B)}{(1-B)(1-\Theta_1 B^s)} a_t$

Problem 29. For an ARMA(p,q) process, invertibility depends ...

a) only on the AR coefficients **b)** only on the MA coefficients
c) on both the AR and MA coefficients **d)** on the values of the v-weights
e) on the variance of the random shocks **f)** on the order of the differencing

Problem 30. If a time series consists of a repeating seasonal pattern plus a linear trend, then seasonal differencing will ...

a) sometimes fail to remove the linear trend and second differencing is needed
b) sometimes fail to remove the seasonal pattern and second differencing is needed
c) remove the seasonal pattern but not the linear trend
d) remove the linear trend but not the seasonal pattern
e) remove both the seasonal pattern and the linear trend