

TEST #2

STA 4853

Name: _____

April 28, 2022

Please read the following directions.

DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- There are **35** multiple choice questions.
- Circle the **single best** answer for each multiple choice question. Your choice should be made clearly.
- Always **circle the correct response**. (Sometimes the question has an empty blank or a box, but this is **NOT** where the answer goes.)
- There is no penalty for guessing.
- The exam has **14** pages.
- Each question is worth equal credit.

Problem 1. The ACF of the process

$$(1 - 0.7B)z_t = (1 - 0.7B)a_t$$

is exactly the same as the ACF of which of the following processes?

- a)** $(1 + 0.7B - 0.7B^2)z_t = a_t$ **b)** $z_t = (1 - 0.7B + 0.7B^2)a_t$ **c)** $(1 + 0.7B)z_t = a_t$
d) $(1 + 0.7B)z_t = (1 - 0.7B)a_t$ **e)** $(1 - 0.7B)z_t = a_t$ **f)★** $z_t = a_t$
g) $(1 - 0.7B)z_t = (1 + 0.7B)a_t$ **h)** $z_t = (1 - 0.7B)a_t$ **i)** $(1 - 0.7B^2)z_t = a_t$
j) $z_t = (1 + 0.7B - 0.7B^2)a_t$ **k)** $(1 - 0.7B + 0.7B^2)z_t = a_t$ **l)** $z_t = (1 + 0.7B)a_t$
-

The next two problems involve the following situation:

Suppose $\{z_t\}$ is a stationary ARMA(3,1) process and let $w_t = \nabla z_t = (1 - B)z_t$.

Problem 2. $\{w_t\}$ is a _____ process.

- a)★** ARMA(3,2) **b)** ARMA(1,2) **c)** ARMA(4,2) **d)** ARMA(4,1)
e) AR(2) **f)** MA(2) **g)** AR(4) **h)** MA(4)
i) ARIMA(3,1,1) **j)** ARIMA(3,1,2) **k)** ARIMA(2,1,1) **l)** ARIMA(1,1,2)

Problem 3. $\{w_t\}$ is _____.

- a)** invertible **b)** seasonal **c)★** non-invertible **d)** non-stationary
e) under-differenced **f)** cross-correlated **g)** deterministic **h)** periodic
-

Problem 4. Which of the following is an example of a regression model with ARMA(2,1) errors?

- a)★** $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \frac{1 - \theta_1 B}{1 - \phi_1 B - \phi_2 B^2} a_t$
b) $y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \frac{1 - \phi_1 B - \phi_2 B^2}{1 - \theta_1 B} a_t$
c) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \frac{1 - \theta_1 B - \theta_2 B^2}{1 - \phi_1 B} a_t$
d) $y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \frac{1 - \theta_1 B - \theta_2 B^2}{1 - \phi_1 B} a_t$
e) $y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \frac{1 - \theta_1 B}{1 - \phi_1 B - \phi_2 B^2} a_t$
f) $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \frac{1 - \phi_1 B - \phi_2 B^2}{1 - \theta_1 B} a_t$

Problem 5. For a stationary ARMA(p, q) process, the ψ -weights ψ_k always satisfy _____.

- a) $\psi_k = 0$ for $k > q$
- b) $\psi_k = 0$ for $k > p$
- c) $\psi_k = 0$ for $k > p + q$
- d) $\sigma_a^2 = 1 + \psi_1^2 + \psi_2^2 + \cdots + \psi_{k-1}^2$
- e) $\psi_k = \theta_k / \phi_k$
- f) $\psi_k = \phi_k / \theta_k$
- g)★ $\psi_k \rightarrow 0$ as $k \rightarrow \infty$
- h) $\psi_k \rightarrow \infty$ as $k \rightarrow \infty$

Problem 6. If one observes an AR(1) process $\{z_t\}$ up to time n and uses this information to compute a forecast of z_{n+k} , the forecast is equal to $\hat{z}_{n+k} =$ _____.

- a) $\phi_1 \hat{a}_{n-1} + \phi_1^2 \hat{a}_{n-2} + \cdots + \phi_1^k \hat{a}_{n-k}$
- b) $C + \phi_1 \hat{a}_{n-1} + \phi_1^2 \hat{a}_{n-2} + \cdots + \phi_1^k \hat{a}_{n-k}$
- c)★ $C + \phi_1 C + \phi_1^2 C + \cdots + \phi_1^{k-1} C + \phi_1^k z_n$
- d) $C + \phi_1 z_{n-1} + \phi_1^2 z_{n-2} + \cdots + \phi_1^k z_{n-k}$
- e) $\hat{a}_n + \phi_1 \hat{a}_{n-1} + \phi_1^2 \hat{a}_{n-2} + \cdots + \phi_1^k \hat{a}_{n-k}$
- f) $C + \phi_1 C + \phi_1^2 C + \cdots + \phi_1^k C + \phi_1^{k+1} z_n$
- g) $C + \phi_1 C + \phi_1^2 C + \cdots + \phi_1^{k-1} C$
- h) $C + \phi_1 C + \phi_1^2 C + \cdots + \phi_1^k C$
- i) $z_n + \phi_1 z_{n-1} + \phi_1^2 z_{n-2} + \cdots + \phi_1^k z_{n-k}$
- j) $\phi_1 z_{n-1} + \phi_1^2 z_{n-2} + \cdots + \phi_1^k z_{n-k}$

Problem 7. Suppose you have a time series z_1, z_2, \dots, z_n , and you use SAS PROC ARIMA to fit an ARIMA(2,1,1) model to this data and specify the NOCONSTANT option in the ESTIMATE statement. The long range forecasts (\hat{z}_{n+k} as $k \rightarrow \infty$) from this model will _____.

- a) converge to zero
- b) converge to the estimated mean $\hat{\mu}_z$ of the process
- c) decay with sinusoidal oscillation
- d) converge to a straight line with a nonzero slope
- e) converge to a repetitive pattern with period S equal to the seasonality of the series
- f) converge to a repetitive pattern added to a straight line with nonzero slope
- g)★ converge to a value which depends mainly on the last few observed values of the series

Problem 8. Suppose $\{z_t\}$ is an ARIMA(0,2,3) process. Then $\nabla^2 z_t$ is an _____ process.

- a)★ MA(3)
- b) MA(1)
- c) ARIMA(2,2,3)
- d) under-differenced
- e) MA(5)
- f) ARIMA(2,1,3)
- g) ARMA(2,3)
- h) over-differenced

Problem 9. Suppose you have a time series z_1, z_2, \dots, z_n , and you use SAS PROC ARIMA to fit an $\text{ARIMA}(2, 0, 0)(0, 1, 1)_{12}$ model to this data and you retain the constant in your model (that is, you do **NOT** use the NOCONSTANT option). The long range forecasts (\hat{z}_{n+k} as $k \rightarrow \infty$) from this model will _____.

- a) converge to a value which depends mainly on the last few observed values of the series
- b) converge to zero
- c) converge to the estimated mean $\hat{m}u_z$ of the process
- d) converge to a straight line with a nonzero slope
- e) converge to a repetitive pattern with period 12
- f)★ converge to a repetitive pattern with period 12 added to a straight line with nonzero slope
- g) decay with sinusoidal oscillation

Problem 10. Based on some available financial information \mathcal{I} , you are asked to forecast the random quantity

X = the closing value of the Dow Jones Index one week from today .

This value is currently in the neighborhood of 34,000 points and can vary by as much as a few hundred points in a day. Suppose you will be paid \$10,000,000 if your forecast is within one point of the correct value. If the conditional distribution of X given \mathcal{I} is skewed, then the best forecast for X is the _____ of the conditional distribution.

- | | | |
|-------------|-----------------------|------------------------|
| a) variance | b) standard deviation | c) interquartile range |
| d)★ mode | e) mean | f) median |

Problem 11. If the variability of a series z_t increases systematically with the level of the series so that the variability is proportional to the level of the series, then it is frequently useful to _____.

- a) use a square-root transform and model the series $y_t = \sqrt{z_t}$ instead of z_t
- b)★ use a log transform and model the series $y_t = \log(z_t)$ instead of z_t
- c) difference the series z_t at lag 1
- d) difference the series z_t at the seasonal lag
- e) model the trend using linear and quadratic functions
- f) model the trend using sine and cosine functions
- g) mean center the series and use \tilde{z}_t instead of z_t

The next three questions ask you to identify three different stationary processes (with seasonality $S = 12$) from plots of their **theoretical** ACF, PACF, and IACF which are given up to lag 48. These plots are given immediately after the next three questions.

Note: In these plots, the PACF does NOT have a spike at lag zero, whereas the ACF and IACF do have a spike at lag zero.

Problem 12. Process #1 is a _____.

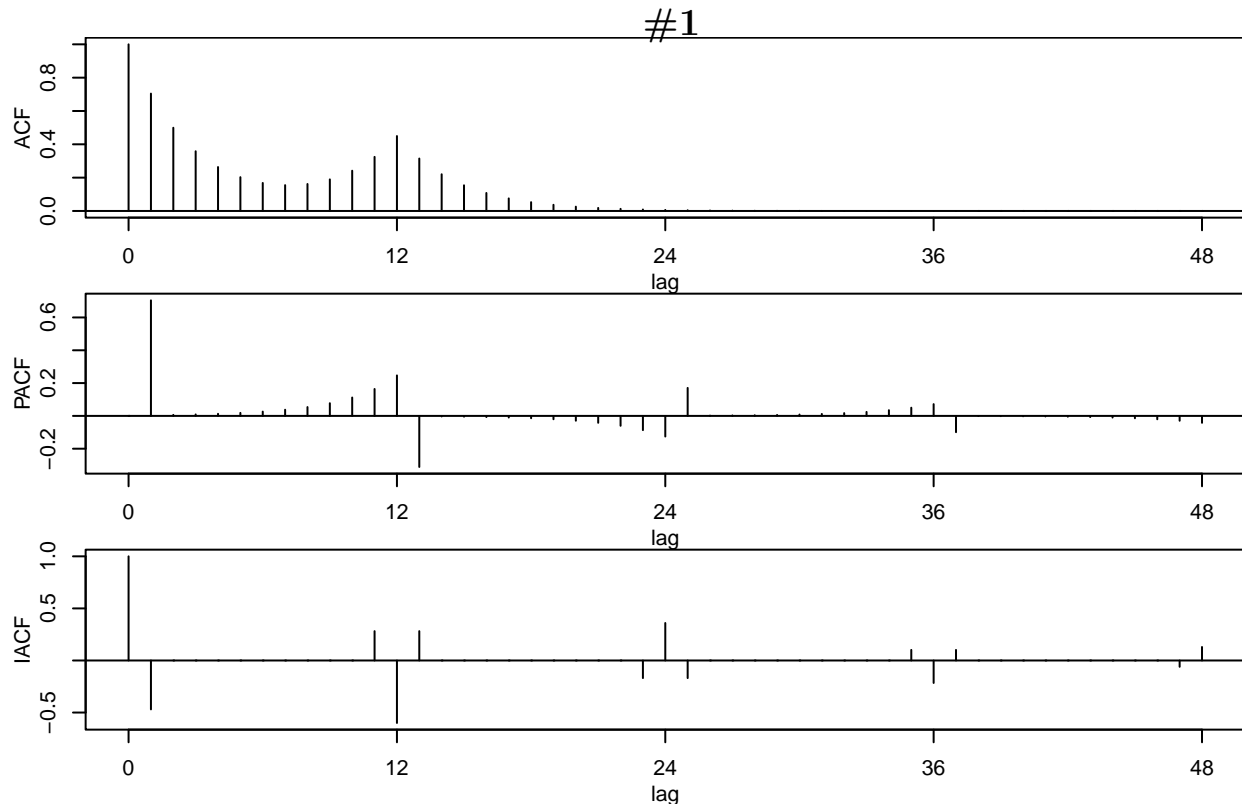
- | | | |
|---|---------------------------------|---------------------------------|
| a)★ ARMA(1,0)(0,1) ₁₂ | b) ARMA(1,0)(1,0) ₁₂ | c) ARMA(0,1)(0,1) ₁₂ |
| d) ARMA(0,1)(1,0) ₁₂ | e) ARMA(2,0)(0,1) ₁₂ | f) ARMA(2,0)(1,0) ₁₂ |
| g) ARMA(0,2)(0,1) ₁₂ | h) ARMA(0,2)(1,0) ₁₂ | i) ARMA(1,1)(0,1) ₁₂ |
| j) Non-Multiplicative Nonseasonal/Seasonal AR model | | |
| k) Non-Multiplicative Nonseasonal/Seasonal MA model | | |

Problem 13. Process #2 is a _____.

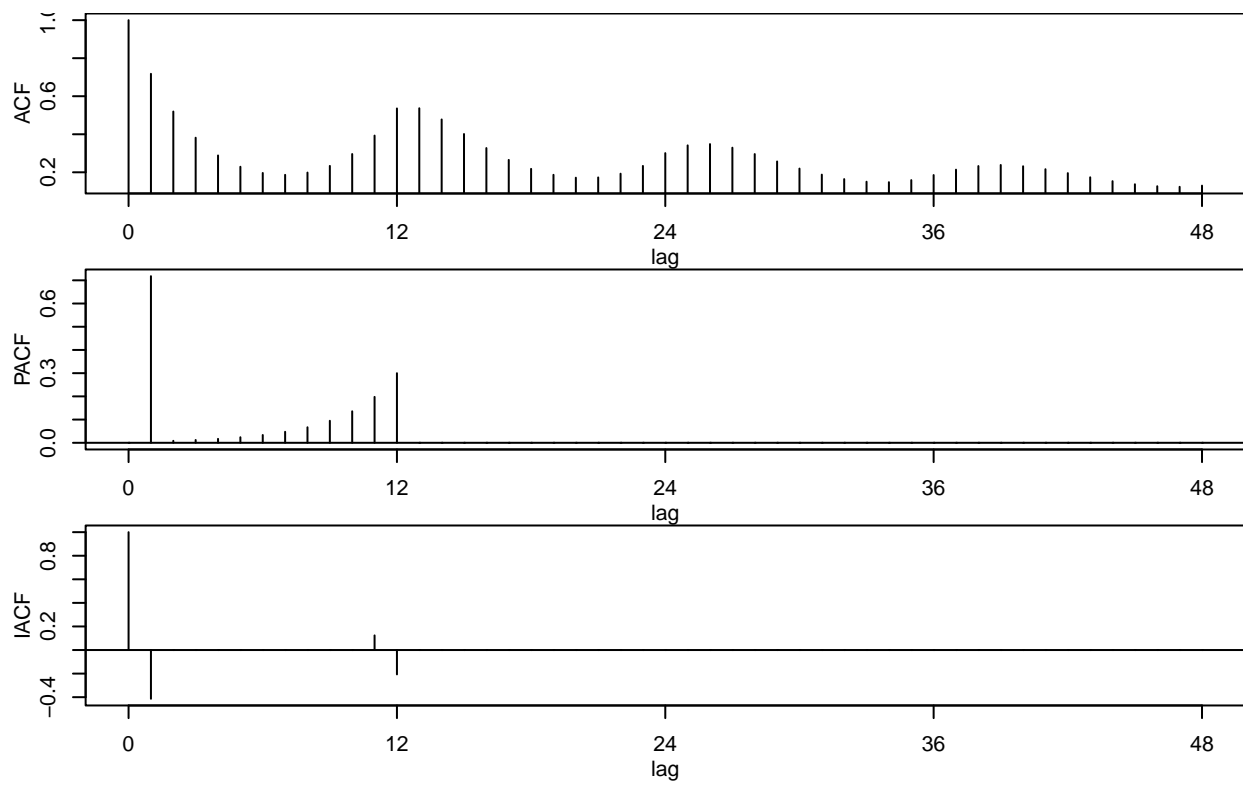
- | | | |
|--|---------------------------------|---------------------------------|
| a) ARMA(1,0)(0,1) ₁₂ | b) ARMA(1,0)(1,0) ₁₂ | c) ARMA(0,1)(0,1) ₁₂ |
| d) ARMA(0,1)(1,0) ₁₂ | e) ARMA(2,0)(0,1) ₁₂ | f) ARMA(2,0)(1,0) ₁₂ |
| g) ARMA(0,2)(0,1) ₁₂ | h) ARMA(0,2)(1,0) ₁₂ | i) ARMA(1,1)(0,1) ₁₂ |
| j)★ Non-Multiplicative Nonseasonal/Seasonal AR model | | |
| k) Non-Multiplicative Nonseasonal/Seasonal MA model | | |

Problem 14. Process #3 is a _____.

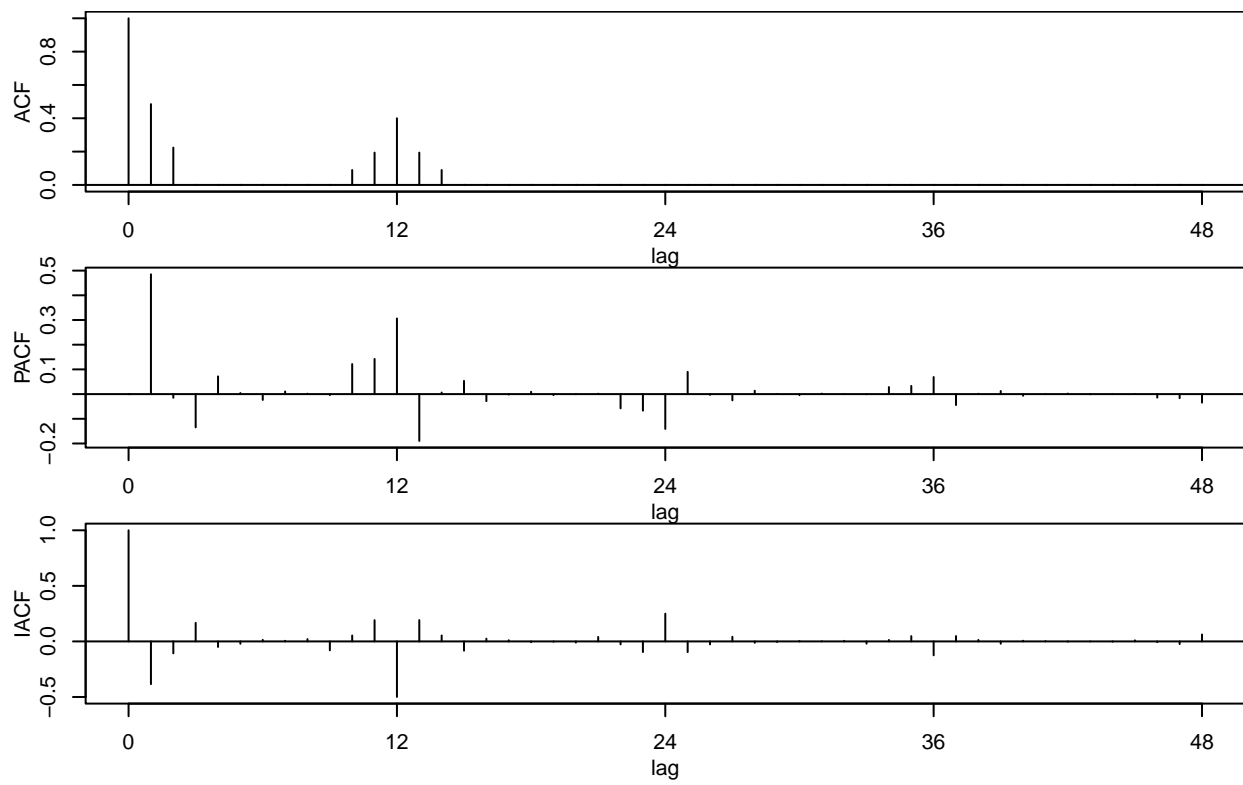
- | | | |
|----------------------------------|---------------------------------|---------------------------------|
| a) ARMA(1,0)(0,1) ₁₂ | b) ARMA(1,0)(1,0) ₁₂ | c) ARMA(0,1)(0,1) ₁₂ |
| d) ARMA(0,1)(1,0) ₁₂ | e) ARMA(2,0)(0,1) ₁₂ | f) ARMA(2,0)(1,0) ₁₂ |
| g)★ ARMA(0,2)(0,1) ₁₂ | h) ARMA(0,2)(1,0) ₁₂ | i) ARMA(1,1)(0,1) ₁₂ |



#2



#3



Problem 15. for the stationary process

$$z_t = C + \Phi_1 z_{t-12} + \Phi_2 z_{t-24} + a_t - \Theta_1 a_{t-12} - \Theta_2 a_{t-24},$$

what is the value of ρ_2 , the autocorrelation at lag 2?

- | | | | | |
|---------------|-------------|-----------------|---------------|---------------|
| a) Φ_1^2 | b) Φ_2 | c) Θ_1^2 | d) Θ_2 | e) Φ_2^2 |
| f)★ 0 | g) 1 | h) -1 | i) 1/2 | j) -1/2 |

Problem 16. Suppose you have time series $\{Y_t\}$ and $\{X_t\}$ and are trying to find a good transfer function model

$$Y_t = C + v(B)X_t + N_t$$

where $v(B) = v_0 + v_1 B + v_2 B^2 + \dots + v_h B^h$. One way to identify a reasonable initial choice of the ARMA(p, q) model for the noise process N_t is to _____.

- a) fit an ARMA(p, q) model for Y_t with relatively large values of p and q , and study the ACF/PACF of the residuals
- b)★ fit a multiple regression model of Y_t on $X_t, X_{t-1}, \dots, X_{t-h}$ with a relatively large h , and study the ACF/PACF of the residuals
- c) fit an ARMA(p, q) model for X_t with relatively large values of p and q , and study the ACF/PACF of the residuals
- d) fit a simple regression model of Y_t on X_t , and study the ACF/PACF of the residuals
- e) use the MINIC option of the IDENTIFY statement in PROC ARIMA to determine the values of p and q which approximately minimize the BIC
- f) study the cross-correlations between the series Y_t and X_t
- g) study the autocovariances between the series Y_t and X_t

Problem 17. Integrating a stationary ARMA process typically produces a _____ ARIMA process.

- | | | |
|--------------------|-------------------|---------------------|
| a)★ non-stationary | b) invertible | c) over-differenced |
| d) stationary | e) non-invertible | f) seasonal |

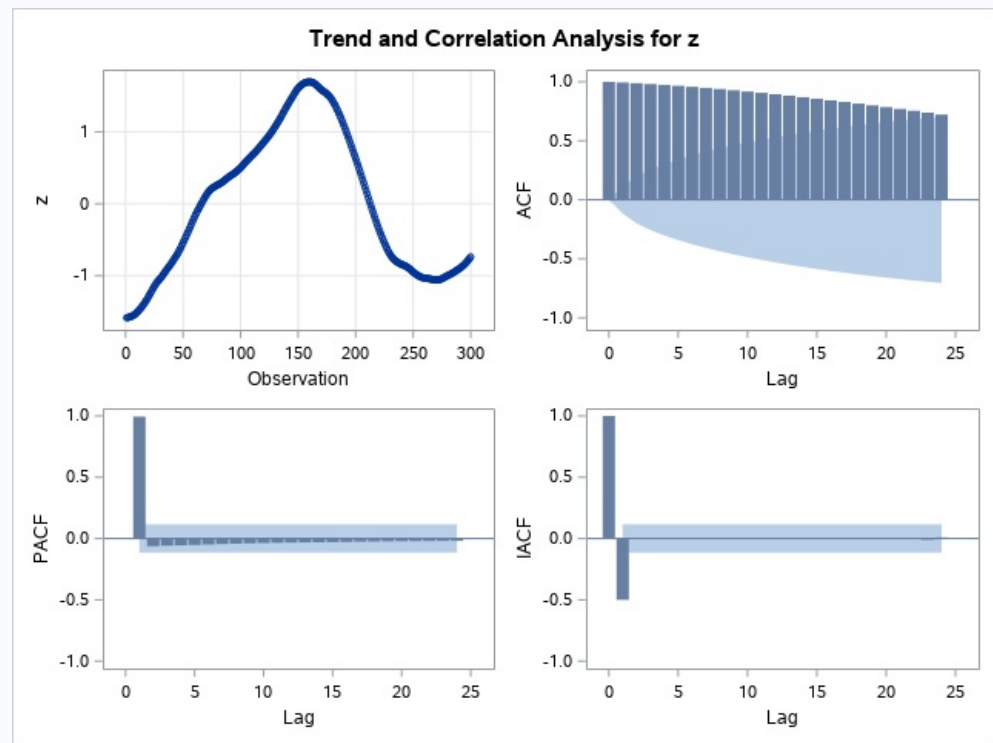
Problem 18. The following two pages give some SAS output produced by the IDENTIFY statement of PROC ARIMA for a time series z_t . Which of the models listed below is the best choice for this series?

- | | | | |
|-----------------|-----------------|------------------|-----------------|
| a) ARIMA(2,1,0) | b) ARIMA(3,1,0) | c) ARIMA(3,1,2) | d) ARIMA(2,1,3) |
| e) ARIMA(2,2,0) | f) ARIMA(0,2,3) | g)★ ARIMA(1,2,1) | h) ARIMA(5,2,6) |
| i) ARIMA(1,0,0) | j) ARIMA(1,0,2) | k) ARIMA(2,0,1) | l) ARIMA(0,0,1) |

The ARIMA Procedure

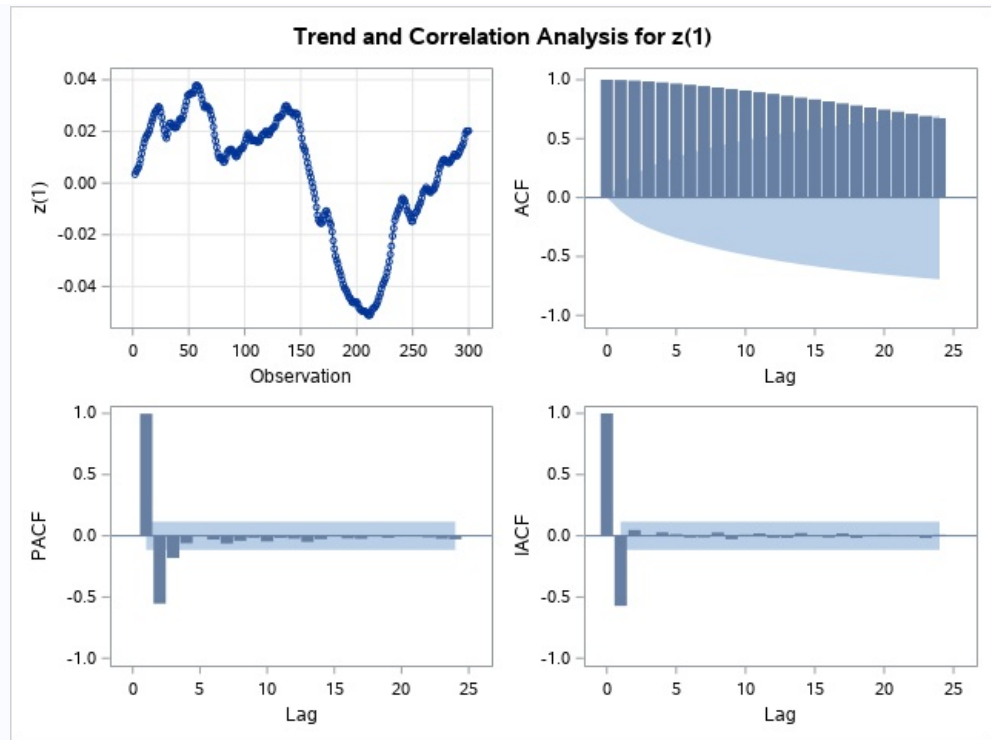
Name of Variable = z	
Mean of Working Series	-2E-6
Standard Deviation	0.998331
Number of Observations	300

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	1751.16	6	<.0001	0.995	0.989	0.982	0.974	0.966	0.958
12	3344.98	12	<.0001	0.949	0.939	0.929	0.918	0.907	0.895
18	4725.68	18	<.0001	0.883	0.870	0.857	0.844	0.830	0.815
24	5860.70	24	<.0001	0.801	0.786	0.770	0.755	0.739	0.723



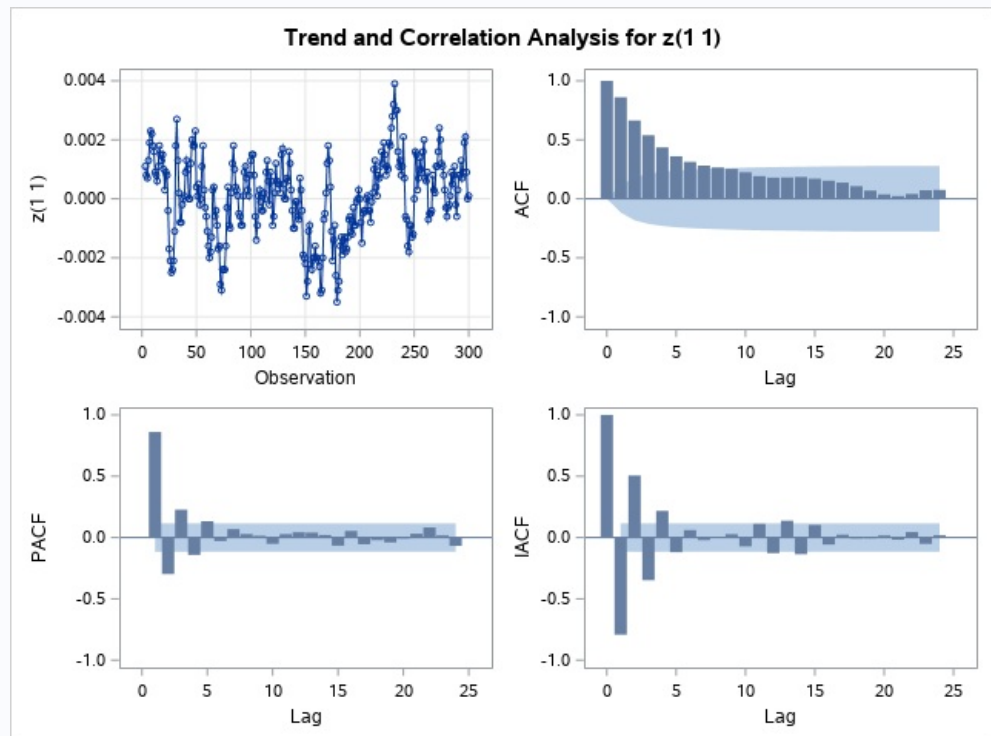
Name of Variable = z	
Period(s) of Differencing	1
Mean of Working Series	0.002829
Standard Deviation	0.024694
Number of Observations	299
Observation(s) eliminated by differencing	1

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	1754.95	6	<.0001	0.998	0.993	0.986	0.978	0.968	0.958
12	3314.18	12	<.0001	0.946	0.934	0.922	0.908	0.894	0.880
18	4608.54	18	<.0001	0.865	0.849	0.833	0.816	0.799	0.782
24	5617.10	24	<.0001	0.764	0.747	0.729	0.711	0.692	0.674



Name of Variable = z	
Period(s) of Differencing	1,1
Mean of Working Series	0.000056
Standard Deviation	0.00135
Number of Observations	298
Observation(s) eliminated by differencing	2

Autocorrelation Check for White Noise									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	571.79	6	<.0001	0.861	0.664	0.539	0.436	0.361	0.313
12	675.23	12	<.0001	0.282	0.266	0.254	0.225	0.193	0.180
18	722.68	18	<.0001	0.181	0.186	0.171	0.154	0.137	0.107
24	729.02	24	<.0001	0.070	0.037	0.023	0.041	0.073	0.076



Problem 19. An ARMA(3,1) model can be written in backshift form as _____.

- a) $(1 - \theta_1 B)z_t = C + (1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)a_t$
- b) $(1 - \theta_1 B^2)z_t = C + (1 - \phi_1 B^2 - \phi_2 B^3 - \phi_3 B^4)a_t$
- c) $(1 - \phi_1 B)z_t = C + (1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3)a_t$
- d) $(B - \theta_1 B^2)z_t = C + (B - \phi_1 B^2 - \phi_2 B^3 - \phi_3 B^4)a_t$
- e) $(B - \phi_1 B^2)z_t = C + (B - \theta_1 B^2 - \theta_2 B^3 - \theta_3 B^4)a_t$
- f) $(\phi_1 B + \phi_2 B^2 + \phi_3 B^3)z_t = C + \theta_1 B a_t$
- g) $\theta_1 B z_t = C + (\phi_1 B + \phi_2 B^2 + \phi_3 B^3)a_t$
- h) $\phi_1 B z_t = C + (\theta_1 B + \theta_2 B^2 + \theta_3 B^3)a_t$
- i) $(1 - \phi_1 B^2 - \phi_2 B^3 - \phi_3 B^4)z_t = C + (1 - \theta_1 B^2)a_t$
- j)★ $(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3)z_t = C + (1 - \theta_1 B)a_t$

Problem 20. Suppose you are trying to find a reasonable ARIMA(p, d, q) model for a series z_t . If you examine the series z_t , the series $(1 - B)z_t$, and the series $(1 - B)^2 z_t$, and all three of the series appear to be stationary, then you should _____.

- a) try a transformation
- b) try $d = 3$
- c)★ choose $d = 0$
- d) try a seasonal model
- e) choose $d = 1$
- f) choose $d = 2$
- g) try a “(trend) + (stationary ARMA process)” model

Problem 21. If you use SAS PROC ARIMA to fit a model to a time series $\{z_t\}$ which is specified by the code given below:

```
IDENTIFY VAR=Z(12) NLAG=36 ;
ESTIMATE P=(12) Q=1 METHOD=ML ;
```

and you use this model to forecast into the far future, then the **confidence interval widths** _____.

- a) will be larger for high forecasts and smaller for low forecasts
- b) converge to a repetitive pattern which repeats with a period of 12
- c) converge to a repetitive pattern added to a straight line with nonzero slope
- d) converge to a limiting value
- e)★ continue to increase and will reach arbitrarily large values
- f) converge to a straight line with a nonzero slope

Problem 22. A time series (such as temperature data) which has an approximately repeating seasonal pattern is _____.

- a) invertible
- b) non-invertible
- c) white noise
- d) over-differenced
- e)★ non-stationary
- f) stationary
- g) auto-regressive
- h) moving average

Problem 23. The sample IACF (Inverse ACF) of a non-invertible process will usually _____

- a) have a cutoff to zero
- b) exhibit sinusoidal decay
- c) go outside the typical bounds of -1 and 1
- d) increase slowly to infinity
- e)★ decay to zero very slowly
- f) decay to zero **not** too slowly

Problem 24. If all the roots of $\theta(B) = 0$ are strictly outside the unit circle in the complex plane, then

- a) $\pi(B) = \frac{\theta(B)}{\phi(B)} = 1 - \sum_{k=1}^{\infty} \pi_k B^k$ and $\pi_k \rightarrow 0$ as $k \rightarrow \infty$
- b)★ $\pi(B) = \frac{\phi(B)}{\theta(B)} = 1 - \sum_{k=1}^{\infty} \pi_k B^k$ and $\pi_k \rightarrow 0$ as $k \rightarrow \infty$
- c) $\psi(B) = \frac{\phi(B)}{\theta(B)} = 1 + \sum_{k=1}^{\infty} \psi_k B^k$ and $\psi_k \rightarrow 0$ as $k \rightarrow \infty$
- d) $\psi(B) = \frac{\theta(B)}{\phi(B)} = 1 + \sum_{k=1}^{\infty} \psi_k B^k$ and $\psi_k \rightarrow 0$ as $k \rightarrow \infty$

Problem 25. Suppose z_t is a **quarterly** series with a **nonstationary** mean. If the first differences ∇z_t appear stationary but have substantial autocorrelations at lags 1, 4, and 8, which of the following options might you wish to pursue? (More than one may be reasonable.)

1. Try a model without any differencing.
2. Try differencing at lag 1 a second time.
3. Try replacing differencing at lag 1 by differencing at lag 4.
4. Try a seasonal model which includes a seasonal term at lag 4.
5. Try an MA(1) model on ∇z_t .
6. Try an AR(1) model on ∇z_t .

Select the pair of options which seem most reasonable and circle your choice **below**. (Do NOT circle items on the list above!)

- | | | | |
|------------|-----------|-----------|-----------|
| a) 1 or 2 | b) 1 or 5 | c) 1 or 6 | d) 2 or 6 |
| e)★ 3 or 4 | f) 3 or 5 | g) 2 or 5 | h) 5 or 6 |

Problem 26. If z_t is an ARIMA(2, 3, 1)(1, 2, 3)₉ process, then $w_t = \underline{\hspace{2cm}}$ will be an ARMA(2, 1)(1, 3)₉ process.

- | | | |
|-------------------------------|--------------------------------|-------------------------------|
| a) $(1 - B^2)(1 - B^3)^9 z_t$ | b) $(1 - 3B - 2B^9)z_t$ | c) $(1 - 3B - 2B^2)^9 z_t$ |
| d) $(1 - B^3 - 9B^2)z_t$ | e) $(1 - B^3 - 2B^9)z_t$ | f) $(1 - B - B^9)^3 z_t$ |
| g) $(1 - B)^2(1 - B^9)^3 z_t$ | h)★ $(1 - B)^3(1 - B^9)^2 z_t$ | i) $(1 - B^3)(1 - B^2)^9 z_t$ |

Note: The series z_t in the next problem has nothing to do with the series z_t in the previous problem.

Problem 27. If we difference a series z_t at lag 1 and then at lag s , we get the series $w_t = \nabla_s \nabla z_t$. This may also be written as _____.

- | | |
|--|---|
| a) $w_t = (z_t - z_{t-s})(z_t - z_{t-1})$ | b) $w_t = (z_t - z_{t+s})(z_t - z_{t+1})$ |
| c) $w_t = z_t - z_{t+1} - z_{t+s} + z_{t+s+1}$ | d)★ $w_t = z_t - z_{t-1} - z_{t-s} + z_{t-s-1}$ |
| e) $w_t = (z_{t+s} - z_t)(z_{t+1} - z_t)$ | f) $w_t = (z_{t+s} - z_{t-s})(z_{t+1} - z_{t-1})$ |
| g) $w_t = z_t - z_{t+1} + z_{t+s} - z_{t+s+1}$ | h) $w_t = -z_t + z_{t-1} + z_{t-s} - z_{t-s+1}$ |

Problem 28. To fit an $\text{ARIMA}(3, 0, 4)(1, 0, 2)_{10}$ model using PROC ARIMA, you would use the code _____ in the ESTIMATE statement.

- | | |
|---|--------------------------------------|
| a) $p=(1, 2, 3)(10, 20) \quad q=(1, 2, 3, 4)(10)$ | b) $p=(3)(1, 10) \quad q=(4)(2, 10)$ |
| c) $p=(1, 2, 3)(1, 10) \quad q=(1, 2, 3, 4)(2, 10)$ | d) $p=(3)(1, 10) \quad q=(4)(2, 10)$ |
| e)★ $p=(1, 2, 3)(10) \quad q=(1, 2, 3, 4)(10, 20)$ | f) $p=(3)(1) \quad q=(4)(2)$ |
| g) $p=(1, 2, 3)(1) \quad q=(1, 2, 3, 4)(2)$ | h) $p=(3)(4) \quad q=(1)(2)$ |

Problem 29. The model

$$(1 - .2B^{12})(1 - .4B - .2B^2 - .3B^3)z_t = 5.0 + (1 - .5B^{12} + .2B^{24})(1 + .2B + .3B^2 - .2B^3 + .1B^4)a_t$$

is a _____.

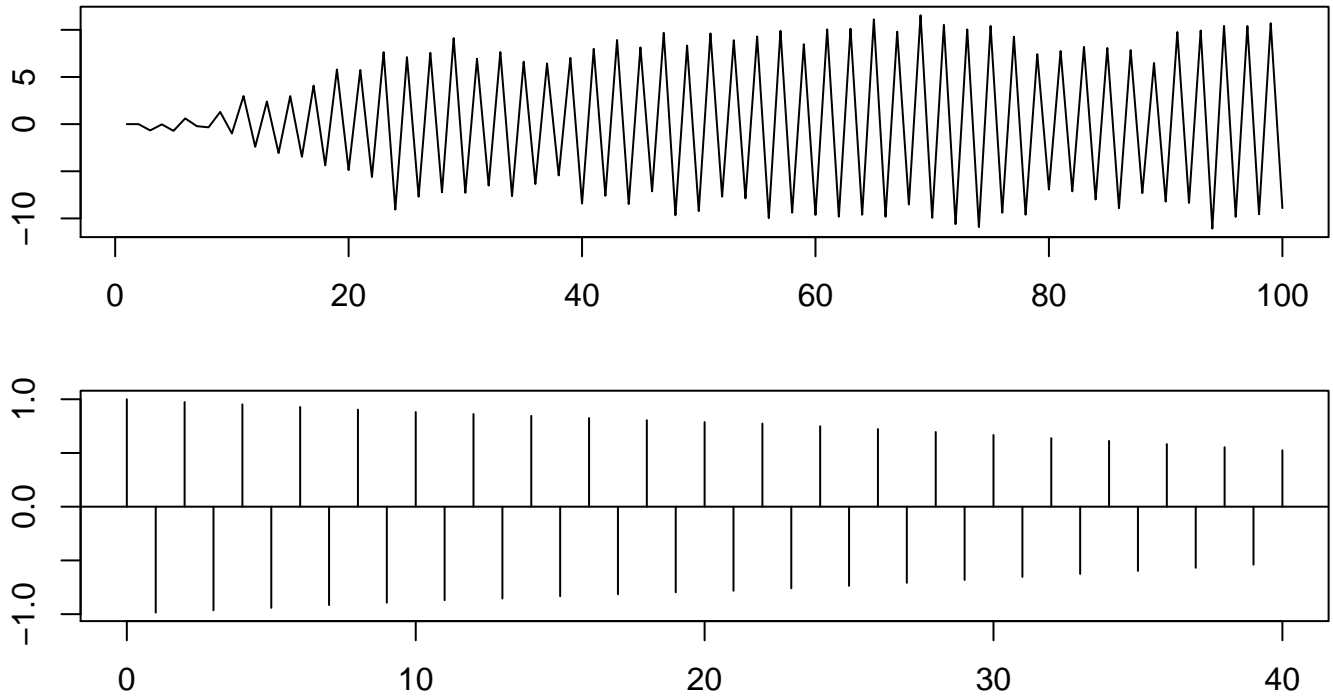
- | | | |
|--|--|---|
| a) $\text{ARIMA}(2, 0, 1)(3, 0, 4)_{12}$ | b) $\text{ARIMA}(3, 0, 1)(4, 0, 2)_{12}$ | c)★ $\text{ARIMA}(3, 0, 4)(1, 0, 2)_{12}$ |
| d) $\text{ARIMA}(4, 0, 3)(2, 0, 1)_{12}$ | e) $\text{ARIMA}(1, 0, 2)(3, 0, 4)_{12}$ | f) $\text{ARIMA}(4, 0, 2)(3, 0, 1)_{12}$ |
| g) $\text{ARIMA}(1, 0, 4)(3, 0, 2)_{12}$ | h) $\text{ARIMA}(1, 0, 2)(4, 0, 3)_{12}$ | i) $\text{ARIMA}(2, 0, 4)(1, 0, 3)_{12}$ |

Problem 30. The backshift expression $B(C + \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1})$ is equal to _____

- a)★ $C + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
b) $C_{t-1} + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$
c) $C_{t+1} + \phi_2 z_t + a_{t+1} - \theta_2 a_t$
d) $C + \phi_2 z_{t-2} + a_{t-1} - \theta_2 a_{t-2}$
e) $0 + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
f) $0 + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$
g) $0 + \phi_2 z_{t+1} + a_{t+1} - \theta_2 a_{t+2}$

Problem 31. For a time series z_1, z_2, \dots, z_{100} of length 100, the two figures below give the time series plot (z_t versus t) and the estimated ACF up to lag 40. Which response best completes this sentence: This time series appears to be a _____ process.

- a) stationary random walk b) MA(1) c) MA(2) d) MA(3)
 e)★ non-stationary f) stationary AR(1) g) stationary AR(2) h) random shock



Problem 32. One general approach to modeling a series with a non-stationary mean which exhibits seasonal patterns or seasonal variation is to _____.

- a)★ make the series stationary by ordinary and/or seasonal differencing, and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the differenced series
 b) choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to eliminate the seasonal effects, and then difference the resulting series (using ordinary and/or seasonal differencing) to eliminate the non-stationarity
 c) transform the series (for example, by using a log or square root transformation), and then choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to model the transformed series
 d) choose an appropriate $\text{ARIMA}(p, 0, q)(P, 0, Q)_s$ process to eliminate the seasonal effects, and then transform the resulting series (for example, by using a log or square root transformation) to eliminate the non-stationarity

Problem 33. If the theoretical **PACF** of a stationary ARMA process has a cutoff (to zero) after lag 3, then the theoretical **IACF** (Inverse Autocorrelation Function) will _____.

- a) undergo sinusoidal decay
- b) undergo alternating exponential decay
- c) decay to zero very rapidly
- d) decay to zero very slowly
- e) decay exponentially
- f) decay exponentially starting at lag 3
- g) undergo sinusoidal decay after lag 3
- h)★ have a cutoff after lag 3

Problem 34. The standard error of the k -step ahead forecast from an ARIMA process involves the ψ -weights and is given by the expression _____.

- | | |
|---|--|
| a) $\sigma_a(1 + \psi_1 + \psi_2 + \cdots + \psi_k)$ | b)★ $\sigma_a\sqrt{1 + \psi_1^2 + \psi_2^2 + \cdots + \psi_{k-1}^2}$ |
| c) $\sigma_a\sqrt{1 + \psi_1 + \psi_2 + \cdots + \psi_k}$ | d) $\sigma_a(1 + \psi_1^2 + \psi_2^2 + \cdots + \psi_{k-1}^2)$ |
| e) $\sigma_a(1 + \psi_1 + \psi_2 + \cdots + \psi_{k-1})$ | f) $\sigma_a(\psi_1^2 + \psi_2^2 + \cdots + \psi_k^2)$ |
| g) $\sigma_a\sqrt{1 + \psi_1 + \psi_2 + \cdots + \psi_{k-1}}$ | h) $\sigma_a\sqrt{\psi_1^2 + \psi_2^2 + \cdots + \psi_k^2}$ |

Problem 35. An MA(1) process with $|\theta_1| < 1$ can be re-written in the form _____.

- a)★ $\tilde{z}_t = a_t - \theta_1\tilde{z}_{t-1} - \theta_1^2\tilde{z}_{t-2} - \theta_1^3\tilde{z}_{t-3} - \cdots$
- b) $\tilde{z}_t = a_t + \theta_1\tilde{z}_{t-1} + \theta_1^2\tilde{z}_{t-2} + \theta_1^3\tilde{z}_{t-3} + \cdots$
- c) $\tilde{z}_t = a_t + \theta_1\tilde{z}_{t-1} + \theta_2\tilde{z}_{t-2} + \theta_3\tilde{z}_{t-3} + \cdots$
- d) $\tilde{z}_t = a_t + \theta_1a_{t-1} + \theta_1^2a_{t-2} + \theta_1^3a_{t-3} + \cdots$
- e) $\tilde{z}_t = a_t - \theta_1a_{t-1} - \theta_1^2a_{t-2} - \theta_1^3a_{t-3} - \cdots$
- f) $\tilde{z}_t = a_t - \theta_1a_{t-1} - \theta_2a_{t-2} - \theta_3a_{t-3} - \cdots$