TEST #2

STA 4853 **Name:**____________________________________

April 29, 2020

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are **30** multiple choice questions.
- Select the **single best** answer for each multiple choice question.
- For each question, type the **single lower case letter** of the correct response into the "Fill in the blank" box in Canvas.
- There is no penalty for guessing.
- The exam has **13** pages.
- Each question is worth equal credit.

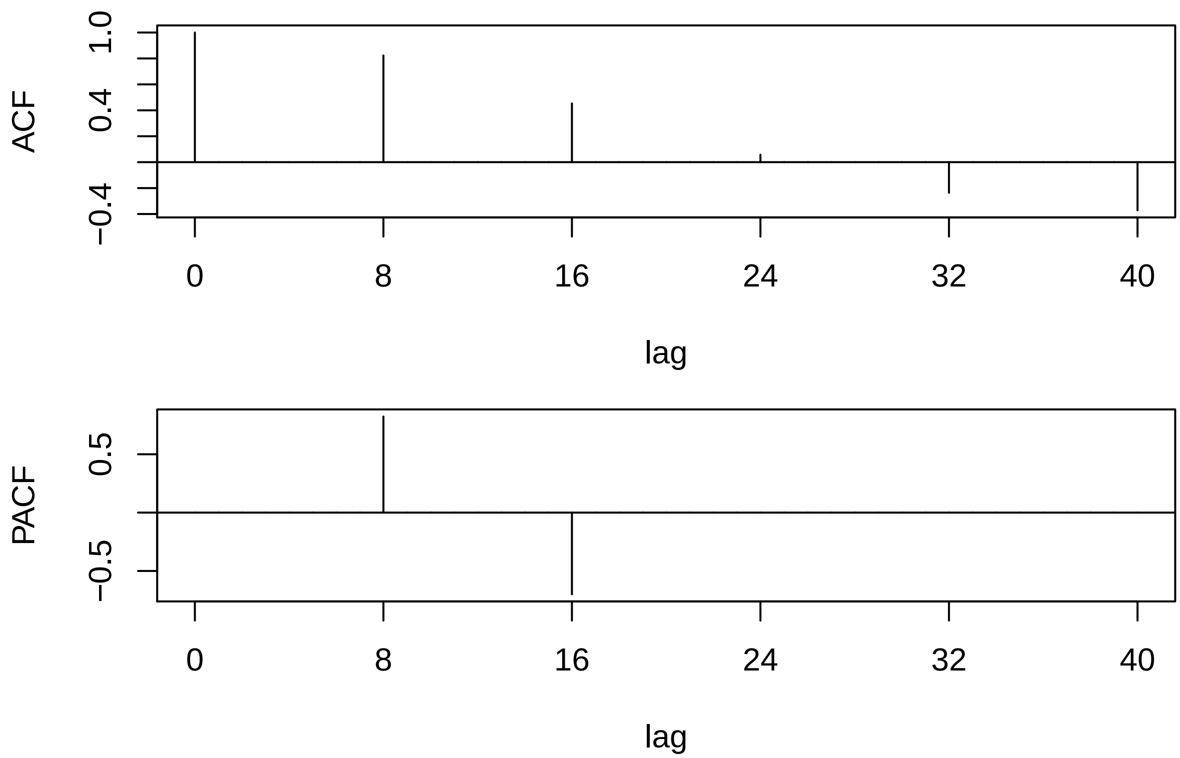
Problem 1. The plots below give the theoretical ACF and PACF of an ARMA process. What is this process?

a) ARMA$(2,0)(0,0)_8$ **b**) ARMA$(0,2)(0,0)_8$ **c**) ARMA$(0,0)(0,2)_8$

d)$\star$ ARMA$(0,0)(2,0)_8$ **e**) ARMA$(0,0)(2,0)_{16}$ **f**) ARMA$(0,0)(0,2)_{16}$

g) ARMA$(2,0)(0,0)_{16}$ **h**) ARMA$(0,2)(0,0)_{16}$ **i**) ARMA$(0,0)(2,2)_8$

j) ARMA$(0,0)(2,2)_{16}$ **k**) ARMA$(2,2)(0,0)_8$ **l**) ARMA$(2,2)(0,0)_{16}$



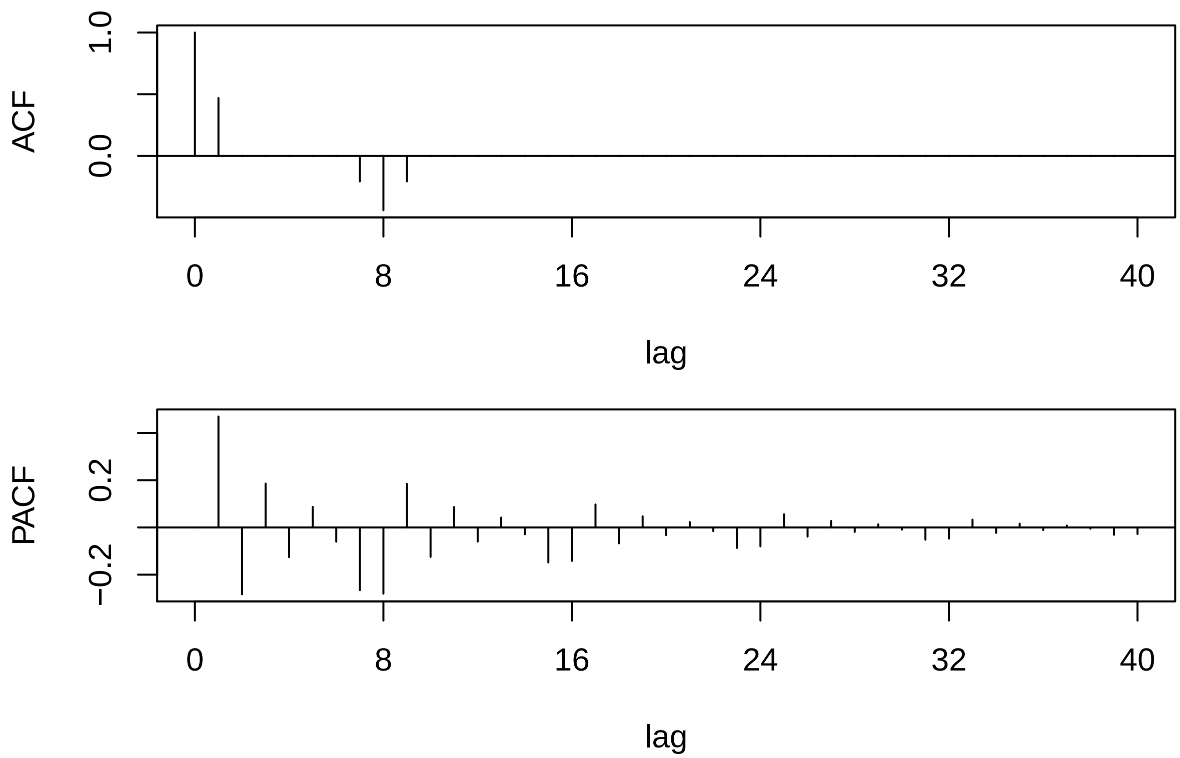
Problem 2. The plots below give the theoretical ACF and PACF of another ARMA process. What is this process?

a) $\mathrm{ARMA}(2,0)(0,1)_8$ **b**) $\mathrm{ARMA}(2,0)(1,0)_{16}$ **c**) $\mathrm{ARMA}(1,0)(0,1)_8$

d) $\mathrm{ARMA}(1,0)(1,0)_8$ **e**) $\mathrm{ARMA}(0,1)(0,1)_{16}$ **f**) $\mathrm{ARMA}(0,1)(1,0)_{16}$

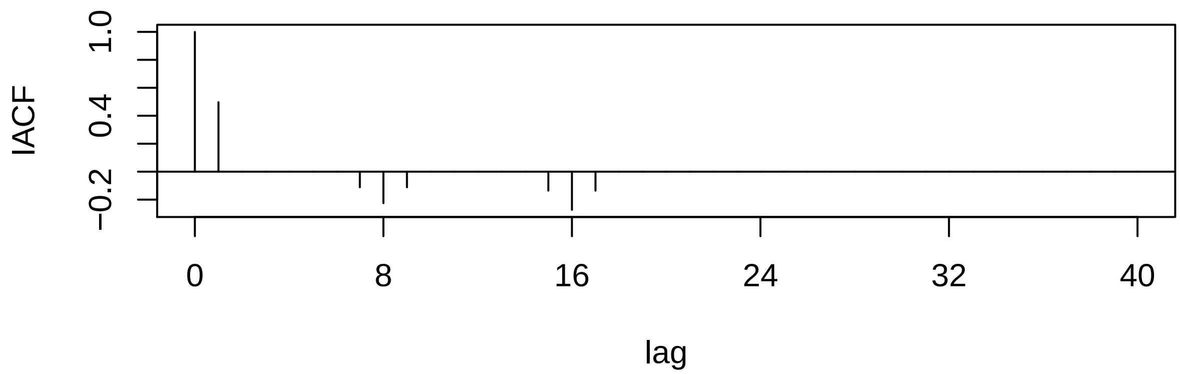
g) $\mathrm{ARMA}(1,0)(0,1)_{16}$ **h**) $\mathrm{ARMA}(1,0)(1,0)_{16}$ **i**) $\mathrm{ARMA}(0,1)(0,2)_8$

j) $\mathrm{ARMA}(0,1)(2,0)_{16}$ **k**)$\star$ $\mathrm{ARMA}(0,1)(0,1)_8$ **l**) $\mathrm{ARMA}(0,1)(1,0)_8$



Problem 3. The plots below give the theoretical **IACF** of yet another ARMA process. What is this process?

a) ARMA$(2,0)(1,0)_8$ **b**) ARMA$(0,2)(1,0)_8$ **c**) ARMA$(2,0)(0,1)_8$
d) ARMA$(0,2)(0,1)_8$ **e**) ARMA$(2,0)(1,0)_{16}$ **f**) ARMA$(0,2)(1,0)_{16}$
g) ARMA$(2,0)(0,1)_{16}$ **h**)$\star$ ARMA$(1,0)(2,0)_8$ **i**) ARMA$(0,1)(2,0)_8$
j) ARMA$(1,0)(0,2)_8$ **k**) ARMA$(0,1)(0,2)_8$ **l**) ARMA$(0,2)(0,1)_{16}$
m) ARMA$(1,0)(0,2)_{16}$ **n**) ARMA$(0,1)(0,2)_{16}$



The following two problems involve this situation:

Suppose $\{z_t\}$ is a realization of a known ARIMA(p,d,q) process; we know the orders p,d,q and the values of all parameters. Suppose we observe the values of z_t and the random shocks a_t for all times t up to time 100. For any time t, let $\widehat{z}_t$ denote the best forecast for z_t based on only the given information up to time 100, that is, $\widehat{z} = E(z_t \,|\, \mathcal{I}_{100})$ where $\mathcal{I}_{100}$ denotes the observed information up to time 100.

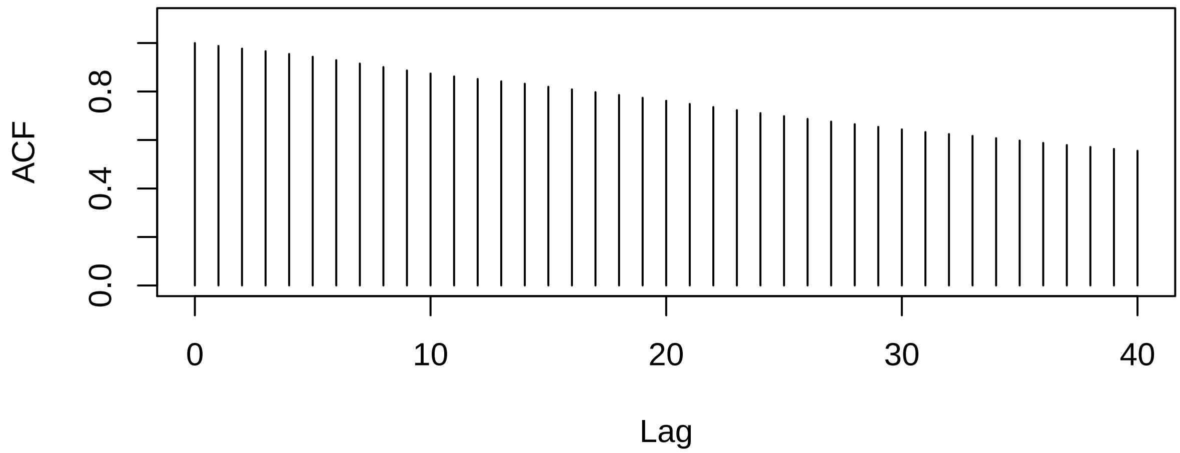
Problem 4. $\widehat{z}_{99} =$ _________.

a) a_{99} **b**) a_{98} **c**) a_{100} **d**) 0
e) $\widehat{z}_{98}$ **f**) $\widehat{z}_{100}$ **g**)$\star$ z_{99} **h**) z_{98}
i) z_{100} **j**) $\widehat{a}_{98}$ **k**) $\widehat{a}_{100}$ **l**) $C + \phi_1 z_{98}$

Problem 5. The best forecast for $2z_{102} + 3a_{101} + 5$ is equal to _________.

a) $2z_{102}$ **b**) $2z_{102} + 3$ **c**) $2z_{101} + 3$ **d**) $2z_{100} + 3$
e)$\star$ $2\widehat{z}_{102} + 5$ **f**) $2\widehat{z}_{101} + 3a_{100} + 5$ **g**) $2\widehat{z}_{100} + 3a_{100} + 5$
h) $2z_{101} + 3a_{100}$ **i**) $2z_{100} + 3a_{100}$

Problem 6. Suppose we wish to find an ARIMA model for a monthly time series of length $n = 500$. The sample ACF of this time series is given below. The appearance of this sample ACF suggests that this time series _________.



a) has non-constant variance and may need a transformation

b) is non-stationary because it has a non-constant ACF

c) has a seasonal tendency and may require a model with a seasonal term

d) has been over-differenced

e) has a downward trend with a roughly constant slope

f)⋆ is non-stationary and may need differencing at lag 1

g) has a strong seasonal pattern and may need differencing at lag 12

Problem 7. A stationary ARMA(p, q) process $\{z_t\}$ may be written as

$$\tilde{z}_t = \sum_{k=0}^{\infty} \psi_k a_{t-k}$$

where the ψ-weights ψ_k may be found by expanding _________ as a series.

a) $\phi(B)\theta(B)$ **b**) $\frac{1}{\phi(B)}$ **c**) $\frac{1}{\theta(B)}$ **d**) $\frac{\theta(B)}{(1-B)^d\phi(B)}$

e) $\frac{(1-B)^d\theta(B)}{\phi(B)}$ **f**) $\frac{\phi(B)}{\theta(B)}$ **g**)⋆ $\frac{\theta(B)}{\phi(B)}$ **h**) $\frac{\phi(B)}{(1-B)^d\theta(B)}$

i) $\frac{(1-B)^d\phi(B)}{\theta(B)}$ **j**) $(1-B)^d\phi(B)\theta(B)$

Problem 8. Let B denote the backshift operator. The expression

$$(4B + 2B^3 + 3B^5)Z_t$$

means the same as _________

a) $4Z_{t+1} + 2Z_{t+3} + 3Z_{t+5}$
b)$\star$ $4Z_{t-1} + 2Z_{t-3} + 3Z_{t-5}$
c) $Z_{t-4} + 3Z_{t-2} + 5Z_{t-3}$
d) $Z_{t+4} + 3Z_{t+2} + 5Z_{t+3}$
e) $Z_{t-4} + Z_{t-2}^3 + Z_{t-3}^5$
f) $Z_{t+4} + Z_{t+2}^3 + Z_{t+3}^5$
g) $Z_{t-1}^4 + Z_{t-3}^2 + Z_{t-5}^3$

Problem 9. If $\{z_t\}$ is an ARIMA(p, d, q) process whose AR coefficients satisfy the stationarity conditions, then _________.

a) differencing z_t d times produces an overdifferenced series
b) integrating z_t d times produces an overdifferenced series
c) integrating z_t d times produces a stationary ARMA(p, q) process
d)$\star$ differencing z_t d times produces a stationary ARMA(p, q) process
e) the theoreticsl ACF of z_t decays rapidly to zero
f) the theoretical ACF of z_t has a cutoff after lag q
g) the theoretical PACF of z_t has a cutoff after lag p

Problem 10. If your goal is to explain a response series $\{y_t\}$ by a linear regression on two input series $\{x_{1,t}\}$ and $\{x_{2,t}\}$, and the regression errors $\{\varepsilon_t\}$ are serially correlated and follow a ARMA(p, q) process

$$\phi(B)\varepsilon_t = \theta(B)a_t\,,$$

then a good model for $\{y_t\}$ is given by _________ where a_t is a sequence of random shocks.

a) $\phi(B)y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \theta(B)a_t$
b) $\phi(B)y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \theta(B)a_t$
c) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + a_t$
d) $\phi(B)y_t = a_t + \theta(B)(\beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t})$
e) $\theta(B)y_t = a_t + \phi(B)(\beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t})$
f) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dfrac{\phi(B)}{\theta(B)} a_t$
g)$\star$ $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dfrac{\theta(B)}{\phi(B)} a_t$

Problem 11. Suppose you wish to apply the methods of ARIMA modeling to a time series. If you observe that the variability of the series increases systematically with the level, then what should you do?

- **a)** Use a seasonal model
- **b)** Try fitting a trend
- **c)**$\star$ Try a transformation
- **d)** Try differencing at lag 1
- **e)** Try differencing at the seasonal lag
- **f)** None of the above

Problem 12. Suppose $\{z_t\}$ is an AR(1) process:

$$z_t = C + \phi_1 z_{t-1} + a_t$$

and you have observed all the values z_t and a_t up to time n. What is the best forecast of z_{n+1}? (The best forecast of z_{n+1} is denoted $\widehat{z}_{n+1}$ or $\widehat{z}_n(1)$ in the lecture notes.)

a) C **b)**$\star$ $C + \phi_1 z_n$ **c)** $C + \phi_1 C + \phi_1^2 z_n$ **d)** $C + \widehat{a}_{n+1}$

e) $\mu_z + \phi_1 \widehat{z}_n$ **f)** $\mu_z \pm 1.96\sigma_z$ **g)** $C + \phi_1 z_n + a_{n+1}$ **h)** μ_z

Problem 13. Continuing the previous problem, if you have observed z_t and a_t up to time n, what is the best forecast of z_{n+2}? (This forecast is denoted $\widehat{z}_{n+2}$ or $\widehat{z}_n(2)$.)

a) $\mu_z \pm 1.96\sigma_z$ **b)** C **c)** $C + \phi_1 z_n$ **d)** $C + \phi_1 z_{n+1}$

e) $\mu_z + \phi_1 \hat{z}_{n+1}$ **f)**$\star$ $C + \phi_1 C + \phi_1^2 z_n$ **g)** $C + \phi_1 z_{n+1} + a_{n+2}$ **h)** μ_z

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Problem 14. The theoretical **Inverse ACF (IACF)** of an **AR**(p) process ...

- **a)**$\star$ is the same as the **ACF** of an **MA**(p) process.
- **b)** is the same as the **IACF** of an **MA**(q) process.
- **c)** is the same as the **PACF** of an **MA**(p) process.
- **d)** is the same as the **PACF** of an **AR**(p) process.
- **e)** is the same as the **ACF** of an **AR**(p) process.
- **f)** is the same as the **IACF** of an **MA**(p) process.

Problem 15. If a time series consists of a repeating seasonal pattern plus a linear trend, then seasonal differencing will ...

a) remove the linear trend but not the seasonal pattern

b)⋆ remove both the seasonal pattern and the linear trend

c) sometimes fail to remove the linear trend and second differencing is needed

d) sometimes fail to remove the seasonal pattern and second differencing is needed

e) remove the seasonal pattern but not the linear trend

Problem 16. If you are unable to find a good ARIMA(p, d, q) model for a non-stationary time series, one alternative is to _________

a) re-express the model in backshift notation

b) check that the model satisfies the stationarity conditions

c) use the Box-Jenkins approach

d)⋆ model the series as (Series) = (Trend) + (Stationary Process)

e) try differencing the series

f) try including additional AR or MA terms

Problem 17. You wish to predict a quantity X on the basis of information $\mathcal{I}$. If you will be forced to pay \$100 unless your prediction $\widehat{X}$ is within ε of X (where ε is small), then your best prediction is the _________ of the conditional distribution of X given $\mathcal{I}$.

a) minimum **b)** P-value **c)**⋆ mode **d)** variance

e) standard deviation **f)** mean **g)** median

Problem 18. If we form a new series $\{z_t\}$ by differencing the series x_t 3 times at lag 1 and then 4 times at lag 12, we may write _________.

a) $z_t = (1 - B^3)^1(1 - B^4)^{12}x_t$

b) $z_t = (1 - 3B)^1(1 - 4B)^{12}x_t$

c)⋆ $z_t = (1 - B)^3(1 - B^{12})^4x_t$

d) $z_t = (1 - 4B)^1(1 - 3B)^{12}x_t$

e) $z_t = (1 - 3B^1)(1 - 4B^{12})x_t$

f) $z_t = (1 - 3B^{12})(1 - 4B^1)x_t$

Problem 19. Suppose you have a monthly time series x_t with seasonal variation at lag 12, and you wish to find a good ARIMA$(p, d, q)(P, D, Q)_{12}$ model for x_t. After finding reasonable orders of differencing d and D, we usually determine plausible initial values for p, q, P, Q by studying the sample ACF, PACF, and IACF of the ________.

a)⋆ differenced series, and identifying p and q using the pattern along the early non-seasonal lags 1, 2, 3, ..., and P and Q from the pattern along the seasonal lags 12, 24, 36, ...

b) differenced series, and identifying p and q using the pattern along the seasonal lags 12, 24, 36, ..., and P and Q from the pattern along the early non-seasonal lags 1, 2, 3, ...

c) original series x_t, and identifying p and q using the pattern along the early non-seasonal lags 1, 2, 3, ..., and P and Q from the pattern along the seasonal lags 12, 24, 36, ...

d) original series x_t, and identifying p and q using the pattern along the seasonal lags 12, 24, 36, ..., and P and Q from the pattern along the early non-seasonal lags 1, 2, 3, ...

e) original series x_t, and then including all possible terms in the model and dropping the non-significant terms one by one

f) original series x_t, and then including all possible terms in the model and dropping the non-significant terms one by one

g) differenced series, and then including all possible terms in the model and dropping the non-significant terms one by one

Problem 20. $Ba_{t-2} =$ ________

a) a_{t+2} **b)** a_{t+1} **c)** a_t **d)** a_{t-2}

e)⋆ a_{t-3} **f)** a_{t-4} **g)** a_{t-5} **h)** a_{t-1}

Problem 21. $(1 + 2B^2)(1 + 3B^{12}) =$ ________

a) $1 + 6B^{24}$ **b)** $2 + 3B^2 + 4B^{12} + 5B^{14}$ **c)**⋆ $1 + 2B^2 + 3B^{12} + 6B^{14}$

d) $2 + 2B^2 + 3B^{12}$ **e)** $2 + 2B^2 + 3B^{12} + 6B^{24}$ **f)** $1 + 3B^2 + 4B^{12} + 5B^{24}$

Problem 22. This problem uses the graphs on the next two pages. These graphs give the time series plot, the sample ACF, and the sample PACF for a series z_t of length $n = 500$, and also similar plots for the differences at lag 1 (∇z_t) and for the second differences ($\nabla^2 z_t$). Using these plots, select a good ARIMA model for the series z_t.

a) ARIMA$(8, 2, 0)$ **b)**⋆ ARIMA$(0, 2, 2)$

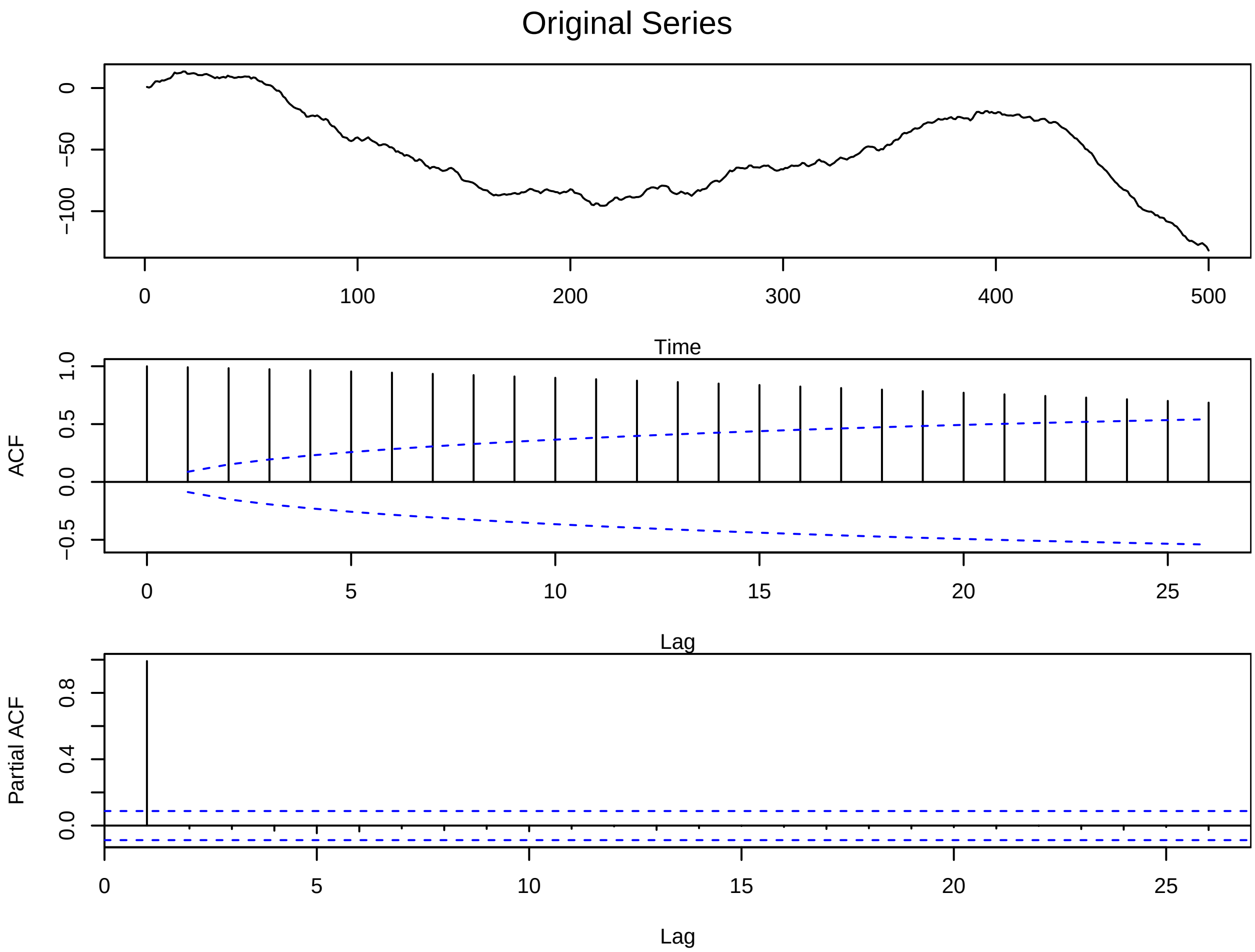
c) ARIMA$(8, 2, 2)$ **d)** ARIMA$(8, 2, 3)$

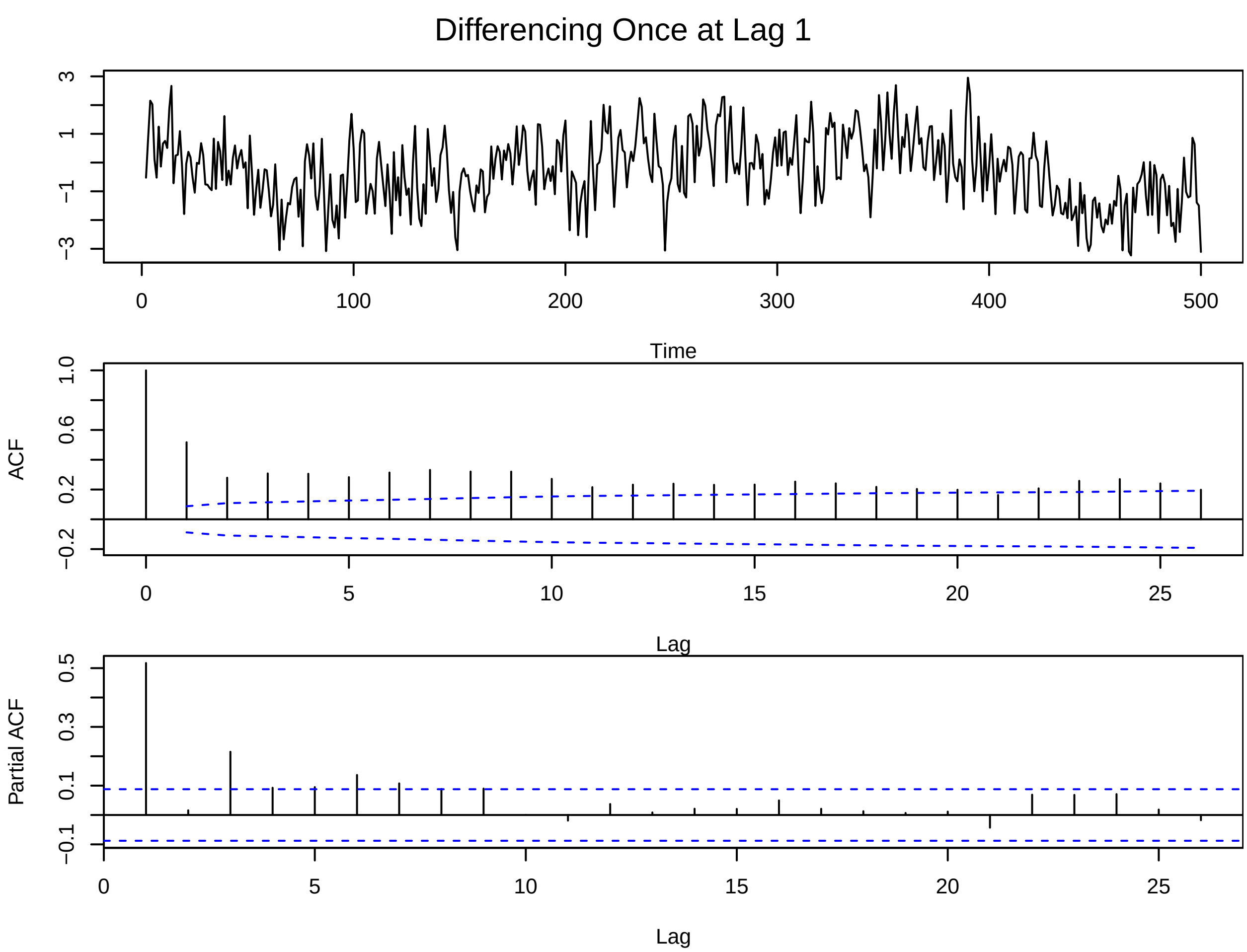
e) ARIMA$(1, 0, 0)$ **f)** ARIMA$(2, 0, 0)$

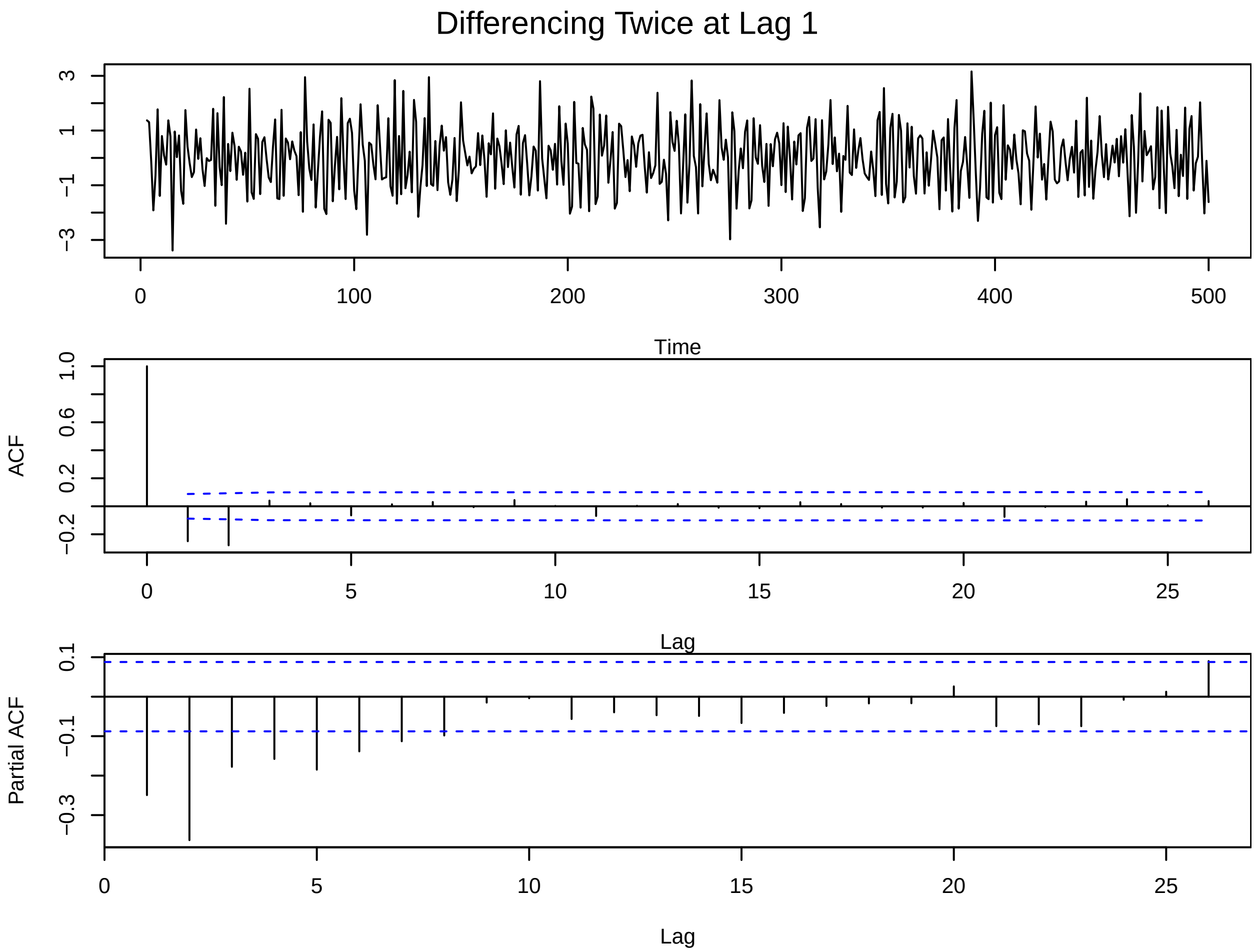
g) ARIMA$(1, 0, 26)$ **h)** ARIMA$(0, 0, 26)$

i) ARIMA$(2, 1, 0)$ **j)** ARIMA$(8, 1, 0)$

k) ARIMA$(8, 1, 1)$ **l)** ARIMA$(0, 1, 1)$


Original Series
Time
ACF
Lag
Partial ACF
Lag


Differencing Once at Lag 1
Time
ACF
Lag
Partial ACF
Lag



Problem 23. The backshift expression $B(C + \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1})$ is equal to ________

a)$\star$ $C + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
b) $C_{t-1} + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$
c) $C_{t+1} + \phi_2 z_t + a_{t+1} - \theta_2 a_t$
d) $C + \phi_2 z_{t-2} + a_{t-1} - \theta_2 a_{t-2}$
e) $0 + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
f) $0 + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$
g) $0 + \phi_2 z_{t+1} + a_{t+1} - \theta_2 a_{t+2}$

Problem 24. Suppose we start with the series x_t, then calculate $y_t = \nabla x_t$, and then $z_t = \nabla_s y_t$. We find that $z_t =$ ________.

a) $x_{t-1} + x_t - x_{t+s+1} + x_{t+s}$
b) $x_t + x_{t+1} - x_{t+s} - x_{t+s+1}$
c) $x_{t-1} - x_t - x_{t-s-1} + x_{t-s}$
d) $x_t + x_{t-1} - x_{t-s} - x_{t-s-1}$
e)$\star$ $x_t - x_{t-1} - x_{t-s} + x_{t-s-1}$
f) $x_t - x_{t+1} - x_{t+s} + x_{t+s+1}$

Problem 25. The process

$$z_t = \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}$$

can be written in backshift form as _________

a)⋆ $(1 - \phi_1 B)z_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)a_t$
b) $(1 - \phi_1 B)z_t = (1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)a_t$
c) $(1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)z_t = (1 - \phi_1 B)a_t$
d) $(1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)z_t = (1 + \phi_1 B)a_t$
e) $(1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)z_t = (1 - \phi_1 B)a_t$
f) $(1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)z_t = (1 + \phi_1 B)a_t$
g) $(1 + \phi_1 B)z_t = (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)a_t$
h) $(1 + \phi_1 B)z_t = (1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3)a_t$

Problem 26. A realization from a process with a **non**-stationary mean will usually _________.

a) have a sample PACF which decays very slowly to zero
b) have a sample IACF which decays very slowly to zero
c) require a log transformation
d) require a square root transformation
e) require a square transformation
f) require a reciprocal transformation
g) require a large AR order p in its model
h) require a large MA order q in its model
i) require large values of both p and q in its model
j)⋆ have a sample ACF which decays very slowly to zero

Problem 27. Which of the following statements is true for an ARIMA$(0,0,0)(0,0,3)_{12}$ process?

a) The PACF will be zero at lags 12, 24, 36.
b) The IACF will be zero at lags 12, 24, 36.
c) The first three MA coefficients will be zero.
d) The first three AR coefficients will be zero.
e) The ACF will decay rapidly along the early lags (1, 2, 3, ...).
f) The PACF will decay rapidly along the early lags (1, 2, 3, ...).
g)⋆ The ACF will be zero everywhere except at lags 12, 24, 36.
h) The PACF will be zero everywhere except at lags 12, 24, 36.
i) The ACF will be zero at lags 12, 24, 36.

The next two questions concern the following situation:

Suppose you are using `proc arima` to fit a ARIMA$(0,1,2)(1,1,0)_{12}$ model to the time series z_t. The SAS code for this will require an `identify` statement and an `estimate` statement.

Problem 28. Which one of the following is a correct way to write the required `identify` statement?

a)⋆ `identify var=z(1,12) ;`

b) `identify var=z(1,1) ;`

c) `identify var=z(12,12) ;`

d) `identify var=z(2) ;`

e) `identify var=z d=1 D=1 ;`

f) `identify var=z d=1 D=12 ;`

g) `identify var=z d=12 D=1 ;`

h) `identify var=z d=1 D=1 s=12 ;`

Problem 29. Which one of the following is a correct way to write the necessary `estimate` statement?

a) `estimate q=(2) p=(12) ;`

b) `estimate q=(2) P=(1) ;`

c) `estimate q=2 P=(1) ;`

d)⋆ `estimate q=(1,2) p=(12) ;`

e) `estimate p=(1,2) q=(12) ;`

f) `estimate p=(1,2)(12) ;`

g) `estimate p=(2)(12) ;`

h) `estimate q=(2)(12) ;`

Problem 30. Suppose you are using `proc arima` to fit a ARIMA$(3,0,0)(2,0,0)_{12}$ model to the time series z_t. Which one of the following is a correct way to write the necessary `estimate` statement?

a) `estimate p=3 q=(12,24) ;`

b) `estimate p=(1,2,3) P=(12,24);`

c) `estimate q=(3)(2) ;`

d) `estimate q=3 Q=2 ;`

e) `estimate p=(3) Q=(12,24) ;`

f)⋆ `estimate p=(1,2,3)(12,24) ;`

g) `estimate p=(3)(2) ;`

h) `estimate p=3 P=2 ;`