TEST #2

STA 4853 **Name:**____

May 4, 2023

Please read the following directions.
DO NOT TURN THE PAGE UNTIL INSTRUCTED TO DO SO

Directions

- This exam is **closed book** and **closed notes**.
- Circle the **single best** answer for each multiple choice question. Your choice should be made clearly.
- Always **circle the correct response**. (Sometimes the question has an empty blank or a box, but this is **NOT** where the answer goes.)
- Each question is worth equal credit.
- There is no penalty for guessing.
- There are **33** multiple choice questions.
- The exam has **12** pages.

Problem 1. Suppose $\{y_t\}$, $\{x_{1,t}\}$, $\{x_{2,t}\}$ are time series stored in variables named `Y`, `X1`, `X2` in the SAS Dataset `STUFF`. Consider the SAS code given below.

```
PROC ARIMA DATA=STUFF;
IDENTIFY VAR=Y CROSSCOR=(X1 X2) NOPRINT;
ESTIMATE P=2 INPUT=(X1 X2) METHOD=ML;
RUN;
```

This code fits which of the following models?

a) $(1-\phi_1 B-\phi_2 B^2)y_t=\beta_0+\beta_1 x_{1,t}+\beta_2 x_{2,t}+a_t$

b) $\dfrac{1}{1-\phi_1 B-\phi_2 B^2}\,y_t=\beta_0+\beta_1 x_{1,t}+\beta_2 x_{2,t}+a_t$

c) $y_t=C+\phi_1 x_{1,t-1}+\phi_2 x_{2,t-2}+a_t$

d) $y_t=C+\phi_1 x_{1,t-1}+\phi_2 x_{1,t-2}+\phi_1 x_{2,t-2}+\phi_2 x_{2,t-2}+a_t$

e) $y_t=C+\phi_1 x_{1,t-1}+\phi_2 x_{2,t-2}+a_t+\phi_1 a_{t-1}+\phi_2 a_{t-2}$

f)$\star$ $y_t=\beta_0+\beta_1 x_{1,t}+\beta_2 x_{2,t}+\dfrac{1}{1-\phi_1 B-\phi_2 B^2}\,a_t$

g) $y_t=\beta_0+\beta_1 x_{1,t}+\beta_2 x_{2,t}+(1-\phi_1 B-\phi_2 B^2)\,a_t$

h) $y_t=C+\phi_1 y_{t-1}+\phi_2 y_{t-2}+\beta_1 x_{1,t}+\beta_2 x_{2,t}+a_t$

Problem 2. Differencing an ARIMA(5,3,7) process 3 times produces a _________ process.

a)$\star$ ARMA(5,7) **b)** ARMA(5,4) **c)** ARMA(2,7) **d)** ARIMA(2,3,7)
e) ARIMA(5,6,7) **f)** ARIMA(5,3,10) **g)** ARIMA(8,3,7) **h)** ARIMA(8,6,10)

Problem 3. Suppose you wish to forecast a time series x_t. You decide to use a square root transformation $y_t=\sqrt{x_t}$, and you model the series y_t and obtain forecasts $\hat{y}_t$ for the series y_t. The forecasts for the original series x_t are then given by $\hat{x}_t=$ _________.

a) $\dfrac{1}{\sqrt{\hat{y}_t}}$ **b)** $\dfrac{1}{\log\hat{y}_t}$ **c)** $\dfrac{1}{\exp\hat{y}_t}$ **d)** $\dfrac{1}{\hat{y}_t^2}$

e) $\sqrt{\hat{y}_t}$ **f)** $\log\hat{y}_t$ **g)** $\exp\hat{y}_t$ **h)**$\star$ $\hat{y}_t^2$

Problem 4. Suppose $\{z_t\}$ is a stationary AR(2) process and we compute $w_t=\nabla z_t$. Then $\{w_t\}$ is _________.

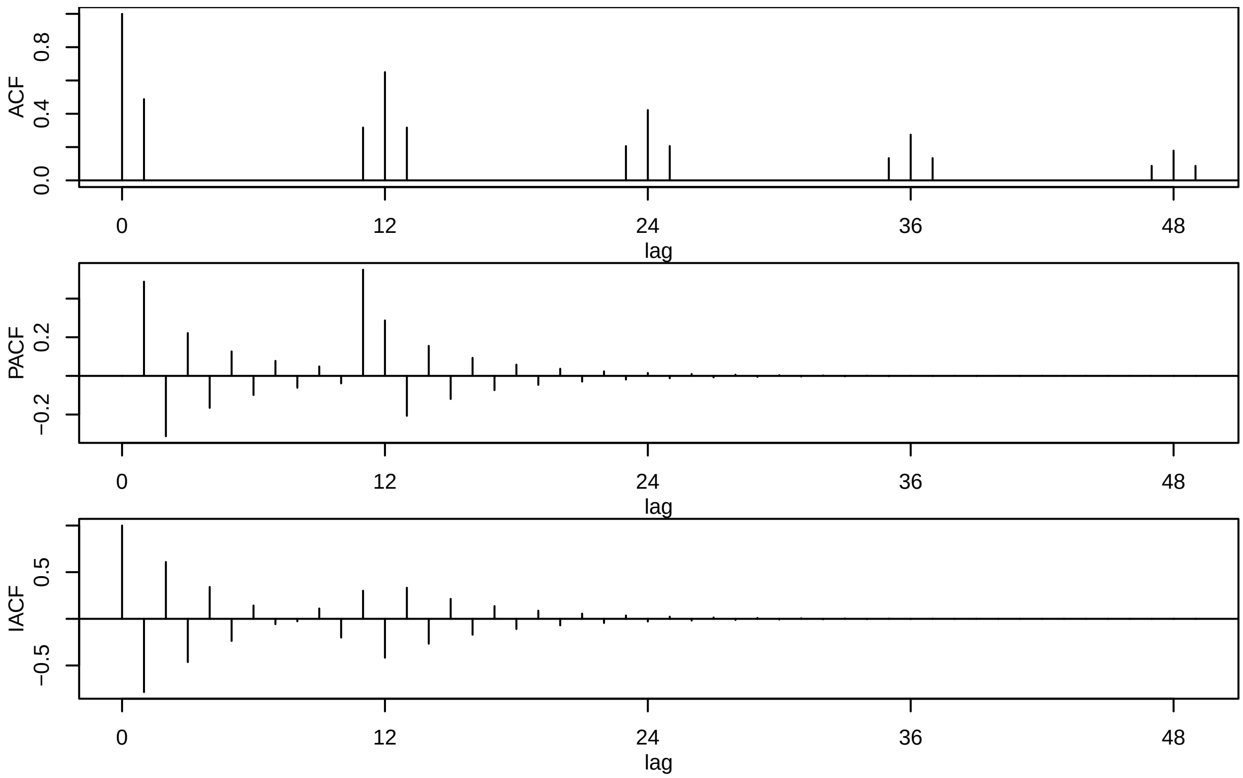
a) an AR(3) process **b)** an ARMA(1,2) process **c)** an ARMA(1,1) process
d)$\star$ non-invertible **e)** non-stationary **f)** invertible

Problem 5. The following plots give the theoretical ACF, PACF, and IACF of a seasonal ARMA process with seasonality $s = 12$ up to lag 49. What is this process?

a) $\text{ARMA}(1,0)(0,1)_{12}$ **b**) $\text{ARMA}(3,0)(4,0)_{12}$ **c**) $\text{ARMA}(3,0)(0,4)_{12}$

d) $\text{ARMA}(0,1)(0,1)_{12}$ **e**) $\text{ARMA}(1,0)(1,0)_{12}$ **f**)$\star$ $\text{ARMA}(0,1)(1,0)_{12}$

g) $\text{ARMA}(0,3)(4,0)_{12}$ **h**) $\text{ARMA}(0,3)(0,4)_{12}$ **i**) $\text{ARMA}(1,0)(0,4)_{12}$



Problem 6. A seasonal ARIMA process $\text{ARIMA}(p,d,q)(P,D,Q)_s$ will be **non**-stationary if _________. (Select the best answer.)

a) the roots of $\theta(B) = 0$ are **not** all strictly outside the unit circle

b) the sample IACF is decaying very slowly

c) the π-weights π_k do **not** decay to zero as $k \to \infty$

d) any of (a) or (b) or (c) is true

e) $d > 0$

f) $D > 0$

g) the AR coefficients violate the stationarity conditions

h)$\star$ any of (e) or (f) or (g) is true

Problem 7. An ARMA(1,1) process is invertible if _________.

a) $|\phi_1| < 1$ **b**) $|\phi_1| > 1$ **c**)$\star$ $|\theta_1| < 1$ **d**) $|\theta_1| > 1$

e) $\phi_1 + \theta_1 < 1$ **f**) $\phi_1 - \theta_1 < 1$ **g**) $|\phi_1| + |\theta_1| < 1$ **h**) $|\phi_1 + \theta_1| < 1$

Problem 8. Suppose z_t is a **quarterly** series with a **non**stationary mean. You wish to find a reasonable ARIMA model (possibly seasonal) for this series. If the first differences ∇z_t appear stationary but have substantial autocorrelations at lags 1, 4, and 8, which of the following options might you wish to pursue? (More than one may be reasonable.)

1. Try a model without any differencing.
2. Try differencing at lag 1 a second time.
3. Try replacing differencing at lag 1 by differencing at lag 4.
4. Try a seasonal model which includes a seasonal term at lag 4.
5. Try an MA(1) model on ∇z_t.
6. Try an AR(1) model on ∇z_t.

Select the pair of options which seem most reasonable and circle your choice **below**. (Do NOT circle items on the list above!)

a) 2 or 5 **b)** 2 or 6 **c)**$\star$ 3 or 4 **d)** 3 or 5

e) 1 or 2 **f)** 1 or 5 **g)** 1 or 6 **h)** 5 or 6

Problem 9. The standard error for k-step-ahead forecasts is $\sigma[e_n(k)] =$ _________.

a) $\left(1+2\sum_{j=1}^{k-1} r_j^2\right)^{1/2} n^{-1/2}$ **b)** $\left(1+2\sum_{j=1}^{k} r_j^2\right)^{1/2} n^{-1/2}$

c) $n^{-1/2}$ **d)** $n^{1/2}$

e) $\dfrac{\sigma_a^2}{1-\phi_1^2}$ **f)** $\dfrac{\sigma_a^2}{1+\phi_1^2}$

g) $\sigma_a\sqrt{1+\psi_1^2+\psi_2^2+\cdots+\psi_k^2}$ **h)**$\star$ $\sigma_a\sqrt{1+\psi_1^2+\psi_2^2+\cdots+\psi_{k-1}^2}$

Problem 10. If the correct model for a time series is an ARIMA model with normally distributed random shocks, then _________.

a) the best prediction of a future value will depend on whether we use the squared error or absolute error loss function

b) the absolute error loss function is preferable to the squared error loss function

c)$\star$ the conditional distribution of a future value is a normal distribution

d) the conditional distribution of a future value is skewed

e) the conditional distribution of a future value is bimodal

Problem 11. Suppose $Y \sim N(\mu, \sigma^2)$ where μ and σ^2 are known values. You wish to forecast the quantity $X = e^Y$, which has a **log**-normal distribution. If your loss function is "squared error loss", then the best forecast for $X = e^Y$ is equal to _________.

a) e^{μ} **b)** $e^{\mu-\sigma^2}$ **c)** μ **d)**$\star$ $e^{\mu+(\sigma^2/2)}$

e) $\log(\mu)$ **f)** $\log(\mu-\sigma^2)$ **g)** σ^2 **h)** $\log(\mu+(\sigma^2/2))$

Problem 12. When choosing (identifying) an appropriate ARIMA(p, d, q) model for a time series, it is best to _________ .

a) first choose p and q and then choose d

b)$\star$ first choose d and then choose p and q

c) choose p, then d, and finally q

d) choose q, then d, and finally p

e) use the MINIC option to simultaneously choose p, d, and q

Problem 13. If you wish to model a time series in which the variability of the series increases systematically with the level of the series, then you should consider _________ .

a) using a seasonal ARIMA model

b) differencing the series at the seasonal lag

c) dividing the series into parts

d)$\star$ transforming the series

e) differencing the series at lag 1

f) modeling the series as (Trend) + (stationary ARMA process)

Problem 14. A time series with an approximately repeating seasonal pattern or tendency has/is _________ .

a) weakly stationary

b) strictly stationary

c) a funnel shape in the residuals versus forecast plot

d) a bend in the residuals versus forecast plot

e)$\star$ a nonstationary mean

f) a nonstationary variance

g) a nonstationary ACF

Problem 15. Suppose we are given a series $x_1, x_2, \dots, x_n$ and we **integrate** this series to obtain the series $w_1, w_2, \dots, w_n$. This means that _________ .

a) $w_t = x_t - x_{t-1}$

b) $w_t = x_{t+1} - x_t$

c) $w_t = \dfrac{\theta(B)}{\phi(B)} x_t$

d) $x_t = \dfrac{\theta(B)}{\phi(B)} w_t$

e)$\star$ $w_t = x_1 + x_2 + \cdots + x_t$

f) $x_t = w_1 + w_2 + \cdots + w_t$

Problem 16. If you take a realization from an already stationary ARMA process and difference it, the sample IACF of the resulting differences will usually _________ .

a)$\star$ decay to zero very slowly
b) decay to zero fairly rapidly
c) have a large spike at lag1 followed by much smaller spikes
d) decays with a damped sine wave pattern
e) exhibit alternating exponential decay
f) have all of the spikes within the two standard error band

Problem 17. The theoretical **I**ACF of the process

$$z_t = C + \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}$$

is equal to the theoretical ACF of the process _________.

a)$\star$ $z_t = C + \theta_1 z_{t-1} + \theta_2 z_{t-2} + \theta_3 z_{t-3} + a_t - \phi_1 a_{t-1}$
b) $z_t = C + \phi_1 z_{t-1} + \theta_3 z_{t-2} + \theta_2 z_{t-3} + a_t - \theta_1 a_{t-1}$
c) $z_t = C + \theta_2 z_{t-1} + \theta_1 z_{t-2} + \phi_1 z_{t-3} + a_t - \theta_3 a_{t-1}$
d) $z_t = C + \theta_1 z_{t-1} + a_t - \phi_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3}$
e) $z_t = C + \phi_1 z_{t-1} + \phi_2 z_{t-2} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$
f) $z_t = C + \phi_1 z_{t-1} + \theta_2 z_{t-2} + a_t - \theta_1 a_{t-1} - \phi_2 a_{t-2}$

Problem 18. A realization from a process with a NON-stationary mean will typically have _________.

a) a residuals versus forecasts plot which does NOT remain centered about zero
b) a residuals versus forecasts plot which does NOT have constant width
c) a normal probability plot which does not follow a straight line
d)$\star$ a sample ACF which decays very slowly to zero
e) a sample PACF which decays very slowly to zero
f) a sample IACF which decays very slowly to zero

Problem 19. An ARMA(1,1) process with $\phi_1 = \theta_1 = 0.8$ has a theoretical ACF which _________.

a) approaches zero at an eventually exponential rate
b) has a cutoff to zero after lag 1
c) has alternating exponential decay
d) decays with a damped sine wave pattern
e)$\star$ has $\rho_k = 0$ for all $k > 0$
f) has $\rho_k = \phi_1^k$ for all $k > 0$

The next two questions concern the long-run behavior of the forecasts and their associated confidence interval widths for the model specified in the following lines of code. The time series `xsin1`, `xcos1`, and `xsin2` appearing in this code are periodic functions of time with period 12 defined in terms of sine and cosine functions.

```
identify var=z crosscor=(xsin1 xcos1 xsin2) noprint;
estimate p=1 input=(xsin1 xcos1 xsin2) method=ml;
```

Problem 20. Describe the long-run behavior of the forecasts.

a)⋆ The forecasts converge to a repetitive pattern which repeats with a period of 12.
b) The forecasts converge to the estimated mean $\hat{\mu}_z$ of the process.
c) The forecasts converge to a value which depends mainly on the last few observed values of the time series.
d) The time series plot of the forecasts converges to a straight line with a nonzero slope.
e) The time series plot of the forecasts converges to a repetitive pattern added to a straight line with nonzero slope.

Problem 21. Describe the long-run behavior of the confidence interval widths.

a) The widths converge to a repetitive pattern which repeats with a period of 12.
b)⋆ The widths converge to a limiting value.
c) The widths continue to gradually increase and will eventually reach arbitrarily large values.

Problem 22. If you write an ARIMA$(0,0,1)(0,0,2)_S$ model in backshift notation, expand the product, and then eliminate the backshift notation, you see it is a special case of a _________ process.

a) MA$(2s-1)$ **b)**⋆ MA$(2s+1)$ **c)** MA$(s+1)$ **d)** MA$(s+2)$
e) MA$(s+3)$ **f)** MA(3) **g)** MA(4) **h)** MA(5)
i) AR$(2s-1)$ **j)** AR$(2s+1)$ **k)** AR$(s+1)$ **l)** AR$(s+2)$
m) AR$(s+3)$ **n)** AR(3) **o)** AR(4) **p)** AR(5)

Problem 23. $\nabla_s z_t =$ _________

a) $(1-B)^s z_t$ **b)** $(B-1)^s z_t$ **c)** $(1-B)^t z_s$ **d)**⋆ $z_t - z_{t-s}$
e) $z_{t+s} - z_{t+s-1}$ **f)** $z_{t+s} - z_t$ **g)** $z_{t-s} - z_{t-s-1}$

Problem 24. $(1+2B^2)(1+3B^{12}) =$ _________

a) $2+3B^2+4B^{12}+5B^{14}$ **b)**⋆ $1+2B^2+3B^{12}+6B^{14}$ **c)** $1+6B^{24}$
d) $2+2B^2+3B^{12}+6B^{24}$ **e)** $2+2B^2+3B^{12}$ **f)** $1+3B^2+4B^{12}+5B^{24}$

Problem 25. The AR(1) process in mean-centered form

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + a_t$$

may be written in backshift notation as

$$\phi(B)\tilde{z}_t = a_t$$

where $\phi(B)$ is the AR polynomial. This AR polynomial has a single zero (or root) which is equal to ________.

a) $\frac{1}{\phi_1 B}$ **b)** $-\phi_1 B$ **c)** $\frac{-1}{B}$ **d)**⋆ $\frac{1}{\phi_1}$ **e)** $\frac{-1}{\phi_1 B}$

f) ϕ_1 **g)** $\frac{1}{B}$ **h)** 1 **i)** B **j)** $\phi_1 B$

Problem 26. How would the ARIMA$(2,0,0)(1,0,0)_{12}$ model be written in backshift notation?

a) $(1-\phi_2 B^2)(1-\Phi_1 B^{12})z_t = C + a_t$
b)⋆ $(1-\phi_1 B-\phi_2 B^2)(1-\Phi_1 B^{12})z_t = C + a_t$
c) $(1-\phi_2 B^2)(1-\Phi_1 B)z_{t-12} = C + a_t$
d) $(1-\phi_2 B^2)z_t = C + (1-\Theta_1 B^{12})a_t$
e) $(1-\phi_2 B^2)z_t = C + (1-\Theta_1 B)a_{t-12}$
f) $(1-\phi_2 B^2)z_{t-12} = C + (1-\Theta_1 B)a_{t-12}$
g) $z_t = C + (1-\theta_1 B-\theta_2 B^2)(1-\Theta_1 B^{12})a_t$
h) $z_t = C + (1-\theta_2 B^2)(1-\Theta_1 B^{12})a_t$
i) $z_t = C + (1-\theta_2 B^2)(1-\Theta_1 B)a_{t-12}$

Problem 27. An $AR(2)_{12}$ or ARIMA$(2,0,0)_{12}$ is a purely seasonal model. It may be written as ________.

a) $z_t = C + \Phi_1 z_{t-1} + \Phi_2 z_{t-12} + a_t$ **b)** $z_t = C + \Phi_1 z_{t-12} + \Phi_2 z_{t-13} + a_t$
c)⋆ $z_t = C + \Phi_1 z_{t-12} + \Phi_2 z_{t-24} + a_t$ **d)** $z_t = C + \Phi_1 z_{t-12} + \Phi_2 z_{t-24} + a_{t-24}$
e) $z_t = C + \Phi_1 z_{t-1} + \Phi_2 z_{t-12} + a_{t-12}$ **f)** $z_t = C + \Phi_1 z_{t-12} + \Phi_2 z_{t-13} + a_{t-12}$

Problem 28. Suppose you wish to explain a response series $\{y_t\}$ by a regression with ARMA errors using k input series $\{x_{1,t}\}$, $\{x_{2,t}\}$, ..., $\{x_{k,t}\}$. That is, you plan to use a multiple regression model

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

in which you assume the error series ε_t is an ARMA(p, q) process. Which of the following is a reasonable way to make an initial choice of p and q?

a) Fit an ordinary multiple regression model and use the p-values from this to drop the non-significant terms.

b)⋆ Fit an ordinary multiple regression model and then study the ACF and PACF of the residuals.

c) Apply the MINIC method to the series y_t.

d) Apply the MINIC method to the series $x_{1,t}, x_{2,t}, \ldots, x_{k,t}$.

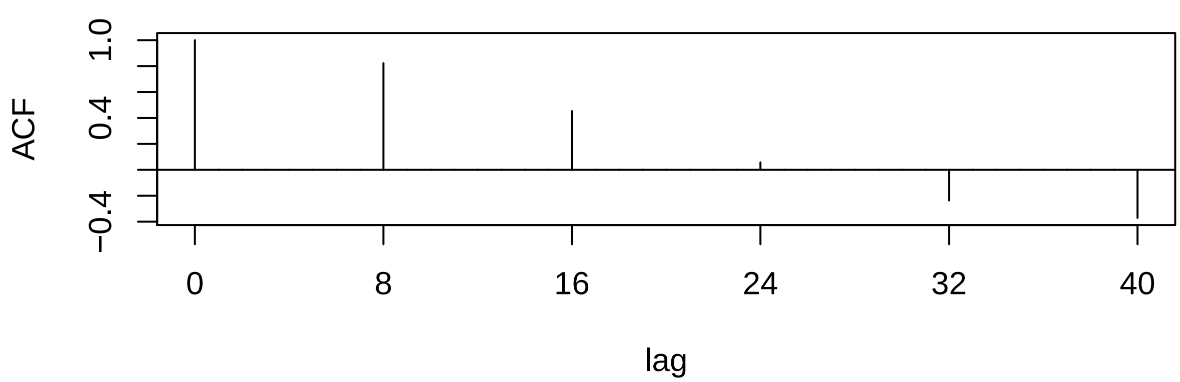
e) Fit an AR(2) model to y_t and then use the residual ACF and PACF to select a better choice of p and q.

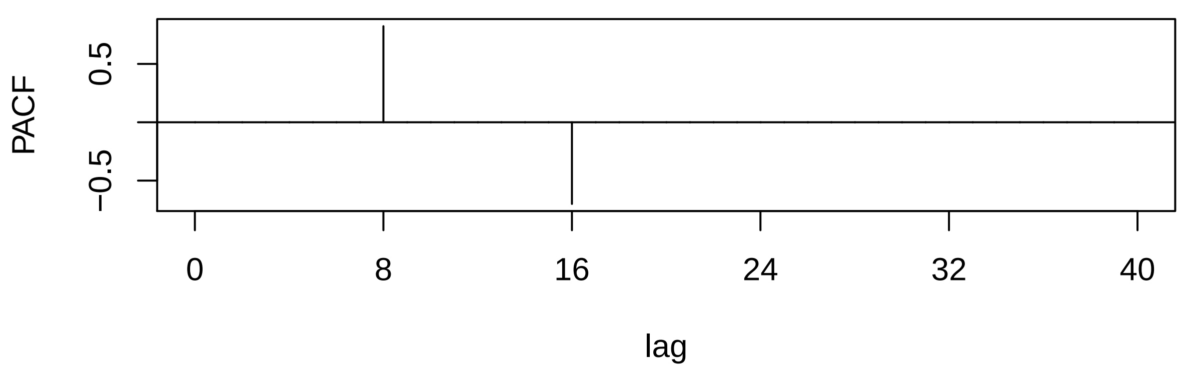
f) Study the ACF and PACF of the series $x_{1,t}, x_{2,t}, \ldots, x_{k,t}$.

g) Study the ACF and PACF of the series y_t.

Problem 29. The plots below give the theoretical ACF and PACF of an ARMA process. What is this process?

a) ARMA$(0,0)(2,0)_{16}$
b)⋆ ARMA$(0,0)(2,0)_{8}$
c) ARMA$(0,0)(0,2)_{16}$
d) ARMA$(0,0)(0,2)_{8}$
e) ARMA$(0,0)(2,2)_{8}$
f) ARMA$(0,0)(2,2)_{16}$
g) ARMA$(2,0)(0,0)_{8}$
h) ARMA$(2,0)(0,0)_{16}$
i) ARMA$(0,2)(0,0)_{8}$
j) ARMA$(0,2)(0,0)_{16}$
k) ARMA$(2,2)(0,0)_{8}$
l) ARMA$(2,2)(0,0)_{16}$





Problem 30. An ARMA(p, q) process is stationary if and only if all the solutions of ________ the unit circle in the complex plane.

a) $\theta(B)/\phi(B) = 0$ lie strictly inside
b) $\theta(B) = 0$ lie strictly inside
c) $\phi(B) = 0$ lie strictly inside
d) $\theta(B)/\phi(B) = 0$ lie on the boundary of
e) $\theta(B) = 0$ lie on the boundary of
f) $\phi(B) = 0$ lie on the boundary of
g) $\theta(B)/\phi(B) = 0$ lie strictly outside
h) $\theta(B) = 0$ lie strictly outside
i)⋆ $\phi(B) = 0$ lie strictly outside

Problem 31. The backshift expression $B(C + \phi_1 z_{t-1} + a_t - \theta_1 a_{t-1})$ is equal to ________

a) $0 + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
b) $0 + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$
c) $0 + \phi_2 z_{t+1} + a_{t+1} - \theta_2 a_{t+2}$
d) $C + \phi_2 z_{t-2} + a_{t-1} - \theta_2 a_{t-2}$
e)⋆ $C + \phi_1 z_{t-2} + a_{t-1} - \theta_1 a_{t-2}$
f) $C_{t+1} + \phi_2 z_t + a_{t+1} - \theta_2 a_t$
g) $C_{t-1} + \phi_0 z_{t-2} + a_{t-1} - \theta_0 a_{t-2}$

Problem 32. By very carefully reading the table given below, you can determine that the ESTIMATE statement which produced this table contained ________

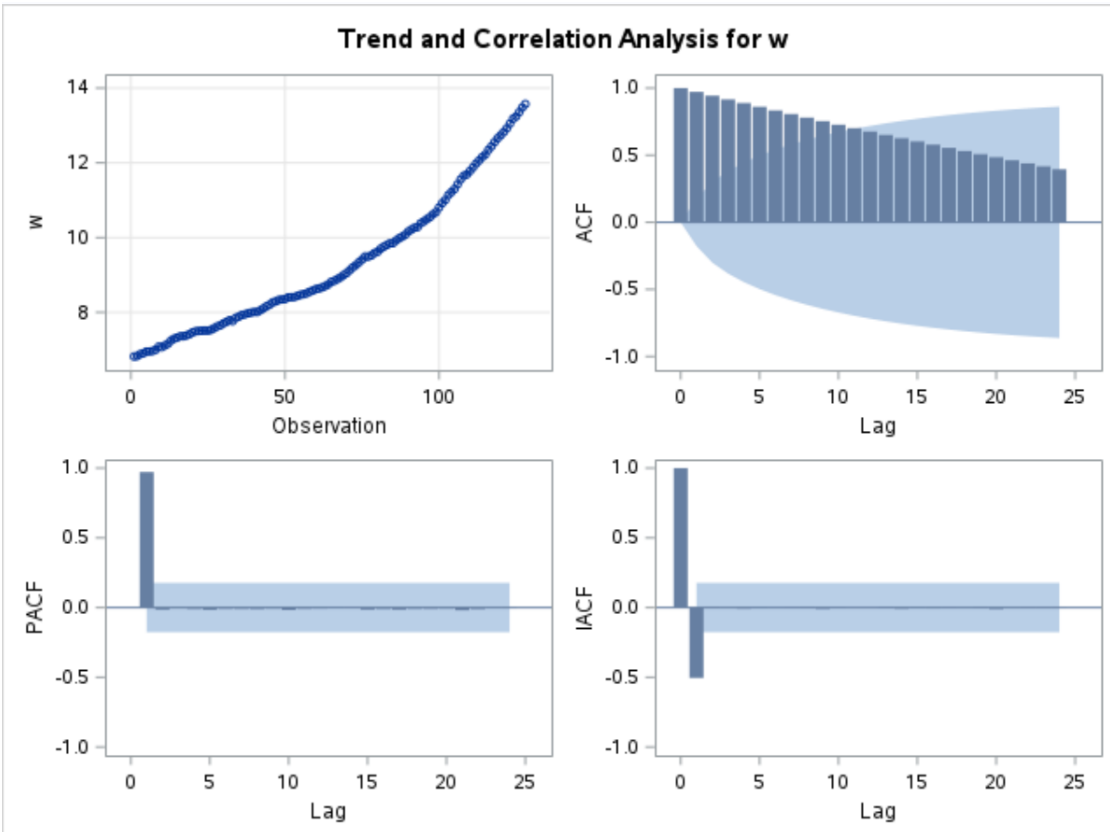
a) `q=(1)(2,12)` b) `q=(1,2)(12)` c) `q=(1,2,12)` d) `q=(2)(12)`
e) `p=(1)(2,12)` f) `p=(1,2)(12)` g)⋆ `p=(1,2,12)` h) `p=(2)(12)`

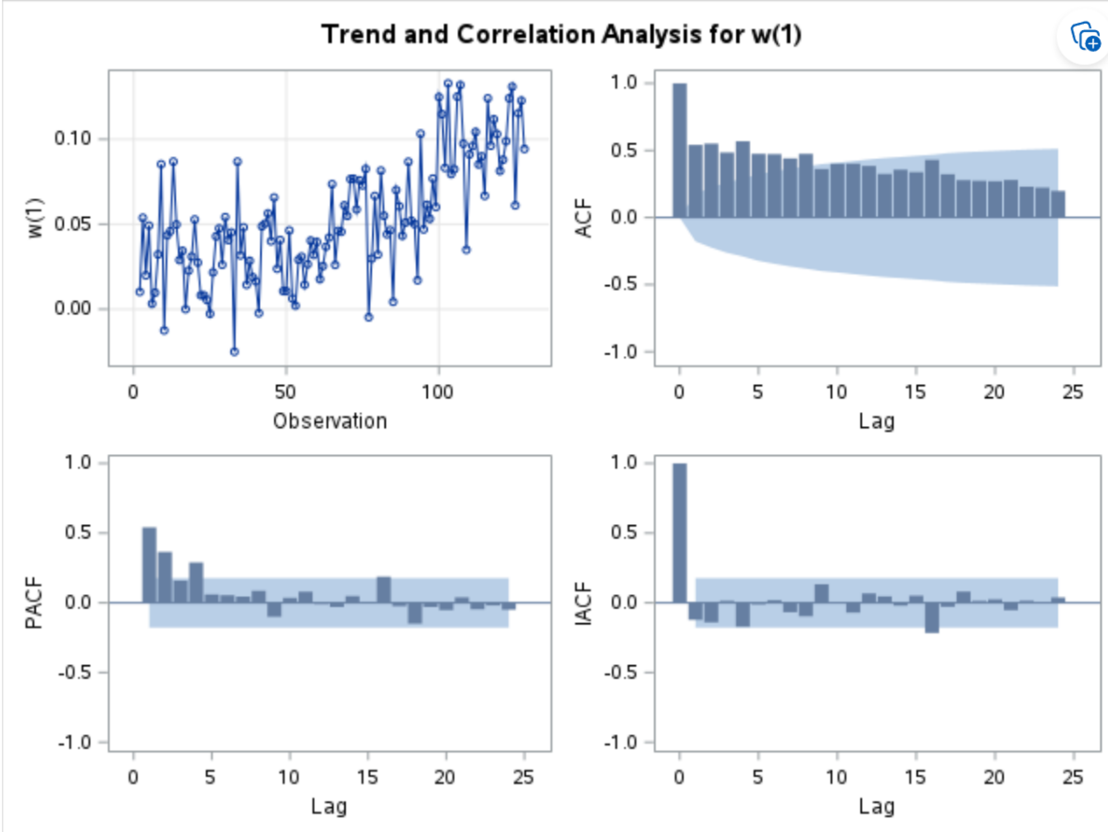
Maximum Likelihood Estimation

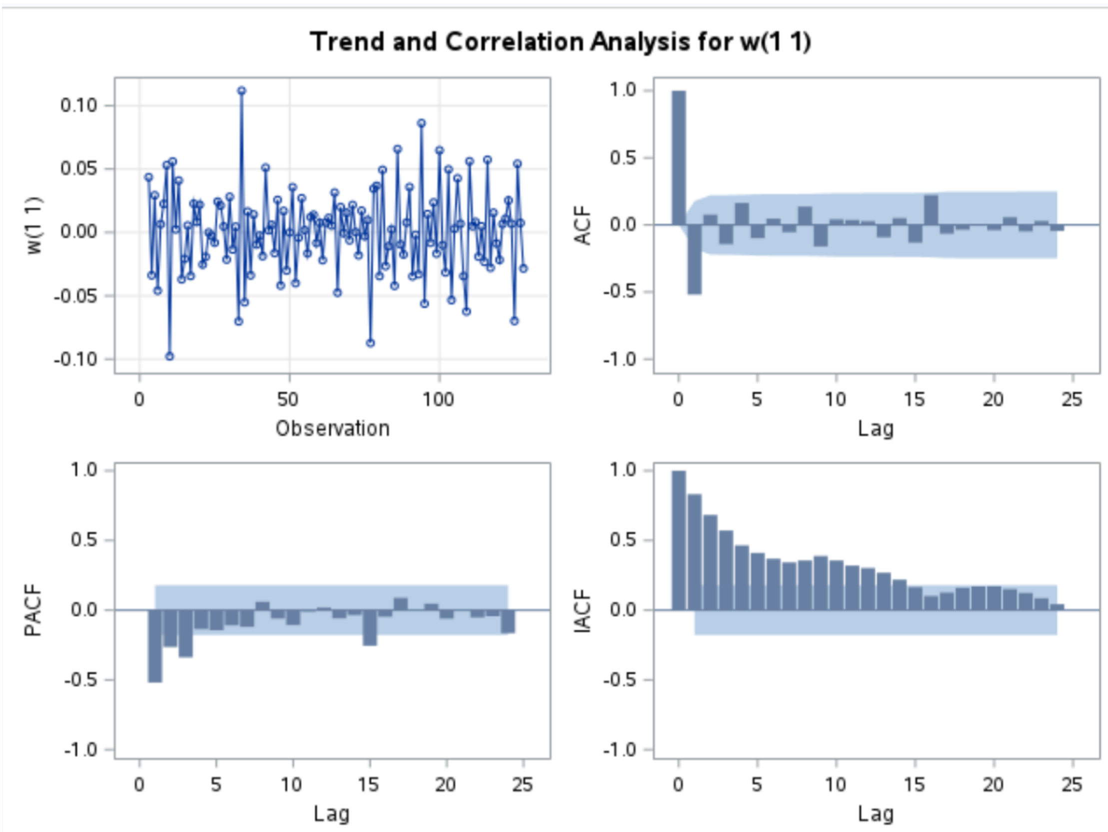
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.28585	0.04178	6.84	<.0001	0
AR1,1	0.34024	0.11096	3.07	0.0022	1
AR1,2	0.31666	0.11044	2.87	0.0041	2
AR1,3	-0.18476	0.09540	-1.94	0.0528	12

Problem 33. The following output gives the ACF/PACF/IACF for a time series w_t and its first and second differences. On the basis of this output, choose a plausible initial model for this time series.

a) ARIMA(1,0,0)	**b**) ARIMA(0,0,1)	**c**) ARIMA(0,0,2)
d)$\star$ ARIMA(0,2,1)	**e**) ARIMA(2,2,0)	**f**) ARIMA(0,2,3)
g) ARIMA(4,1,0)	**h**) ARIMA(0,1,4)	**i**) ARIMA(0,1,1)




Trend and Correlation Analysis for w(1)
w(1)
Observation
ACF
PACF
IACF
Lag


Trend and Correlation Analysis for w(1 1)
w(1 1)
Observation
ACF
PACF
IACF
Lag