Durbin-Watson (DW) Test

The DW test should be used whenever regression models are applied to data which are time ordered.

The Durbin-Watson Test is a test for serial correlation (also called auto-correlation) in the regression errors $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$. This is a type of dependence or relationship among the errors ε_t, which is a violation of the assumption of independence.

Positive serial correlation occurs when neighboring errors (in time) ε_t and ε_{t+1} tend to be similar. With positive serial correlation, positive errors tend to be followed by positive errors, and negative errors followed by negative errors.

Negative serial correlation occurs when neighboring errors (in time) tend to be dissimilar. With negative serial correlation, positive errors tend to be followed by negative errors, and negative errors followed by positive errors.

Positive serial correlation is very common, negative serial correlation less so.

The DW test is based on residuals: $e_t = \hat{\varepsilon}_t$, $t =$

$1, \ldots, n$

Durbin-Watson Statistic:

$$DW = \frac{\sum_{t=2}^{n}(e_t - e_{t-1})^2}{\sum_{t=1}^{n} e_t^2}$$

DW is used to test

$H_0 : \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are independent

versus

$H_1 : \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ display positive serial correlation

or

$H_1 : \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ display negative serial correlation

Under H_0, typically $DW \approx 2$.

Positive serial correlation leads to $DW \ll 2$.

Negative serial correlation leads to $DW \gg 2$.

SAS prints two different p-values:

$P(DW < DW_{\text{observed}})$ and $P(DW > DW_{\text{observed}})$ under H_0.

The first is for testing for positive serial correlation, and the second for negative serial correlation. If the first p-value is small, you reject H_0 in favor of the alternative of

positive serial correlation. If the second p-value is small, you reject H_0 in favor of the alternative of negative serial correlation.

SAS also displays the **first order autocorrelation** of the residuals:

$$r_1 = \frac{\sum_{t=2}^{n} e_t e_{t-1}}{\sum_{t=1}^{n} e_t^2}$$

The first order autocorrelation is positive when there is positive serial correlation, and negative for negative serial correlation. It is a correlation coefficient and takes values between -1 and 1. Values close to 1 indicate strong positive serial correlation. Values close to -1 indicate strong negative serial correlation.