

TEST #2

STA 4853

Name: _____

May 3, 2018

Please read the following directions.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are **35** multiple choice questions.
- Circle the **single best** answer for each multiple choice question. Your choice should be made clearly.
- Always **circle the correct response**. (Sometimes the question has an empty blank or a box, but this is **NOT** where the answer goes.)
- There is no penalty for guessing.
- The exam has **14** pages.
- Each question is worth equal credit.

Problem 1. Suppose you have data on a series Y_t up to time $t = 84$ and use this data to fit an ARIMA model and forecast future values of the series Y_t for times greater than 84. Some of these forecasts are given in the table below. Assuming the model is valid, which of the following statements are true. Given the information about Y_t up to time 84, _____

- a) Y_{88} will lie in the interval $(10.0286 - 0.2051, 10.0286 + 0.2051)$.
- b) Y_{88} will lie in the interval $(9.6267, 10.4306)$.
- c)★ Y_{88} has a normal distribution with mean 10.0286 and standard deviation 0.2051.
- d) Y_{88} has a log-normal distribution with median 10.0286 and standard deviation 0.2051.
- e) The forecast for Y_{88} has a 95% chance of being inside the interval $(9.6267, 10.4306)$.
- f) Y_{88} has a skewed distribution with median 10.0286 and standard deviation 10.4306.

Obs	Forecast	Std Error	95% Confidence Limits	
85	9.5783	0.1718	9.2415	9.9150
86	9.7548	0.1819	9.3984	10.1113
87	10.1862	0.1991	9.8959	10.6765
88	10.0286	0.2051	9.6267	10.4306

Problem 2. For the model used in the series Y_t of the previous problem, use the information in the table above to determine the value of $\hat{\sigma}_a$, the estimate of the standard deviation of the random shocks. The value of $\hat{\sigma}_a$ is _____.

- a)★ 0.1718 b) 0.1819 c) 0.1991 d) 0.2051 e) 9.5783 f) 9.7548
- g) 10.2862 h) 10.0286 i) 0.6735 j) 0.7129 k) 0.7806 l) 0.8039

Problem 3. A transfer function model has the general form

$$Y_t = C + v(B)X_t + N_t.$$

If the noise process N_t is a stationary ARMA(p, q) process with AR polynomial $\phi(B)$ and MA polynomial $\theta(B)$, then the noise process N_t can be written as

- a) $\frac{\phi(B)}{\theta(B)}a_t$ b)★ $\frac{\theta(B)}{\phi(B)}a_t$ c) $\frac{B^b\theta(B)}{\phi(B)}a_t$ d) $\frac{B^b\phi(B)}{\theta(B)}a_t$
- e) $\frac{\theta(B)}{(1-B)\phi(B)}a_t$ f) $\frac{\phi(B)}{(1-B)\theta(B)}a_t$ g) $\frac{(1-B)\theta(B)}{\phi(B)}a_t$ h) $\frac{(1-B)\phi(B)}{\theta(B)}a_t$

Problem 4. For the transfer function

$$v(B) = B^3(0.4 + 0.5B + 0.3B^2),$$

the lag of the last nonzero v -weight will be _____

- a) 0 b) 1 c) 2 d) 3 e) 4 f)★ 5 g) 6 h) 7 i) ∞

Problem 5. Suppose we use the ALTPARM option when specifying our transfer function model by using the code below:

ESTIMATE INPUT=(3\$(1,2)/(1)X) Q=(1) ALTPARM METHOD=ML;

Then _____

- a) the MA factor will be $\theta_0 - \theta_1 B$.
 - b) the MA factor will be $\theta_0(1 - \theta_1 B)$.
 - c) the AR factor will be $\phi_0 - \phi_1 B$.
 - d) the AR factor will be $\phi_0(1 - \phi_1 B)$.
 - e)★ the numerator factor will be $\omega_0(1 - \omega_1 B - \omega_2 B^2)$.
 - f) the numerator factor will be $\omega_0 - \omega_1 B - \omega_2 B^2$.
 - g) the denominator factor will be $\delta_0 - \delta_1 B$.
 - h) the denominator factor will be $\delta_0(1 - \delta_1 B)$.
-

Suppose you wish to predict the random variable X based on the information $\mathcal{I}$. Let $\hat{X}$ denote your prediction for X . Suppose you pay a penalty depending on the accuracy of your prediction $\hat{X}$.

Problem 6. If you will have to pay a penalty of $|X - \hat{X}|$, then the best prediction for X is the _____ of the conditional distribution of X given $\mathcal{I}$.

- a) mean b)★ median c) mode d) standard error e) variance

Problem 7. If you will have to pay a penalty of \$100 unless $\hat{X}$ is within ε of X (where ε is small), then the best prediction for X is the _____ of the conditional distribution of X given $\mathcal{I}$.

- a) mean b) median c)★ mode d) standard error e) variance

Problem 8. If you will have to pay a penalty of $(X - \hat{X})^2$, then the best prediction for X is the _____ of the conditional distribution of X given $\mathcal{I}$.

- a)★ mean b) median c) mode d) standard error e) variance
-

Problem 9. An ARMA(2,2) process will be **invertible** if ...

- a) $|\theta_1| < 1$
- b) $|\phi_1| < 1$
- c) $|\theta_2| < 1$
- d) $|\phi_2| < 1$
- e) it can be written as an MA(∞) process
- f) the long run mean is constant
- g) $|\phi_2| < 1$, $\phi_2 + \phi_1 < 1$, and $\phi_2 - \phi_1 < 1$
- h)★ $|\theta_2| < 1$, $\theta_2 + \theta_1 < 1$, and $\theta_2 - \theta_1 < 1$

The next two problems concern this situation: Suppose $\{z_t\}$ is an ARIMA process generated by the random shocks $\{a_t\}$. We observe **all** the values z_t and a_t up to time n . Call this set of information $\mathcal{I}_n$:

$$\mathcal{I}_n = \{z_n, z_{n-1}, z_{n-2}, \dots, a_n, a_{n-1}, a_{n-2}, \dots\}$$

We define $\hat{z}_t = E(z_t | \mathcal{I}_n)$ and $\hat{a}_t = E(a_t | \mathcal{I}_n)$.

Problem 10. Which of the following statements about $\hat{a}_t$ is always true?

- a) $\hat{a}_t = z_t$ for $t \leq n$ and $\hat{a}_t = a_t$ for $t > n$
- b) $\hat{a}_t = \sigma_a^2$ for $t \leq n$ and $\hat{a}_t = 0$ for $t > n$
- c) $\hat{a}_t = 0$ for $t \leq n$ and $\hat{a}_t = \sigma_a^2$ for $t > n$
- d) $\hat{a}_t = a_t$ for $t \leq n$ and $\hat{a}_t = z_t$ for $t > n$
- e) $\hat{a}_t = \sigma_a^2$ for $t = n$ and $\hat{a}_t = 0$ for $t \neq n$
- f)★ $\hat{a}_t = a_t$ for $t \leq n$ and $\hat{a}_t = 0$ for $t > n$
- g) $\hat{a}_t = 0$ for $t \leq n$ and $\hat{a}_t = a_t$ for $t > n$

Problem 11. Which of the following statements about $\hat{z}_t$ is always true?

- a)★ $\hat{z}_t = z_t$ for $t \leq n$
- b) $\hat{z}_t = \mu_z$ for $t > n$
- c) $\hat{z}_t = 0$ for $t \leq n$
- d) $\hat{z}_t = 0$ for $t > n$
- e) $\hat{z}_t = z_t$ for $t > n$
- f) $\hat{z}_t = \mu_z$ for $t \leq n$

Problem 12. One general approach to modeling non-stationary series which exhibit seasonal patterns or seasonal variation is to:

- a) Pre-whiten the series and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the pre-whitened series.
- b) Mean-center the series and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the mean-centered series.
- c) Transform the series (by taking logs or square roots or some other transformation) and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the transformed series.
- d)★ Make the series stationary by differencing (either ordinary or seasonal differencing or some combination), and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the differenced series.
- e) Integrate the series (compute cumulative sums) and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the integrated series.

Problem 13. If we define $v(B) = v_0 + v_1B + v_2B^2 + \cdots + v_hB^h$, then $v(B)X_t = \underline{\hspace{2cm}}$.

- a) $\frac{\theta(B)}{\phi(B)}a_t$ b) $\frac{\omega(B)}{\delta(B)}a_t$ c) $(1 - B)^hX_t$ d) $v_0X_t + v_1X_{t+1} + v_2X_{t+2} + \cdots + v_hX_{t+h}$
- e) $(1 - v_0B)^hX_t$ f)★ $v_0X_t + v_1X_{t-1} + v_2X_{t-2} + \cdots + v_hX_{t-h}$
- g) $v_0X_t - v_1X_t - v_2X_t - \cdots - v_hX_t$ h) $(1 - B^h)X_t$

Problem 14. For **non**-stationary ARIMA processes, as you forecast further and further into the future, the confidence interval **widths** for the forecasts ...

- a) converge to a repetitive pattern which repeats with a period of S (= the seasonality)
- b) converge to a repetitive pattern added to a straight line with nonzero slope
- c) converges to a straight line with a nonzero slope
- d) converge to a limiting value
- e)★ continue to gradually increase and will eventually reach arbitrarily large values

Problem 15. If a time series consists of a repeating seasonal pattern plus a linear trend, then seasonal differencing will ...

- a) remove the seasonal pattern but not the linear trend
- b) remove the linear trend but not the seasonal pattern
- c)★ remove both the seasonal pattern and the linear trend
- d) sometimes fail to remove the linear trend and second differencing is needed
- e) sometimes fail to remove the seasonal pattern and second differencing is needed

Problem 16. The terms in the summation _____ are periodic functions of (integer-valued) time t with period 12, and any periodic function with period 12 can be represented as a constant plus a summation of this form for some values of α_j and ξ_j .

$$\begin{array}{ll} \text{a)} \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{\pi t}{12j} \right) + \xi_j \cos \left(\frac{\pi t}{12j} \right) \right\} & \text{b)} \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{2\pi t}{12j} \right) + \xi_j \cos \left(\frac{2\pi t}{12j} \right) \right\} \\ \text{c)} \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{\pi j t}{12} \right) + \xi_j \cos \left(\frac{\pi j t}{12} \right) \right\} & \text{d)} \star \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{2\pi j t}{12} \right) + \xi_j \cos \left(\frac{2\pi j t}{12} \right) \right\} \\ \text{e)} \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{2\pi t}{12} \right) + \xi_j \cos \left(\frac{2\pi t}{12} \right) \right\} & \text{f)} \sum_{j=1}^6 \left\{ \alpha_j \sin \left(\frac{2\pi j}{12} \right) + \xi_j \cos \left(\frac{2\pi j}{12} \right) \right\} \end{array}$$

Problem 17. After fitting a time series model, the plot of the residuals versus the one-step-ahead forecasts is often used to determine if the _____

- a) ACF of the series varies with time
- b) residuals are normally distributed
- c) mean of the series varies with time
- d) $\star$ variability of the residuals changes with the level of the series
- e) series needs further differencing
- f) one-step-ahead forecasts are normally distributed
- g) one-step-ahead forecasts change with the level of the series

Problem 18. For an AR(1) process $z_t = C + \phi_1 z_{t-1} + a_t$, if you are given $\mathcal{I}_n$ (information up to time n), the forecast $\hat{z}_{n+2}$ is _____

- a) $C - \phi_1 a_n - \phi_1 a_{n-1}$
- b) $C - \phi_1 a_n$
- c) $C + z_n$
- d) $\star C + \phi_1 C + \phi_1^2 z_n$
- e) C
- f) z_n
- g) $\phi_1 z_n$

Problem 19. Suppose you wish to construct a transfer function model explaining the response series Y_t in terms of the input series X_t . You find that X_t and Y_t are not jointly stationary, but, after differencing, the differenced series $X_t^* = (1 - B)X_t$ and $Y_t^* = (1 - B)Y_t$ are jointly stationary. You then identify a transfer function and ARMA noise model for the differenced series which is

$$Y_t^* = \frac{B^b \omega(B)}{\delta(B)} X_t^* + \frac{\theta(B)}{\phi(B)} a_t.$$

In terms of the original series X_t and Y_t , this model may be written as _____

- a)★ $Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{\theta(B)}{(1 - B)\phi(B)} a_t.$
- b) $Y_t = \frac{B^b \omega(B)}{\delta(B)} (1 - B)X_t + \frac{\theta(B)}{\phi(B)} a_t.$
- c) $Y_t = \frac{B^b \omega(B)}{(1 - B)\delta(B)} X_t + \frac{\theta(B)}{\phi(B)} a_t.$
- d) $(1 - B)Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{\theta(B)}{\phi(B)} a_t.$
- e) $\frac{Y_t}{1 - B} = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{\theta(B)}{\phi(B)} a_t.$
- f) $Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{(1 - B)\theta(B)}{\phi(B)} a_t.$

Problem 20. Suppose now that the series $\{X_t\}$ and $\{Y_t\}$ are jointly stationary and we wish to construct a transfer function model $Y_t = C + v(B)X_t + N_t$. In the pre-whitening process, the goal is to apply a filter $f(B)$ to both X_t and Y_t to produce new series $X'_t = f(B)X_t$ and $Y'_t = f(B)Y_t$ such that _____.

- a) X'_t and Y'_t are independent
- b) their CCF is close to zero
- c) both X'_t and Y'_t are white noise
- d)★ X'_t is white noise
- e) Y'_t is white noise
- f) X'_t and Y'_t are uncorrelated
- g) X'_t and Y'_t are jointly stationary

Problem 21. An MA(1) process with $|\theta_1| < 1$ can be re-written in the form _____.

- a) $\tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_1^2 a_{t-2} - \theta_1^3 a_{t-3} - \dots$
- b) $\tilde{z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \dots$
- c) $\tilde{z}_t = a_t + \theta_1 \tilde{z}_{t-1} + \theta_2 \tilde{z}_{t-2} + \theta_3 \tilde{z}_{t-3} + \dots$
- d) $\tilde{z}_t = a_t + \theta_1 a_{t-1} + \theta_1^2 a_{t-2} + \theta_1^3 a_{t-3} + \dots$
- e)★ $\tilde{z}_t = a_t - \theta_1 \tilde{z}_{t-1} - \theta_1^2 \tilde{z}_{t-2} - \theta_1^3 \tilde{z}_{t-3} - \dots$
- f) $\tilde{z}_t = a_t + \theta_1 \tilde{z}_{t-1} + \theta_1^2 \tilde{z}_{t-2} + \theta_1^3 \tilde{z}_{t-3} + \dots$

Problem 22. The AR-polynomial of an ARIMA(2, 0, 1)(1, 0, 2)₁₂ process is _____

- a) $1 - \phi_1 B - \Phi_1 B^{12} - \Phi_2 B^{24}$
- b) $(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^{12} - \Phi_2 B^{24})$
- c) $1 - \phi_1 B - \phi_2 B^2 - \Phi_1 B^{12} - \Phi_2 B^{24}$
- d) $(1 - B)^2(1 - B^{12})(1 - \phi_1 B - \phi_2 B^2)$
- e)★ $(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^{12})$
- f) $1 - \phi_1 B - \phi_2 B^2 - \Phi_1 B^{12}$
- g) $(1 - \phi_1 B)(1 - \Phi_1 B^{12} - \Phi_2 B^{24})$

Problem 23. The SAS code given below _____

```
PROC ARIMA DATA=STUFF;
  IDENTIFY VAR=Y CROSSCOR=(X1 X2 X3) NOPRINT;
  ESTIMATE INPUT=(X1 X2 X3) METHOD=ML;
  QUIT;
```

- a) fits a multiple regression model for Y_t on the regressors $X_{t-1}, X_{t-2}, X_{t-3}$.
- b) uses pre-whitening to identify a transfer function for Y_t on $X_{1,t}, X_{2,t}, X_{3,t}$.
- c)★ fits a multiple regression model for Y_t on the regressors $X_{1,t}, X_{2,t}, X_{3,t}$.
- d) fits a transfer function model for Y_t on $X_{t-1}, X_{t-2}, X_{t-3}$.
- e) fits a transfer function model for Y_t on $X_{1,t}, X_{2,t}, X_{3,t}$.
- f) uses pre-whitening to identify a transfer function for Y_t on $X_{t-1}, X_{t-2}, X_{t-3}$.
- g) fits a transfer function model for Y_t on $X_{t-1}, X_{t-2}, X_{t-3}$ using a proxy AR(2) model for the noise.
- h) fits a transfer function model for Y_t on $X_{1,t}, X_{2,t}, X_{3,t}$ using a proxy AR(2) model for the noise.

Problem 24. The model

$$(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^s)z_t = C + a_t$$

may be re-written as _____.

- a) $z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-1} z_{t-s} - \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$
- b) $z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-1} z_{t-s} + \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$
- c) $z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-1} z_{t-s} + \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$
- d) $z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-s-1} - \phi_2 \Phi_1 z_{t-s-2} + a_t$
- e) $z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-s-1} + \phi_2 \Phi_1 z_{t-s-2} + a_t$
- f)★ $z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-s-1} - \phi_2 \Phi_1 z_{t-s-2} + a_t$
- g) $z_t = C - \phi_1 z_{t-1} - \phi_2 z_{t-2} - \Phi_1 z_{t-s} + \phi_1 \Phi_1 z_{t-s-1} + \phi_2 \Phi_1 z_{t-s-2} + a_t$
- h) $z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \Phi_1 z_{t-s} - \phi_1 \Phi_1 z_{t-1} z_{t-s} - \phi_2 \Phi_1 z_{t-2} z_{t-s} + a_t$

Problem 25. If you difference a series and then observe that the sample ACF of the differenced series decays very slowly, what should you do?

- a) Use an MA(q) model with a large value of q .
- b) Conclude that the series has been over-differenced.
- c) Try a transformation, perhaps a log or square root.
- d) Use a mixed model with both $p > 0$ and $q > 0$.
- e) Conclude that the series is non-invertible.
- f)★ Try differencing the series again.

Problem 26. To estimate the transfer function model

$$Y_t = C + \frac{B^3 \omega_0}{1 - \delta_1 B} X_t + \frac{1}{(1 - \phi_{1,1} B)(1 - \phi_{2,1} B^{12})} a_t,$$

you use the SAS code _____.

- a) ESTIMATE INPUT=(\$3)/(0,1)X Q=(1)(12) ;
- b) ESTIMATE INPUT=(\$3)/(1)X Q=(1,12) ;
- c) ESTIMATE INPUT=(\$3)/(1)X P=(1)(12) ;
- d)★ ESTIMATE INPUT=(3\$)/(1)X P=(1)(12) ;
- e) ESTIMATE INPUT=(3\$)/(1)X P=(1,12) ;
- f) ESTIMATE INPUT=(3\$(0))/(0,1)X P=(1)(12) ;
- g) ESTIMATE INPUT=(3\$(0))/(0,1)X P=(1,12) ;
- h) ESTIMATE INPUT=(3\$(0))/(0,1)X Q=(1,12) ;

Problem 27. Suppose z_t is a stationary ARMA process with psi-weights $\psi_1, \psi_2, \psi_3, \dots$. Let $e_n(3) = z_{n+3} - \hat{z}_{n+3}$ be the 3-step-ahead forecast error based on information about the series up to time n . The forecast error $e_n(3)$ is equal to _____.

- a) $\sigma_a \sqrt{1 + \psi_1^2 + \psi_2^2 + \psi_3^2}$
- b) $a_n + \psi_1 a_{n-1} + \psi_2 a_{n-2}$
- c)★ $a_{n+3} + \psi_1 a_{n+2} + \psi_2 a_{n+1}$
- d) $a_{n+3} + \psi_1 a_{n+2} + \psi_2 a_{n+1} + \psi_3 a_n$
- e) $a_n + \psi_1 a_{n-1} + \psi_2 a_{n-2} + \psi_3 a_{n-3}$
- f) $\sigma_a^2(1 + \psi_1^2 + \psi_2^2)$
- g) $\sigma_a^2(1 + \psi_1^2 + \psi_2^2 + \psi_3^2)$
- h) $\sigma_a \sqrt{1 + \psi_1^2 + \psi_2^2}$

Problem 28. A series which approximately repeats a consistent seasonal pattern is _____.

- a) invertible b) non-invertible c) multiplicative d) non-multiplicative e) integrated
- f) differenced g) over-differenced h) autoregressive i) stationary j)★ non-stationary

Problem 29. The long range forecasts from the model

$$(1 - B)(1 - .7B)z_t = .5 + (1 - .5B^{12} + .2B^{24})a_t$$

will _____.

- a) converge to a straight line with a positive slope plus a repeating seasonal pattern
- b) converge to a straight line with a negative slope plus a repeating seasonal pattern
- c) converge to a repeating seasonal pattern
- d) converge to the overall mean of the process
- e)★ converge to a straight line with a positive slope
- f) converge to a straight line with a negative slope
- g) converge to a straight line with zero slope

Problem 30. Suppose you are attempting to identify a plausible *initial* choice of ARIMA(p, d, q) model for a time series z_t . After examining the data, you decided not to use a transformation. Then you selected a particular value of d , that is, you decided that the series $w_t = \nabla^d z_t$ obtained by differencing z_t d times is stationary. Now you are going to choose reasonable *initial* values for p and q . Which of the following will you use in making this decision?

- a)★ The sample ACF and PACF of w_t
- b) The sample ACF and PACF of z_t
- c) The sample IACF of z_t
- d) The residual ACF and PACF
- e) The normal probability plot (the QQ-Plot)
- f) The auto-regressive parameters
- g) The moving average parameters
- h) The AIC or SBC
- i) The correlations between the parameter estimates

Attached to the end of the exam are two pages entitled “Transfer Function Plots”. These plots give plots of transfer function v -weights for transfer functions having the form

$$v(B) = \frac{B^b \omega(B)}{\delta(B)} \quad \text{with}$$
$$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_h B^h$$
$$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \delta_r B^r$$

Problem 31. Which of the plots illustrates a case with $b = 0, r = 1, h = 1$?

- a) b) c) d) e) f) g) h) i) j)★ k) l)

Problem 32. Which of the plots illustrates a case with $b = 3, r = 1, h = 0$?

- a) b) c) d) e)★ f) g) h) i) j) k) l)

Problem 33. Which of the plots illustrates a case with $b = 2, r = 0, h = 2$?

- a) b) c) d) e) f) g) h) i)★ j) k) l)

Attached to the end of the exam is a page entitled “Theoretical ACF and PACF for Two Seasonal ARIMA Processes.” These processes are referred to as #1 and #2. Each of these processes is one of the following:

- | | |
|---|---|
| a) ARIMA(1, 0, 0)(1, 0, 0) ₈ | b) ARIMA(1, 0, 0)(0, 0, 1) ₈ |
| c) ARIMA(0, 0, 1)(1, 0, 0) ₈ | d) ARIMA(0, 0, 1)(0, 0, 1) ₈ |
| e) ARIMA(2, 0, 0)(1, 0, 0) ₈ | f) ARIMA(2, 0, 0)(0, 0, 1) ₈ |
| g) ARIMA(0, 0, 2)(1, 0, 0) ₈ | h) ARIMA(0, 0, 2)(0, 0, 1) ₈ |
| i) ARIMA(4, 0, 0)(2, 0, 0) ₈ | j) ARIMA(4, 0, 0)(0, 0, 2) ₈ |
| k) ARIMA(0, 0, 4)(2, 0, 0) ₈ | l) ARIMA(0, 0, 4)(0, 0, 2) ₈ |

Answer the following two questions by circling the letter corresponding to the correct choice from the list above.

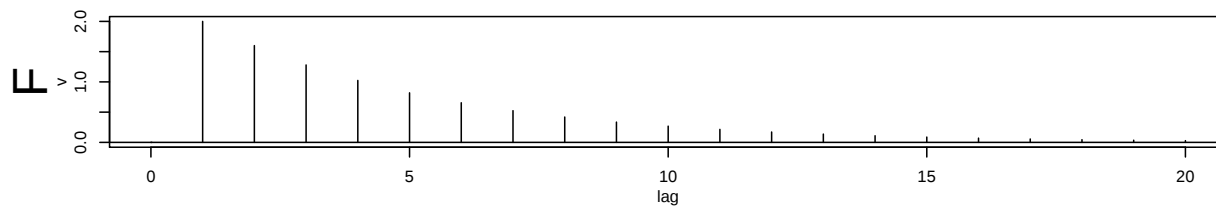
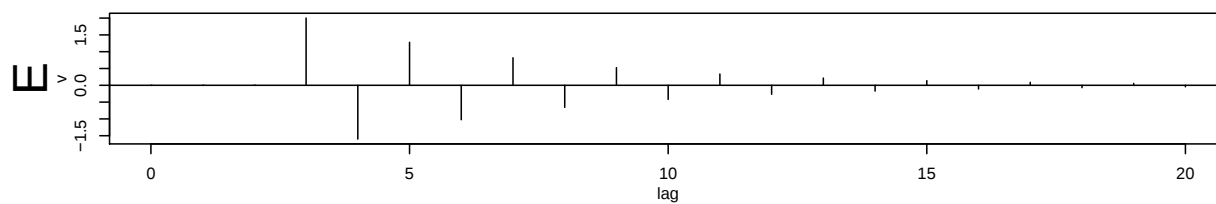
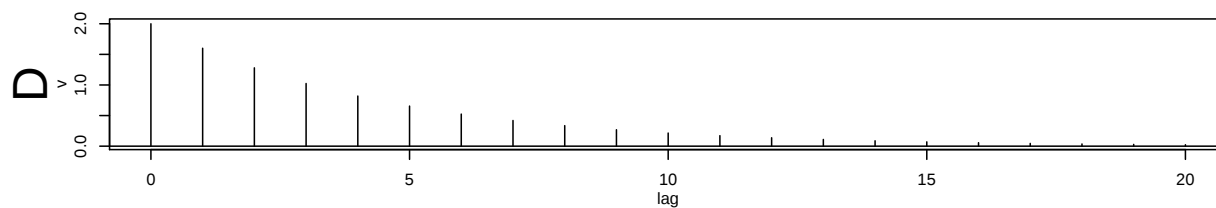
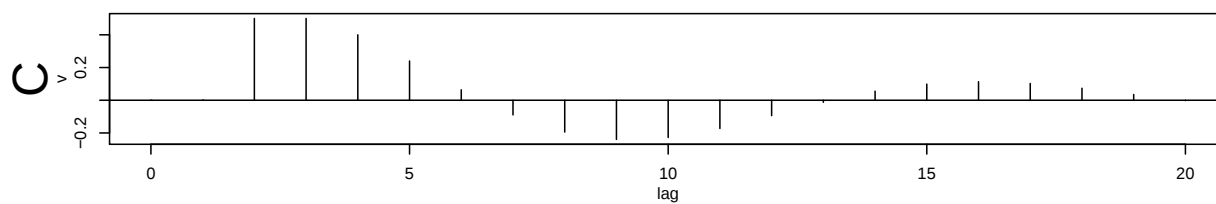
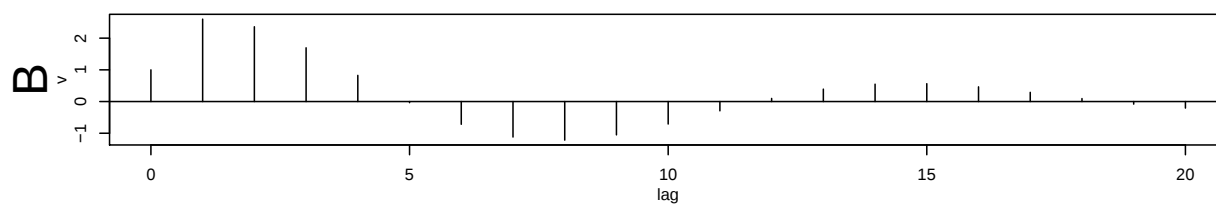
Problem 34. Identify the simplest reasonable choice of a process for #1.

- | | | | | | |
|----|-----|----|----|----|----|
| a) | b)★ | c) | d) | e) | f) |
| g) | h) | i) | j) | k) | l) |

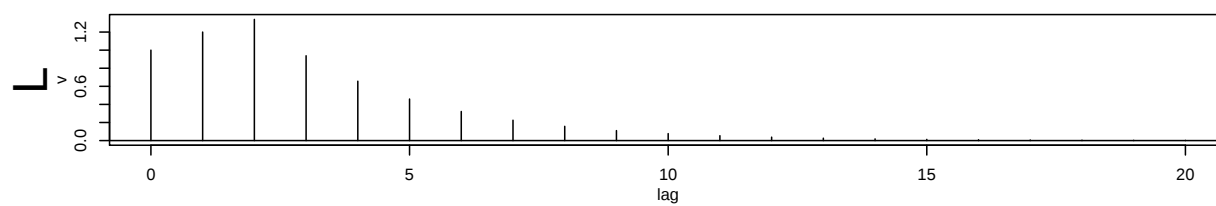
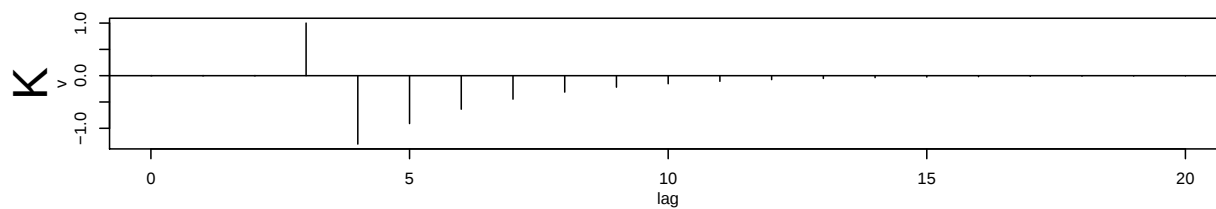
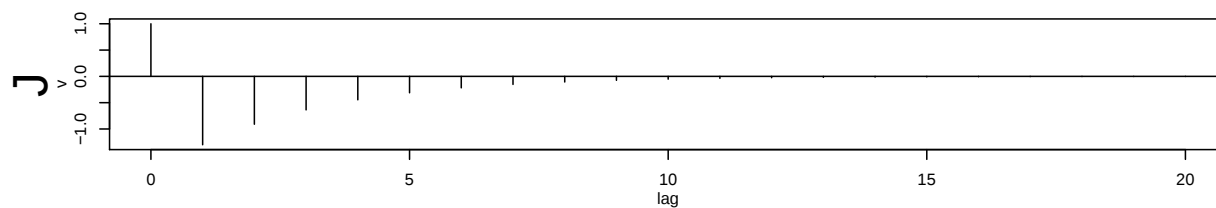
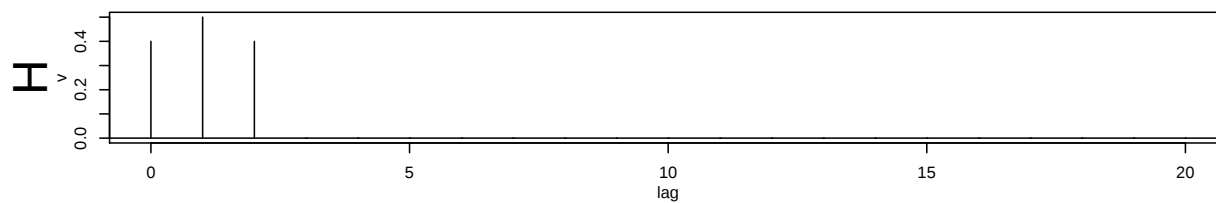
Problem 35. Identify the simplest reasonable choice of a process for #2.

- | | | | | | |
|-----|----|----|----|----|----|
| a) | b) | c) | d) | e) | f) |
| g)★ | h) | i) | j) | k) | l) |

Transfer Function Plots (page 1)



Transfer Function Plots (page 2)



Theoretical ACF and PACF for Two Seasonal ARIMA Processes

#1			#2		
LAG	ACF	PACF	LAG	ACF	PACF
1	-0.680477	-0.680477	1	-0.593939	-0.593939
2	0.448444	-0.027200	2	0.242424	-0.170478
3	-0.274069	0.038913	3	0.000000	0.107058
4	0.134930	-0.055755	4	0.000000	0.139839
5	-0.013140	0.080136	5	0.000000	0.056075
6	-0.106960	-0.115922	6	0.145455	0.223094
7	0.240813	0.169973	7	-0.356364	-0.297598
8	-0.405627	-0.256642	8	0.600000	0.381590
9	0.283939	-0.298273	9	-0.356364	0.340515
10	-0.198757	-0.014980	10	0.145455	0.106021
11	0.139130	0.021409	11	0.000000	-0.059646
12	-0.097391	-0.030612	12	0.000000	-0.083460
13	0.068174	0.043812	13	0.000000	-0.034765
14	-0.047722	-0.062824	14	0.087273	0.009001
15	0.033405	0.090444	15	-0.213818	0.020196
16	-0.023384	-0.131280	16	0.360000	0.010539
17	0.016369	-0.163050	17	-0.213818	-0.000700
18	-0.011458	-0.008730	18	0.087273	-0.004706
19	0.008021	0.012474	19	0.000000	-0.003014
20	-0.005614	-0.017825	20	0.000000	-0.000227
21	0.003930	0.025480	21	0.000000	0.001046
22	-0.002751	-0.036446	22	0.052364	0.000823
23	0.001926	0.052202	23	-0.128291	0.000158
24	-0.001348	-0.074973	24	0.216000	-0.000219
25	0.000944	-0.094789	25	-0.128291	-0.000216
26	-0.000661	-0.005185	26	0.052364	-0.000064
27	0.000462	0.007407	27	0.000000	0.000042
28	-0.000324	-0.010583	28	0.000000	0.000055
29	0.000227	0.015121	29	0.000000	0.000022
30	-0.000159	-0.021612	30	0.031418	-0.000007
31	0.000111	0.030902	31	-0.076975	-0.000013
32	-0.000078	-0.044228	32	0.129600	-0.000007
33	0.000054	-0.056244	33	-0.076975	0.000001
34	-0.000038	-0.003099	34	0.031418	0.000003
35	0.000027	0.004428	35	0.000000	0.000002
36	-0.000019	-0.006326	36	0.000000	0.000000
37	0.000013	0.009037	37	0.000000	-0.000001
38	-0.000009	-0.012913	38	0.018851	-0.000001
39	0.000006	0.018453	39	-0.046185	0.000000
40	-0.000004	-0.026378	40	0.077760	0.000000