TEST #2 STA 4853 April 22, 2021

Name:

Please read the following directions. DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are **35** multiple choice questions.
- Select the **single best** answer for each multiple choice question.
- For each question, type the **single lower case letter** of the correct response into the "Fill in the blank" box in Canvas.
- There is no penalty for guessing.
- The exam has **14** pages.
- Each question is worth equal credit.
- The exam is closed book and closed notes. NO books, notes, computers, or internet resources are allowed. No communication with other humans about the exam is allowed during the exam.
- You must attend the Zoom meeting and remain unmuted with your video on during the entire exam. You must remain visible on Zoom during the entire exam.

Problem 1. The plots below give the Theoretical ACF, PACF, and IACF of a seasonal ARIMA process. What is this process?



Problem 2. If a process has _____, then differencing one or more times may produce a stationary process.

- a) a nonstationary variance
 b) a nonstationary ACF
 c) a nonstationary mean
 d) non-significant terms
 e) a very large AIC
 f) a very large SBC
- ${\bf g})$ variance which increases with the level ${\bf h})\,$ a slowly decaying Inverse ACF (IACF)

The time series plots below show the series Z_t and its first differences ∇Z_t and Problem 3. second differences $\nabla^2 Z_t$. Based on these plots, which of the following statements is true?

- **a**) The variability of this series increases with the level so that a transformation should be tried.
- **b**) Differencing and then transforming this series will make it stationary.
- c) Differencing twice and then transforming this series will make it stationary.
- d) A combination of ordinary and seasonal differencing will make this series stationary.
- e) Seasonal differencing sufficiently many times will make this series stationary.
- f) The variability of this series increases with time so that a transformation is not likely to help.
- g) The variability of this series increases with the level so that differencing sufficiently many times will make it stationary.



Time Series Plot of Z

Problem 4. Suppose you are modeling a series which requires a transformation, but you do not realize this and fit an ARIMA model to the raw data instead. Which of the following items will be most useful in alerting you that a transformation is needed?

- a) The residual ACF
- **b**) The residual PACF
- c) The White Noise Probability plot
- d) The AIC
- e) The SBC
- f) The time series plot of the residuals
- g) The normal probability plot (Q-Q Plot) of the residuals
- h) The residuals versus forecasts plot

Problem 5. When we fit an ARIMA model using SAS PROC ARIMA, the output includes a table with an estimate, standard error, *t*-value, and *p*-value for every parameter in the model. Suppose θ_2 is a parameter in our model. If the *p*-value for θ_2 is small (say, less than 0.05), then we usually _____

- **a**) drop θ_2 from the model
- **b**) keep θ_2 in the model
- c) try adding other parameters to the model
- d) try removing other parameters from the model
- e) try adding a seasonal term to the model
- f) use MINIC to choose a better model
- **g**) use AIC to choose a better model
- **h**) try additional differencing
- i) try a transformation of the time series

Problem 6. The IACF (Inverse Autocorrelation Function) of an ARIMA $(0, 0, 0)(0, 0, 1)_s$ process _____

- **a**) decays exponentially along the seasonal lags $0, s, 2s, 3s, \ldots$ and is zero otherwise
- **b**) decays exponentially along the lags $0, 1, 2, 3, \ldots$
- \mathbf{c}) has a cutoff to zero after lag 1
- **d**) is non-zero only at lags 0, s, and 2s
- **e**) is non-zero only at lags 0, 1 and s
- **f**) is non-zero only at lags 0, 1, s 1, s and s + 1
- \mathbf{g}) is non-zero only at lags 0 and s
- **h**) has a cutoff to zero after lag s

Problem 7. The plots below give the Theoretical ACF, PACF, and IACF of a seasonal ARIMA process. What is this process?

- **a**) ARIMA $(2, 0, 0)(1, 0, 0)_{12}$
- **c**) ARIMA $(0, 0, 2)(1, 0, 0)_{12}$
- e) $ARIMA(1,0,0)(2,0,0)_{12}$
- **g**) ARIMA $(0, 0, 1)(2, 0, 0)_{12}$

- **b**) ARIMA $(2, 0, 0)(0, 0, 1)_{12}$
- d) $ARIMA(0, 0, 2)(0, 0, 1)_{12}$
- f) $ARIMA(1,0,0)(0,0,2)_{12}$
- **h**) ARIMA $(0, 0, 1)(0, 0, 2)_{12}$





Problem 9. If a series $\{y_t\}$ is created by differencing the series $\{z_t\}$ d times at lag 1 and then D times at lag s, we may write $y_t =$ _____.

a)
$$(1-B)^d (1-B^s)^D z_t$$

b) $(1-B)^s (1-B^d)^D z_t$
c) $(1-B^s)(1-B^D)^d z_t$
d) $(1-B^D)(1-B^s)^d z_t$
e) $(1-B^d)(1-B^D)^s z_t$
f) $(1-B)^D (1-B^s)^d z_t$

Problem 10. Suppose $\{z_t\}$ is a realization of an MA(2) process with known parameter values. We observe all the values z_t and a_t (the random shocks) up to time 100, and use this information to compute \hat{z}_{101} , the best forecast for z_{101} . What is the value of Var $(z_{101} - \hat{z}_{101})$, the variance of the prediction error?

a)
$$\mu_z$$
 b) $1.96\sigma_z$ c) σ_a^2
d) $C - \theta_1 a_{100} - \theta_2 a_{99}$ e) σ_z^2 f) $1.96\sigma_a$
g) $C - \theta_2 a_{100}$ h) $\sigma_a^2(\psi_0^2 + \psi_1^2 + \psi_2^2)$ i) $\sigma_a^2(\psi_0\psi_1 + \psi_1\psi_2)$

Problem 11. The AR(1) process in mean-centered form

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + a_t$$

may be written in backshift notation as

$$\phi(B)\tilde{z}_t = a_t$$

where $\phi(B)$ is the AR polynomial. This AR polynomial has a single zero (or root) which is equal to _____.

a)
$$\frac{-1}{\phi_1 B}$$
 b) ϕ_1 **c**) $\frac{1}{B}$ **d**) 1 **e**) B

f)
$$\phi_1 B$$
 g) $\frac{1}{\phi_1 B}$ **h**) $-\phi_1 B$ **i**) $\frac{-1}{B}$ **j**) $\frac{1}{\phi_1}$

Problem 12. The solutions of the equation

$$1 - \phi_1 B - \phi_2 B^2 = 0$$

are strictly outside the unit circle if and only if _____.

 $\begin{array}{l} \mathbf{a}) \ |\phi_2| < 1, \ \phi_2 + \phi_1 < 1, \ \phi_2 - \phi_1 < 1 \\ \mathbf{b}) \ |\phi_2| > 1, \ \phi_2 + \phi_1 > 1, \ \phi_2 - \phi_1 > 1 \\ \mathbf{c}) \ \phi_2 + \phi_1 > 1, \ \phi_2 - \phi_1 > 1 \\ \mathbf{d}) \ \phi_1 + \phi_2 \ge 1 \\ \mathbf{e}) \ |\phi_1| < 1 \\ \mathbf{f}) \ |\phi_2| < 1 \end{array}$

Problem 13. The process

$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

can be written in backshift form as

$$\phi(B)z_t = C + \theta(B)a_t$$

where _____.

a) $\phi(B) = 1 - \phi_1 B - \phi_2 B^2$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3$ **b**) $\phi(B) = 1 + \phi_1 B + \phi_2 B^2$ and $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + \theta_3 B^3$ **c**) $\phi(B) = 1 - \phi_1 B - 2\phi_2 B - 3\phi_3 B$ and $\theta(B) = 1 - \theta_1 B - 2\theta_2 B$ **d**) $\phi(B) = 1 + \phi_1 B + 2\phi_2 B + 3\phi_3 B$ and $\theta(B) = 1 + \theta_1 B + 2\theta_2 B$ **e**) $\phi(B) = 1 - \phi_1 B - 2\phi_2 B$ and $\theta(B) = 1 - \theta_1 B - 2\theta_2 B - 3\theta_3 B$ **f**) $\phi(B) = 1 + \phi_1 B + 2\phi_2 B$ and $\theta(B) = 1 - \theta_1 B - 2\theta_2 B - 3\theta_3 B$ **g**) $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3$ and $\theta(B) = 1 - \theta_1 B - \theta_2 B^2$ **h**) $\phi(B) = 1 + \phi_1 B + \phi_2 B^2 + \phi_3 B^3$ and $\theta(B) = 1 + \theta_1 B + \theta_2 B^2$

Problem 14. Suppose we have a series z_1, z_2, \ldots, z_n . If we difference this series d times, we **always** end up with a series _____.

(Here "differencing" is ordinary differencing, and d is any positive integer less than n.)

- **a**) which is stationary
- **b**) which is non-stationary
- c) with a slowly decaying IACF
- d) with a slowly decaying ACF
- e) with a seasonal pattern
- \mathbf{f}) with d missing values at the beginning
- \mathbf{g}) with d missing values at the end
- **h**) with no seasonal pattern
- i) with d roots strictly outside the unit circle
- \mathbf{j}) with d roots strictly inside the unit circle

Problem 15. A stationary ARMA(p,q) process with mean zero can be expressed as _____

$$\begin{array}{ll} \mathbf{a}) & (1-B)\phi(B)z_t = \theta(B)a_t & \mathbf{b}) & (1-B)\theta(B)z_t = \phi(B)a_t \\ \mathbf{c}) & \frac{\theta(B)}{(1-B)\phi(B)}a_t & \mathbf{d}) & \frac{\phi(B)}{(1-B)\theta(B)}a_t & \mathbf{e}) & \frac{(1-B)\phi(B)}{\theta(B)}a_t \\ \mathbf{f}) & \frac{\phi(B)}{\theta(B)}a_t & \mathbf{g}) & \frac{\theta(B)}{\phi(B)}a_t & \mathbf{h}) & \frac{1}{\psi(B)}a_t \end{array}$$

Problem 16. Suppose you are trying to find a good model for a time series. If more than one model has reasonable residual diagnostics (or if all the models have problems), then one good strategy is to choose the model _____.

- **a**) with the largest White Noise Probabilities
- **b**) with the smallest White Noise Probabilities
- c) with the largest *p*-values for the parameter estimates
- \mathbf{d}) with the smallest *p*-values for the parameter estimates
- e) with the largest value of the Variance Estimate $\hat{\sigma}_a^2$
- ${\bf f})$ with the smallest value of the Constant Estimate.
- **g**) with the smallest AIC or SBC
- **h**) with the largest AIC or SBC

Problem 17. However, if you are employing the strategy which is the correct answer to the previous problem, it is important to only use this strategy to compare _____.

- **a**) models which have the same number of seasonal terms (if any are used)
- **b**) models of the same type; all are multiplicative models or all are non-multiplicative models
- c) models which involve the same transformation (if one is used) and the same degree of differencing
- d) models which have the same number of parameters
- e) autoregressive (AR) models
- f) moving average (MA) models
- g) mixed models
- **h**) stationary models
- i) non-stationary models

Problem 18. If $\{z_t\}$ is an ARIMA(2, 2, 2) process, then $\nabla^2 z_t$ is an _____ process.

a) ARIMA $(4, 2, 2)$	b) ARIMA $(2, 2, 0)$	c) $ARIMA(2,2,4)$
$\mathbf{d}) \ \mathrm{ARIMA}(2,4,4)$	e) ARIMA $(4, 4, 2)$	f) ARIMA $(4, 2, 4)$
$\mathbf{g}) \text{ ARIMA}(2,4,2)$	h) ARIMA $(2, 0, 2)$	i) ARIMA $(0, 2, 2)$

Problem 19. Suppose you are trying to predict X given the information \mathcal{I} . If the conditional distribution of X given \mathcal{I} is Normal with mean 200 and variance 100, then a 95% confidence interval for X is _____.

$\mathbf{a})$	(4, 396)	b) $(172.3, 227.7)$	$\mathbf{c}) \ (198.04, 201.96)$	$\mathbf{d}) \ (180.4, 219.6)$
$\mathbf{e})$	(121.6, 278.4)	f) $(102, 298)$	\mathbf{g}) (160.8, 239.2)	h) $(141.2, 258.8)$

Problem 20. When applying regression to time series:

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dots + x_{k,t} + \varepsilon_t ,$$

the errors ε_t are often found to be serially correlated and can be modeled as an ARMA(p,q) process. An initial choice for the ARMA(p,q) process for the errors can be made by _____.

- a) trying a variety of transformations and examining the plot of residuals versus predicted values
- **b**) trying different orders of differencing for y_t and examining the time series plot and ACF to choose the best one
- c) fitting an ordinary multiple regression model and studying the ACF and PACF of the residuals
- **d**) studying the ACF and PACF of the series y_t
- e) applying the MINIC option of the IDENTIFY statement to the series y_t
- **f**) fitting a sequence of autoregressive models to y_t and choosing the one with the best AIC value

Problem 21. The theoretical IACF (Inverse ACF) of the process

$$z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + \phi_3 z_{t-3} + a_t - \theta_1 a_{t-1}$$

will be the same as the theoretical ACF of the process _____.

$$\begin{array}{l} \mathbf{a}) \ z_t = C + \theta_1 z_{t-1} + \theta_2 z_{t-2} + \theta_3 z_{t-3} + a_t - \phi_1 a_{t-1} \\ \mathbf{b}) \ z_t = C + \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} \\ \mathbf{c}) \ z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \\ \mathbf{d}) \ z_t = C + \theta_1 z_{t-1} + \theta_2 z_{t-2} + a_t - \phi_1 a_{t-1} - \phi_2 a_{t-2} \\ \mathbf{e}) \ z_t = C + \theta_1 z_{t-1} + a_t - \phi_1 a_{t-1} - \phi_2 a_{t-2} - \phi_3 a_{t-3} \\ \end{array}$$

The next two questions concern the situation below:

Suppose you want to fit an ARIMA $(1, 1, 2)(0, 1, 1)_{12}$ model to the time series $\{z_t\}$ using SAS PROC ARIMA.

Problem 22. Your IDENTIFY statement should contain var =

a) z(1)	b) z(12)	$\mathbf{c})$ z	d) z(1,1,12)
e) z(1,1)	f) z(12,12)	g) z(1,12)	h) z(2)

Problem 23. Your ESTIMATE statement should contain _____.

a) p=(12) q=(2)(1)	b) p=(12) q=(2)(1)	c) p=(1) q=(1,2,12)
d) p=(1) q=(2)(12)	e) p=(1,0) q=(2,1)	f) p=(1) q=(1,2)(12)
g) p=(1,1,2) q=(0,1,1)	h) p=(1,1,2) q=(0,12,12)	i) p=(2)(12) q=(1)

Problem 24. Consider the MA(3) process given below:

$$z_t = C + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}$$

Suppose we know all the values of z_t and a_t up to time n. Call this set of information \mathcal{I}_n . What is \hat{z}_{n+2} , the best forecast of z_{n+2} given \mathcal{I}_n ?

a)
$$C$$
 b) $C - \theta_1 a_n - \theta_2 a_{n-1} - \theta_3 a_{n-2}$ c) $C - \theta_3 a_n$
d) $C - \theta_1 a_{n+1} - \theta_2 a_n - \theta_3 a_{n-1}$ e) $C - \theta_1 a_{n+1} - \theta_2 a_n$ f) $C - \theta_3 a_{n+1}$
g) $C - \theta_2 a_n$ h) $C - \theta_1 a_n - \theta_2 a_{n-1}$ i) $C - \theta_2 a_n - \theta_3 a_{n-1}$

Problem 25. Suppose the time series z_t has the form ..., 1, 3, 5, 7, 1, 3, 5, 7, 1, 3, 5, 7, ... and continues repeating the same pattern forever into both the future and the past. What is $\nabla_4 z_t$?

Problem 26. The model

$$(1 - \phi_1 B - \phi_2 B^2)(1 - \Phi_1 B^{12})z_t = C + a_t$$

is (after expanding the product, eliminating the backshift notation, and re-arranging the terms) equivalent to a special case of a ______ process.

Problem 27. Suppose $\{z_t\}$ is a realization of a **known** ARIMA(p, d, q) process; we know the orders p, d, q and the values of all parameters. Suppose also that we observe **all** the values z_t and a_t (the random shocks) up to time n. Call this set of information \mathcal{I}_n . Given \mathcal{I}_n , the forecast of z_{n+k} is $\hat{z}_{n+k} = E(z_{n+k} | \mathcal{I}_n)$. Which of the following statements is always true?

a) $E(z_t \mathcal{I}_n) = 0$ for $t \ge n$	b) $E(z_t \mathcal{I}_n) = \mu_z$ for $t \ge n$
c) $E(z_t \mathcal{I}_n) = a_t$ for $t \ge n$	d) $E(z_t \mathcal{I}_n) = z_{\mathcal{H}}$ for $t \ge n$
e) $E(z_t \mathcal{I}_n) = \mu_z$ for $t \le n$	f) $E(z_t \mathcal{I}_n) = a_t$ for $t \le n$
g) $E(z_t \mathcal{I}_n) = z \boldsymbol{\eta}$ for $t \leq n$	h) $E(z_t \mathcal{I}_n) = z_t$ for $t \ge n$
i) $E(z_t \mathcal{I}_n) = z_t$ for $t \leq n$	j) $E(z_t \mathcal{I}_n) = 0$ for $t \le n$

Problem 28. Suppose you are trying to find a good time series model for a series x_t . You first decide to use a log transformation $y_t = \log(x_t)$, and then you decide to difference the transformed series two times, obtaining the series $z_t = \nabla^2 y_t$. You then observe that the IACF of z_t is slowly decaying. This suggests _____.

- **a**) differencing three times
- **b**) differencing only once
- c) not transforming the series at all
- d) trying a different transformation
- e) dropping the constant from the model
- \mathbf{f}) adding a seasonal term

Problem 29. The model

 $(1 - .2B^{12})(1 - .4B - .2B^2 - .3B^3)z_t = 5.0 + (1 - .5B^{12} + .2B^{24})(1 + .2B + .3B^2 - .2B^3 + .1B^4)a_t$ is a ______.

a) ARIMA $(4, 0, 3)(2, 0, 1)_{12}$	b) ARIMA $(1, 0, 2)(3, 0, 4)_{12}$	c) ARIMA $(4, 0, 2)(3, 0, 1)_{12}$
d) ARIMA $(2, 0, 1)(3, 0, 4)_{12}$	e) ARIMA $(3, 0, 4)(1, 0, 2)_{12}$	f) ARIMA $(3, 0, 1)(4, 0, 2)_{12}$
g) ARIMA $(1, 0, 4)(3, 0, 2)_{12}$	h) ARIMA $(1, 0, 2)(4, 0, 3)_{12}$	i) ARIMA $(2, 0, 4)(1, 0, 3)_{12}$

Problem 30. An AR(2)₁₂ or ARIMA(2, 0, 0)₁₂ is a purely seasonal model. It may be written as _____.

$$\begin{aligned} \mathbf{a}) \ z_t &= C + \Phi_1 z_{t-12} + \Phi_2 z_{t-13} + a_{t-12} \\ \mathbf{b}) \ z_t &= C + \Phi_1 z_{t-12} + \Phi_2 z_{t-24} + a_t \\ \mathbf{c}) \ z_t &= C + \Phi_1 z_{t-1} + \Phi_2 z_{t-12} + a_t \\ \mathbf{e}) \ z_t &= C + \Phi_1 z_{t-12} + \Phi_2 z_{t-24} + a_{t-24} \\ \end{aligned}$$

$$\begin{aligned} \mathbf{b}) \ z_t &= C + \Phi_1 z_{t-12} + \Phi_2 z_{t-13} + a_t \\ \mathbf{d}) \ z_t &= C + \Phi_1 z_{t-12} + \Phi_2 z_{t-13} + a_t \\ \mathbf{f}) \ z_t &= C + \Phi_1 z_{t-1} + \Phi_2 z_{t-12} + a_{t-12} \\ \end{aligned}$$

Problem 31. If we difference the series z_t at lag 1 and then at lag s, we get the series $w_t = \nabla_s \nabla z_t$. This may also be written as _____.

 $\begin{aligned} \mathbf{a}) \ w_t &= z_t - z_{t-1} - z_{t-s} + z_{t-s-1} \\ \mathbf{b}) \ w_t &= z_t - z_{t+1} - z_{t+s} + z_{t+s+1} \\ \mathbf{c}) \ w_t &= (z_{t+s} - z_t)(z_{t+1} - z_t) \\ \mathbf{e}) \ w_t &= z_t - z_{t+1} + z_{t+s} - z_{t+s+1} \\ \mathbf{g}) \ w_t &= (z_t - z_{t-s})(z_t - z_{t-1}) \\ \end{aligned}$ $\begin{aligned} \mathbf{b}) \ w_t &= z_t - z_{t+1} - z_{t+s} + z_{t+s+1} \\ \mathbf{d}) \ w_t &= (z_{t+s} - z_{t-s})(z_{t+1} - z_{t-1}) \\ \mathbf{f}) \ w_t &= -z_t + z_{t-1} + z_{t-s} - z_{t-s+1} \\ \mathbf{g}) \ w_t &= (z_t - z_{t-s})(z_t - z_{t-1}) \\ \end{aligned}$

Problem 32. By very carefully reading the table given below, you can determine that the ESTIMATE statement which produced this table contained ______

a) p=(1,2,12)	b) q=(1)(2,12)	c) q=(1,2,12)	d) q=(1,2)(12)
e) q=(2)(12)	f) p=(1)(2,12)	g) p=(1,2)(12)	h) p=(2)(12)

Maximum Likelihood Estimation						
	Standard Approx					
Parameter	Estimate	Error	t Value	$\mathbf{Pr} > t $	Lag	
MU	0.28585	0.04178	6.84	<.0001	0	
AR1,1	0.34024	0.11096	3.07	0.0022	1	
AR1,2	0.31666	0.11044	2.87	0.0041	2	
AR1,3	-0.18476	0.09540	-1.94	0.0528	12	

Problem 33. Another example: By very carefully reading the table given below, you can determine that the ESTIMATE statement which produced this table contained _____.

a) q=(1)(2,12)	b) q=(1,2)(12)	c) q=(1,2,12)	d) q=(2)(12)
e) p=(1)(2,12)	f) p=(1,2,12)	g) p=(1,2)(12)	h) p=(2)(12)

Maximum Likelihood Estimation						
	Standard Approx					
Parameter	Estimate	Error	t Value	$\mathbf{Pr} > t $	Lag	
MU	0.27419	0.02149	12.76	<.0001	0	
MA1,1	-0.33681	0.09313	-3.62	0.0003	1	
MA1,2	-0.57181	0.10020	-5.71	<.0001	2	
MA2,1	0.54778	0.14127	3.88	0.0001	12	

Problem 34. For a stationary time series with seasonality s, if the values of the ACF at lags s + 1 and s - 1 differ greatly, the correct model for the time series might be _____.

- a) ARIMA $(0, 0, 0), (0, 0, 1)_s$
- **b**) ARIMA $(0, 0, 0), (0, 0, 2)_s$
- c) ARIMA $(0, 0, 0), (1, 0, 0)_s$
- **d**) ARIMA $(0, 0, 0), (2, 0, 0)_s$
- e) ARIMA $(0, 0, 0), (1, 0, 1)_s$
- **f**) ARIMA $(0, 0, 1), (0, 0, 1)_s$
- **g**) ARIMA $(0, 0, 1), (0, 1, 1)_s$
- **h**) ARIMA $(0, 1, 1), (0, 0, 1)_s$
- i) a model with random shocks which are not normally distributed
- \mathbf{j}) a non-multiplicative seasonal model

Problem 35. An AR(1) process $\{z_t\}$ has $\phi_1 = 0.8$ and mean $\mu_z = 0$. We observe this process from time 1 until time 100:

$$z_1, z_2, z_3, \ldots, z_{99}, z_{100}$$

and then compute the best forecasts of this process for times 101 to 150:

$$\widehat{z}_{101}, \widehat{z}_{102}, \widehat{z}_{103}, \dots, \widehat{z}_{149}, \widehat{z}_{150}$$

One of the six time series plots given on the next page displays the observed values (in black) followed by the correct forecasts (in red). Which plot gives the correct forecasts? (The observed values in black are the same in all six plots; only the forecasts in red differ.)

Note: Number the plots 1 to 6 from top to bottom.

a) 1 **b**) 2 **c**) 3 **d**) 4 **e**) 5 **f**) 6

