TEST #2
STA 4853
April 28, 2016

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

## **Directions**

- This exam is **closed book** and **closed notes**.
- There are 30 multiple choice questions.
- Circle the **single best** answer for each multiple choice question. Your choice should be made clearly.
- Always **circle the correct response**. (Sometimes the question has an empty blank or a box, but this is **NOT** where the answer goes.)
- There is no penalty for guessing.
- The exam has 11 pages.
- Each question is worth equal credit.

<b>Problem 1.</b> One general approach to modeling a series with a non-stationary mean which exhibits seasonal patterns or seasonal variation is to
a) make the series stationary by ordinary and/or seasonal differencing, and then choose an appropriate $ARIMA(p, 0, q)(P, 0, Q)_s$ process to model the differenced series
b) choose an appropriate $ARIMA(p, 0, q)(P, 0, Q)_s$ process to eliminate the seasonal effects, and then transform the resulting series (for example, by using a log or square root transformation) to eliminate the non-stationarity
c) choose an appropriate $ARIMA(p, 0, q)(P, 0, Q)_s$ process to eliminate the seasonal effects, and then difference the resulting series (using ordinary and/or seasonal differencing) to eliminate the non-stationarity
d) transform the series (for example, by using a log or square root transformation), and then choose an appropriate $ARIMA(p, 0, q)(P, 0, Q)_s$ process to model the transformed series
<b>Problem 2.</b> Transforming a series $z_t$ (that is, modeling $y_t = f(z_t)$ instead of $z_t$ for some appropriately chosen function $f$ such as log or square root) is useful when
a) the level of the series changes systematically with time.
<b>b</b> ) the level of the series $z_t$ changes periodically with time.
$\mathbf{c}$ ) the level of the series $z_t$ changes repeatedly over time.
<b>d</b> ) the variability of the series $z_t$ changes systematically with time.
$\mathbf{e}$ ) the variability of the series $z_t$ changes systematically with the level of the series.
f) the variability of the series $z_t$ changes periodically with time.
Problem 3. A series which approximately repeats a consistent seasonal pattern is
${f a}$ ) stationary ${f b}$ ) non-stationary ${f c}$ ) invertible ${f d}$ ) non-invertible ${f e}$ ) multiplicative
$\mathbf{f}) \   \text{non-multiplicative}  \mathbf{g}) \   \text{integrated}  \mathbf{h}) \   \text{differenced}  \mathbf{i}) \   \text{over-differenced}  \mathbf{j}) \   \text{autoregressive}$
Problem 4. If you model a time series $z_t$ using a stationary ARMA process with a non-zero constant $(C \neq 0)$ and use it to forecast future values of $z_t$ , then as you forecast further and further into the future, the confidence interval widths for your forecasts will
a) continue to increase and eventually reach arbitrarily large values
b) follow a straight line with a non-zero slope
c) gradually decay to zero
$\mathbf{d}$ ) cutoff to zero after some lag
e) converge to a non-zero limiting value
$\mathbf{f}$ ) converge to a repeating pattern with period $C$

Problem 5. The sample IACF of an invertible process will

- a) have all of its roots strictly inside the unit circle
- **b)** have all of its roots strictly outside the unit circle
- c) decay to zero at a reasonable rate
- d) decay to zero very slowly
- e) indicate there has been overdifferencing
- f) be the same as the PACF of the dual process
- g) depend only on the MA coefficients

Which one of the following choices for v(B) will produce a transfer function Problem 6.  $v(B)X_t$  with v-weights which decay sinusoidally? (Note: alternating exponential decay is **not** considered to be sinusoidal.)

a) 
$$v(B) = \frac{B(1 - 0.8B)}{1 - 1.6B - (-0.8)B^2}$$

**b**) 
$$v(B) = \frac{1 - 1.6B - (-0.8)B^2}{1 - 0.8B}$$

c) 
$$v(B) = \frac{1 - 1.6B - (-0.8)B^2}{1 - (-0.8)B}$$

**d**) 
$$v(B) = \frac{B^2(1 - 1.6B - (-0.8)B^2)}{1 - 0.8B}$$

e) 
$$v(B) = \frac{B^2}{1 - (-0.8)B}$$

$$f) v(B) = B(1 - 1.6B - (-0.8)B^2)$$

Problem 7. An MA(1) process with  $|\theta_1| < 1$  can be re-written in the form \_\_\_\_\_\_

a) 
$$\tilde{z}_t = a_t + \theta_1 \tilde{z}_{t-1} + \theta_2 \tilde{z}_{t-2} + \theta_3 \tilde{z}_{t-3} + \cdots$$

**b**) 
$$\tilde{z}_t = a_t + \theta_1 a_{t-1} + \theta_1^2 a_{t-2} + \theta_1^3 a_{t-3} + \cdots$$

$$\mathbf{c}) \ \tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_1^2 a_{t-2} - \theta_1^3 a_{t-3} - \cdots$$

$$\mathbf{d}) \ \tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \cdots$$

e) 
$$\tilde{z}_t = a_t - \theta_1 \tilde{z}_{t-1} - \theta_1^2 \tilde{z}_{t-2} - \theta_1^3 \tilde{z}_{t-3} - \cdots$$

$$\mathbf{f}) \ \tilde{z}_t = a_t + \theta_1 \tilde{z}_{t-1} + \theta_1^2 \tilde{z}_{t-2} + \theta_1^3 \tilde{z}_{t-3} + \cdots$$

Suppose that Y has a Normal distribution with mean  $\mu$  and variance  $\sigma^2$  (that is,  $Y \sim N(\mu, \sigma^2)$ ). Then  $X = e^Y$  has a log-normal distribution with **median** equal to \_\_\_\_

$$\mathbf{a}$$
)  $\log(\mu)$ 

$$\mathbf{b}$$
)  $\log(\sigma^2)$ 

**a**) 
$$\log(\mu)$$
 **b**)  $\log(\sigma^2)$  **c**)  $\log(\mu + (\sigma^2)/2)$  **d**)  $e^{\mu + (\sigma^2)/2}$  **e**)  $e^{\mu}$  **f**)  $e^{\sigma^2}$ 

**d**) 
$$e^{\mu + (\sigma^2)/2}$$

$$e) e^{\mu}$$

$$\mathbf{f}$$
)  $e^{\sigma^2}$ 

$$Y_t = C + v(B)X_t + N_t$$

where the noise process  $N_t$  is an ARIMA $(p, d, q)(P, D, Q)_s$  process, then you can write the noise process as \_\_\_\_\_, where as usual  $a_t$  denotes a random shock process, and  $\phi(B)$  and  $\theta(B)$  are the AR and MA polynomials, respectively.

a) 
$$N_t = \frac{(1-B)^d (1-B^s)^D \phi(B)}{\theta(B)} a_t$$

**b**) 
$$N_t = \frac{(1 - B^d)(1 - B^D)^s \phi(B)}{\theta(B)} a_t$$

c) 
$$N_t = \frac{\phi(B)}{(1-B)^d(1-B^s)^D\theta(B)} a_t$$

**d**) 
$$N_t = \frac{\theta(B)}{(1-B)^d (1-B^s)^D \phi(B)} a_t$$

e) 
$$N_t = \frac{\phi(B)}{(1 - B^d)(1 - B^D)^s \theta(B)} a_t$$

**f**) 
$$N_t = \frac{(1 - B^d)(1 - B^D)^s \theta(B)}{\phi(B)} a_t$$

Problem 10. The SAS code given below

PROC ARIMA DATA=STUFF; IDENTIFY VAR=Y CROSSCOR=(X1 X2 X3) NOPRINT; ESTIMATE INPUT=(X1 X2 X3) METHOD=ML; QUIT;

- a) fits a transfer function model for  $Y_t$  on  $X_{t-1}, X_{t-2}, X_{t-3}$ .
- **b**) fits a transfer function model for  $Y_t$  on  $X_{1,t}$ ,  $X_{2,t}$ ,  $X_{3,t}$ .
- c) fits a multiple regression model for  $Y_t$  on the regressors  $X_{1,t}, X_{2,t}, X_{3,t}$ .
- d) fits a multiple regression model for  $Y_t$  on the regressors  $X_{t-1}, X_{t-2}, X_{t-3}$ .
- e) uses pre-whitening to identify a transfer function for  $Y_t$  on  $X_{1,t}$ ,  $X_{2,t}$ ,  $X_{3,t}$ .
- f) uses pre-whitening to identify a transfer function for  $Y_t$  on  $X_{t-1}, X_{t-2}, X_{t-3}$ .
- **g**) fits a transfer function model for  $Y_t$  on  $X_{t-1}, X_{t-2}, X_{t-3}$  using a proxy AR(2) model for the noise.
- h) fits a transfer function model for  $Y_t$  on  $X_{1,t}$ ,  $X_{2,t}$ ,  $X_{3,t}$  using a proxy AR(2) model for the noise.

Suppose we wish to forecast an AR(1) process  $z_t = C + \phi_1 z_{t-1} + a_t$  and we know all the values of  $z_t$  and  $a_t$  up to time n. What is the best forecast for  $z_{n+2}$ ?

$$\mathbf{a}) \; \mu_z$$

$$\mathbf{b}) z_n$$

**b**) 
$$z_n$$
 **c**)  $\mu_z + \phi_1^2 z_n$ 

$$\mathbf{d}) \ z_n + \phi_1^2 a_n$$

e) 
$$C + \phi_1 C + \phi_1^2 z$$

$$C + \phi_1 C + \phi_1^2 a_1$$

$$\mathbf{g}) \ z_n + \phi_1 z_n + \phi_1^2 a_r$$

e) 
$$C + \phi_1 C + \phi_1^2 z_n$$
 f)  $C + \phi_1 C + \phi_1^2 a_n$  g)  $z_n + \phi_1 z_n + \phi_1^2 a_n$  h)  $z_n + \phi_1 C + \phi_1^2 a_n$ 

## Problem 12. The model

$$(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^s)z_t = C + a_t$$

may be re-written as \_\_\_\_\_.

a) 
$$z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-s-1} - \phi_2 \Phi_1 z_{t-s-2} + a_t$$

**b**) 
$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-s-1} + \phi_2 \Phi_1 z_{t-s-2} + a_t$$

c) 
$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-s-1} - \phi_2 \Phi_1 z_{t-s-2} + a_t$$

d) 
$$z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-s-1} + \phi_2 \Phi_1 z_{t-s-2} + a_t$$

e) 
$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-1} z_{t-s} - \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$$

$$\mathbf{f}) \ z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-1} z_{t-s} - \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$$

g) 
$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-1} z_{t-s} + \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$$

**h**) 
$$z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-1} z_{t-s} + \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$$

Problem 13. If you difference a series and then observe that the sample ACF of the differenced series decays very slowly, what should you do?

- a) Conclude that the series has been over-differenced.
- **b)** Try a transformation, perhaps a log or square root.
- c) Use a mixed model with both p > 0 and q > 0.
- **d**) Conclude that the series is non-invertible.
- e) Try differencing the series again.
- f) Use an MA(q) model with a large value of q.

Suppose the conditional distribution of X given the information  $\mathcal{I}$  is Problem 14.

Normal with mean 
$$= E(X \mid \mathcal{I})$$
 and variance  $= \text{Var}(X \mid \mathcal{I})$ ,

and we are using the squared error loss function. Then our best forecast of X is  $\underline{\hspace{1cm}}$ 

a) 
$$E(X \mid \mathcal{I})$$

**b**) 
$$Var(X \mid \mathcal{I})$$

$$\mathbf{c}) \ X - \widehat{X}$$

a) 
$$E(X \mid \mathcal{I})$$
 b)  $Var(X \mid \mathcal{I})$  c)  $X - \widehat{X}$  d)  $E[(X - \widehat{X})^2 \mid \mathcal{I}]$ 

e) 
$$E\left(|X-\widehat{X}|\,|\,\mathcal{I}\right)$$

$$\mathbf{f}) \ \sqrt{\operatorname{Var}(X \mid \mathcal{I})}$$

e) 
$$E\left(|X-\widehat{X}|\,|\,\mathcal{I}\right)$$
 f)  $\sqrt{\operatorname{Var}(X\,|\,\mathcal{I})}$  g)  $(\widehat{X}-1.96\operatorname{SE},\widehat{X}+1.96\operatorname{SE})$ 

Problem 15. The theoretical IACF of an AR(p) process

- a) is the same as the PACF of an MA(p) process.
- **b**) is the same as the ACF of an MA(p) process.
- c) is the same as the ACF of an AR(p) process.
- d) eventually decays to zero (perhaps in a complicated way).
- e) decays to zero very slowly.
- f) gets closer and closer to a non-zero limiting value for large lags.
- g) converges to a value depending mainly on the last few observed values.

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The next three problems involve this situation: Suppose

$$v(B) = \frac{B^b \omega(B)}{\delta(B)} \quad \text{with}$$

$$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \dots - \omega_h B^h \quad \text{and}$$

$$\delta(B) = \delta_0 - \delta_1 B - \delta_2 B^2 - \dots - \delta_r B^r.$$

Problem 16. If r = 1, then the **first** non-zero v-weight is at lag \_\_\_\_\_.

- **a**) b + 1
- **b**) b+h **c**) b+h+1 **d**) b-1
- **f**) b+h-1 **g**) h **h**) h-1 **i**) h+1 **j**) b-h+1

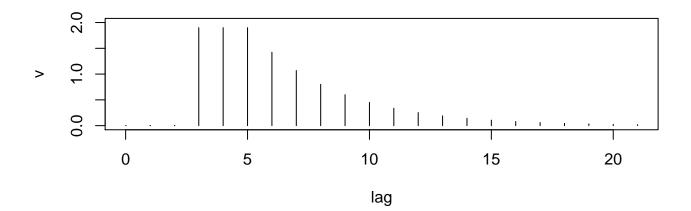
If r=0, then the **last** non-zero v-weight is at lag

- **a**) *b*
- **b**) h **c**) b-h **d**) b+h **e**) h-b **f**) bh **g**) b/h

- $\mathbf{h}) h/b$

Problem 18. What is the value of h for the transfer function with v-weights given in the plot below?

- $\mathbf{a}) 0$
- **b**) 1 **c**) 2
- **d**) 3 **e**) 4
- **f**) 5
- $\mathbf{g})$  6



Suppose  $w_t = \nabla^2 z_t$ . If  $w_t$  appears to be stationary, and the sample ACF of  $w_t$ has a single large spike at lag 1 followed by much smaller non-significant spikes, then a reasonable model for the original series  $z_t$  is \_\_\_\_\_

- **a)** ARIMA(1,1,0)
- **b)** ARIMA(0,1,1) **c)** ARIMA $(0,0,1)_2$
- **d**) ARIMA $(1,0,0)_2$

- $\mathbf{e}$ ) ARIMA(1,2,0)

- f) ARIMA(0,2,1) g) ARIMA(0,1,2) h) ARIMA(2,1,0)

Problem 20. For a time series with a non-stationary mean, it is common for the PACF to

- a) decay very slowly.
- **b**) have a single large spike at lag 1 with  $\hat{\phi}_{11}$  close to 1.
- c) have several large spikes followed by an approximate cutoff to zero.
- d) display strong sinusoidal oscillations.
- e) decay exponentially.
- f) exhibit alternating exponential decay.
- g) exhibit variability which changes with the level.

**Problem 21.** To estimate the transfer function model

$$Y_t = C + \frac{B^3 \omega_0}{1 - \delta_1 B} X_t + \frac{1}{(1 - \phi_{1,1} B)(1 - \phi_{2,1} B^{12})} a_t,$$

you use the SAS code \_\_\_\_\_.

- a) ESTIMATE INPUT=(3\$/(1)X) P=(1)(12);
- **b**) ESTIMATE INPUT=(3\$/(1)X) P=(1,12);
- c) ESTIMATE INPUT=(3\$(0)/(0.1)X) P=(1)(12);
- d) ESTIMATE INPUT=(3\$(0)/(0.1)X) P=(1,12);
- e) ESTIMATE INPUT=(3\$(0)/(0.1)X) Q=(1.12);
- f) ESTIMATE INPUT=(\$(3)/(0,1)X) Q=(1)(12);
- $\mathbf{g}) \ \mathrm{ESTIMATE} \ \mathrm{INPUT}{=}(\$(3)/(1)\mathrm{X}) \ \mathrm{Q}{=}(1{,}12) \ ;$
- **h**) ESTIMATE INPUT=(\$(3)/(1)X) P=(1)(12);

**Problem 22.** Suppose  $z_t$  is a stationary ARMA process with psi-weights  $\psi_1, \psi_2, \psi_3, \dots$  Let  $e_n(3) = z_{n+3} - \hat{z}_{n+3}$  be the 3-step-ahead forecast error based on information about the series up to time n. The forecast error  $e_n(3)$  is equal to \_\_\_\_\_\_.

- a)  $\sigma_a^2(1+\psi_1^2+\psi_2^2)$
- **b**)  $\sigma_a^2(1+\psi_1^2+\psi_2^2+\psi_3^2)$
- c)  $\sigma_a \sqrt{1 + \psi_1^2 + \psi_2^2}$
- d)  $\sigma_a \sqrt{1 + \psi_1^2 + \psi_2^2 + \psi_3^2}$
- e)  $a_n + \psi_1 a_{n-1} + \psi_2 a_{n-2}$
- $\mathbf{f)} \ a_{n+3} + \psi_1 a_{n+2} + \psi_2 a_{n+1}$
- $\mathbf{g}) \ a_{n+3} + \psi_1 a_{n+2} + \psi_2 a_{n+1} + \psi_3 a_n$
- h)  $a_n + \psi_1 a_{n-1} + \psi_2 a_{n-2} + \psi_3 a_{n-3}$

**Problem 23.** Suppose we start with the series  $x_t$ , then calculate  $y_t = \nabla x_t$ , and then  $z_t = \nabla_s y_t$ . We find that  $z_t = \underline{\hspace{1cm}}$ .

a) 
$$x_{t-1} - x_t - x_{t-s-1} + x_{t-s}$$

**b**) 
$$x_t + x_{t-1} - x_{t-s} - x_{t-s-1}$$

c) 
$$x_t - x_{t-1} - x_{t-s} + x_{t-s-1}$$

**d**) 
$$x_t - x_{t+1} - x_{t+s} + x_{t+s+1}$$

e) 
$$x_{t-1} + x_t - x_{t+s+1} + x_{t+s}$$

f) 
$$x_t + x_{t+1} - x_{t+s} - x_{t+s+1}$$

**Problem 24.** A stationary ARMA process  $z_t$  with psi-weights  $\psi_1, \psi_2, \psi_3, \ldots$  can be written as

a) 
$$z_t = \mu_z + \frac{1}{1 - \psi_1 B - \psi_2 B^2 - \psi_3 B^3 - \dots} a_t$$

**b**) 
$$z_t = C + a_t + \frac{\omega_0 - \omega_1 B - \omega_2 B^2 - \omega_3 B^3 - \dots}{1 - \psi_1 B - \psi_2 B^2 - \psi_3 B^3 - \dots} z_t$$

c) 
$$z_t = C + \frac{1 - \psi_1 B - \psi_2 B^2 - \psi_3 B^3 - \dots}{1 - \delta_1 B - \delta_2 B^2 - \delta_3 B^3 - \dots} a_t$$

$$\mathbf{d}) \ z_t = C + a_t + \psi_1 z_{t-1} + \psi_2 z_{t-2} + \psi_3 z_{t-3} + \dots$$

e) 
$$z_t = \mu_z + a_t + \psi_1 a_{t+1} + \psi_2 a_{t+2} + \psi_3 a_{t+3} + \dots$$

$$\mathbf{f}) \ z_t = \mu_z + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \psi_3 a_{t-3} + \dots$$

$$\mathbf{g}) \ z_t = C + a_t + \psi_1 z_{t+1} + \psi_2 z_{t+2} + \psi_3 z_{t+3} + \dots$$

**Problem 25.** If we form a new series  $w_t$  by differencing the series  $x_t$  3 times at lag 1 and then 4 times at lag 6, we may write  $w_t = \underline{\hspace{1cm}}$ .

a) 
$$(1-B^3)(1-B^4)^6x_t$$

**b**) 
$$(1-B^3)(1-B^6)^4x_t$$

c) 
$$(1-B)^4(1-B^6)^3x_t$$

**d**) 
$$(1-B)^4(1-B^3)^6x_t$$

e) 
$$(1-B^6)(1-B^3)^4x_t$$

f) 
$$(1-B^6)(1-B^4)^3x_t$$

$$\mathbf{g}$$
)  $(1-B)^3(1-B^6)^4x_t$ 

**h**) 
$$(1-B)^3(1-B^4)^6x_t$$

**Problem 26.** The ACF and PACF of a stationary ARIMA $(p, 0, q)(P, 0, Q)_8$  process is given on the next page. What is this process?

a) 
$$ARIMA(2,0,0)(1,0,0)_8$$

**b**) ARIMA
$$(1,0,0)(0,0,2)_8$$

c) ARIMA
$$(1,0,0)(2,0,0)_8$$

**d**) ARIMA
$$(0,0,2)(0,0,1)_8$$

e) ARIMA
$$(2,0,0)(0,0,1)_8$$

**f**) ARIMA
$$(0,0,2)(1,0,0)_8$$

**g**) ARIMA
$$(0,0,1)(0,0,2)_8$$

**h**) ARIMA
$$(0,0,1)(2,0,0)_8$$

i) 
$$ARIMA(0,0,2)(2,0,0)_8$$

LAG	ACF	PACF	PSI	PI
1	-0.593939	-0.593939	-0.700000	-0.700000
2	0.242424	-0.170478	0.400000	-0.090000
3	0.0	0.107058	0.0	0.217000
4	0.0	0.139839	0.0	0.187900
5	0.0	0.056075	0.0	0.044730
6	0.145455	0.223094	0.0	-o.o43849
7	-0.356364	-0.297598	0.0	-o.o48586
8	0.600000	0.381590	0.600000	0.583529
9	-0.356364	0.340515	-0.420000	0.427905
10	0.145455	0.106021	0.240000	0.066122
11	0.0	-0.059646	0.0	-0.124877
12	0.0	-0.083460	0.0	-0.113862
13	0.0	-o.o34765	0.0	-o.o29753
14	0.087273	0.009001	0.0	0.024718
15	-0.213818	0.020196	0.0	0.029204
16	0.360000	0.010539	0.360000	0.010555
17	-0.213818	-0.000700	-0.252000	-0.004293
18	0.087273	-0.004706	0.144000	-0.007227
19	0.0	-0.003014	0.0	-0.003342
20	0.0	-0.000227	0.0	0.000552
21	0.0	0.001046	0.0	0.001723
22	0.052364	0.000823	0.0	0.000985
23	-0.128291	0.000158	0.0	0.000001
24	0.216000	-0.000219	0.216000	-0.000394
25	-0.128291	-0.000216	-0.151200	-0.000276
26	0.052364	-0.000064	0.086400	-0.000036
27	0.0	0.000042	0.0	0.000085
28	0.0	0.000055	0.0	0.000074
29	0.0	0.000022	0.0	0.000018
30	o.o31418	-0.000007	0.0	-0.000017
31	-o.o76975	-0.000013	0.0	-0.000019
32	0.129600	-0.000007	0.129600	-0.00006
33	-o.o76975	0.000001	-0.090720	0.000003
34	o.o31418	0.000003	0.051840	0.000005
35	0.0	0.000002	0.0	0.000002
36	0.0	0.0	0.0	0.0
37	0.0	-0.000001	0.0	-0.000001
38	0.018851	-0.000001	0.0	-0.000001
39	-o.o46185	0.0	0.0	0.0
40	0.077760	0.0	0.077760	0.0

**Problem 27.** In addition to the ACF and PACF, the table on the previous page also has columns giving the  $\psi$ -weights and  $\pi$ -weights (PSI and PI). Is the ARMA process described in the table **invertible**? And why?

- a) Yes, because the  $\psi$ -weights are decaying to zero.
- **b**) Yes, because the  $\pi$ -weights are decaying to zero.
- c) Yes, because the ACF is decaying to zero.
- d) No, because the  $\psi$ -weights are decaying to zero.
- e) No, because the  $\pi$ -weights are decaying to zero.
- f) No, because the ACF is decaying to zero.

The last page of the exam contains some bits of PROC ARIMA output obtained from fitting a model to a time series  $y_t$  with 84 observations. Use this output to answer the following questions.

**Problem 28.** What is the estimated model?

a) 
$$(1 - 0.34695B - 0.35167B^2)(1 - B^{12})y_t = (1 - 0.52048B^{12})a_t$$

**b**) 
$$(1 - 0.34695B - 0.35167B^2)(1 - B)y_t = (1 - 0.52048B)a_t$$

c) 
$$(1 - 0.34695B - 0.35167B^2)y_t = (1 - 0.52048B^{12})a_t$$

d) 
$$(1 - 0.34695B - 0.35167B^2)(1 - B^{12})y_t = 0.086079 + (1 - 0.52048B^{12})a_t$$

e) 
$$(1 - 0.34695B - 0.35167B^2)(1 - B)y_t = 0.086079 + (1 - 0.52048B)a_t$$

f) 
$$(1 - 0.34695B - 0.35167B^2)y_t = 0.086079 + (1 - 0.52048B^{12})a_t$$

g) 
$$(1 - 0.34695B - 0.35167B^2)(1 - B)^{12}y_t = 0.086079 + (1 - 0.52048B)a_t$$

**Problem 29.** What is the standard error (that is, the square root of the variance) of the one-step-ahead prediction error?

 $\mathbf{a)} \ \ 0.17182 \quad \ \mathbf{b)} \ \ 0.221759 \quad \ \mathbf{c)} \ \ 0.03657 \quad \ \mathbf{d)} \ \ 0.13629 \quad \ \mathbf{e)} \ \ 0.11029 \quad \ \mathbf{f)} \ \ 0.11058 \quad \ \mathbf{g)} \ \ 0.1819$ 

**Problem 30.** As you predict further and further into the future, the long-range forecasts from this model converge to \_\_\_\_\_\_.

- ${\bf a})$  a straight line with a nonzero slope
- $\mathbf{b})$  the estimated mean of the process
- $\mathbf{c})$  a repeating pattern added to a straight line with nonzero slope
- d) a repeating pattern
- e) a value which depends mainly on the last few observed values

Name of Variable = y		
Period(s) of Differencing	12	
Mean of Working Series	0.288627	
Standard Deviation	0.221759	
Number of Observations	72	
Observation(s) eliminated by differencing	12	

Maximum Likelihood Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr >  t	Lag
MU	0.28562	0.03657	7.81	<.0001	0
MA1,1	0.52048	0.13629	3.82	0.0001	12
AR1,1	0.34695	0.11029	3.15	0.0017	1
AR1,2	0.35167	0.11058	3.18	0.0015	2

<b>Constant Estimate</b>	0.086079
Variance Estimate	0.029522
Std Error Estimate	0.17182
AIC	-41.0894
SBC	-31.9828
Number of Residuals	72

Forecasts for variable y				
Obs	Forecast	Std Error	95% Confidence Limits	
85	9.5783	0.1718	9.2415	9.9150
86	9.7548	0.1819	9.3984	10.1113
87	10.2862	0.1991	9.8959	10.6765
88	10.0286	0.2051	9.6267	10.4306
89	9.9509	0.2101	9.5391	10.3627