The Lynx time series

Annual numbers of lynx trappings for 1821–1934 in Canada.
(Length $n = 114$)

Read across.

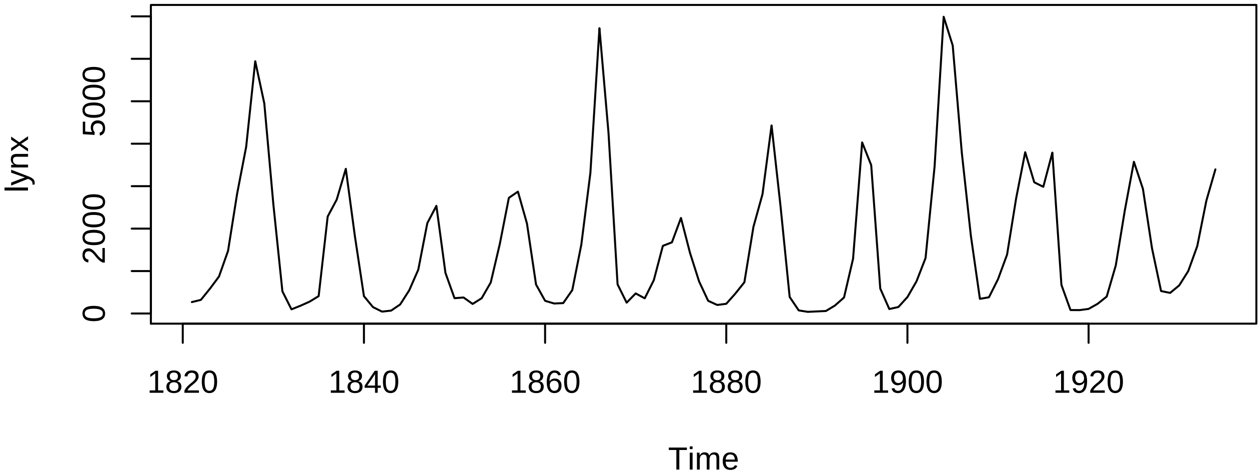
269	321	585	871	1475	2821	3928	5943	4950	2577	523
98	184	279	409	2285	2685	3409	1824	409	151	45
68	213	546	1033	2129	2536	957	361	377	225	360
731	1638	2725	2871	2119	684	299	236	245	552	1623
3311	6721	4254	687	255	473	358	784	1594	1676	2251
1426	756	299	201	229	469	736	2042	2811	4431	2511
389	73	39	49	59	188	377	1292	4031	3495	587
105	153	387	758	1307	3465	6991	6313	3794	1836	345
382	808	1388	2713	3800	3091	2985	3790	674	81	80
108	229	399	1132	2432	3574	2935	1537	529	485	662
1000	1590	2657	3396							

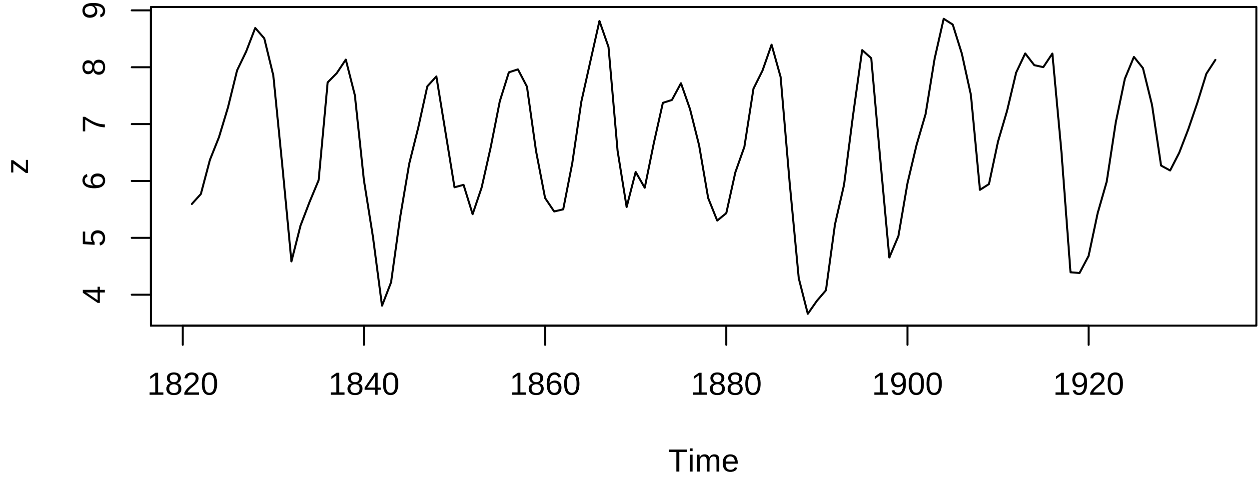
The (natural) log of the lynx series.

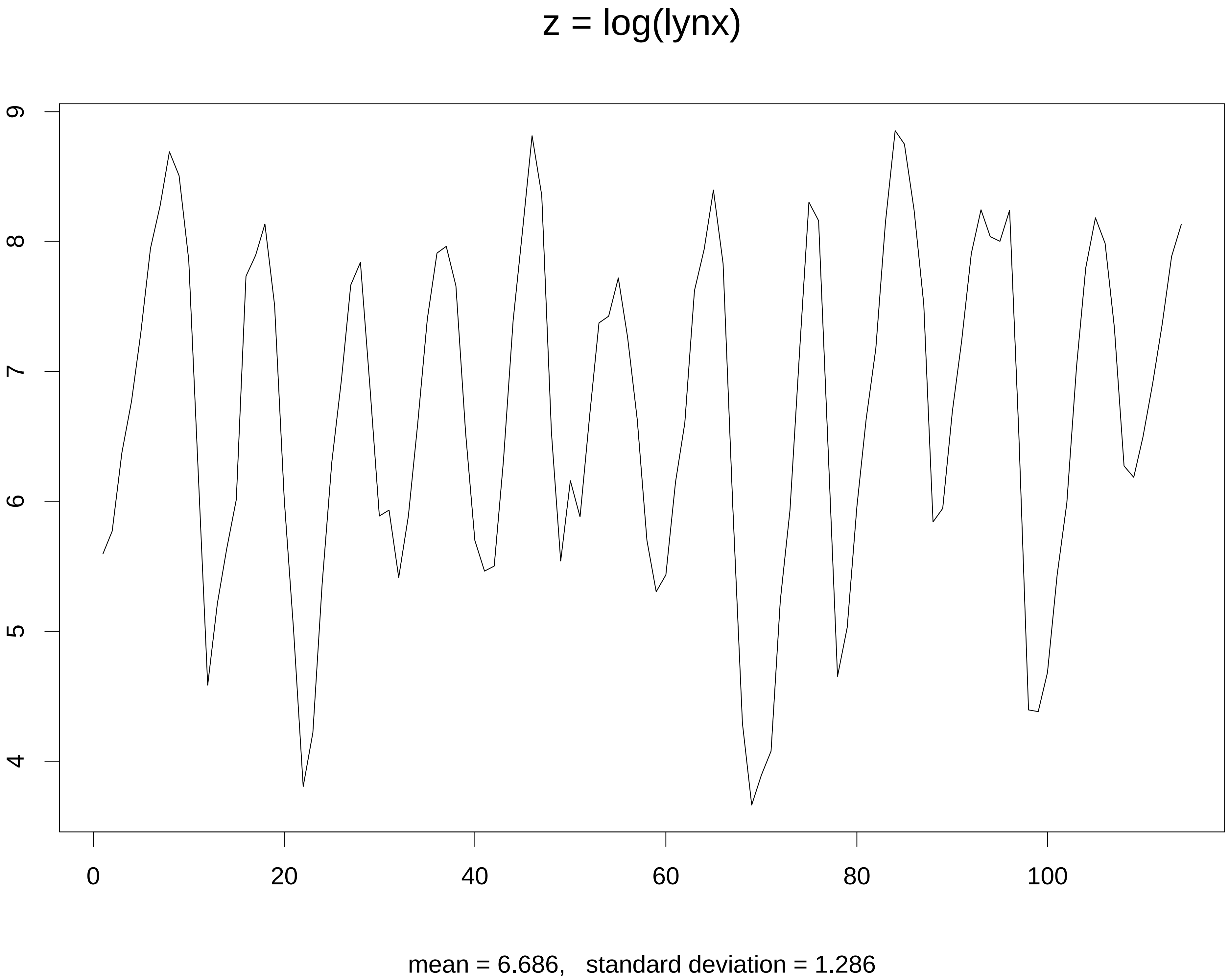
$z = \log(\text{lynx})$

```
 5.59 5.77 6.37 6.77 7.30 7.94 8.28 8.69 8.51 7.85 6.26
 4.58 5.21 5.63 6.01 7.73 7.90 8.13 7.51 6.01 5.02 3.81
 4.22 5.36 6.30 6.94 7.66 7.84 6.86 5.89 5.93 5.42 5.89
 6.59 7.40 7.91 7.96 7.66 6.53 5.70 5.46 5.50 6.31 7.39
 8.11 8.81 8.36 6.53 5.54 6.16 5.88 6.66 7.37 7.42 7.72
 7.26 6.63 5.70 5.30 5.43 6.15 6.60 7.62 7.94 8.40 7.83
 5.96 4.29 3.66 3.89 4.08 5.24 5.93 7.16 8.30 8.16 6.38
 4.65 5.03 5.96 6.63 7.18 8.15 8.85 8.75 8.24 7.52 5.84
 5.95 6.69 7.24 7.91 8.24 8.04 8.00 8.24 6.51 4.39 4.38
 4.68 5.43 5.99 7.03 7.80 8.18 7.98 7.34 6.27 6.18 6.50
 6.91 7.37 7.88 8.13
```

```
mean of z = 6.686
standard deviation of z = 1.286
```


lynx
0
2000
5000
1820
1840
1860
1880
1900
1920
Time


z
4
5
6
7
8
9
1820
1840
1860
1880
1900
1920
Time


z = log(lynx)
4
5
6
7
8
9
0
20
40
60
80
100
mean = 6.686,   standard deviation = 1.286

Lagged Variables and Lagged Scatter Plots Illustrated on the Lynx Data

z = log(lynx)

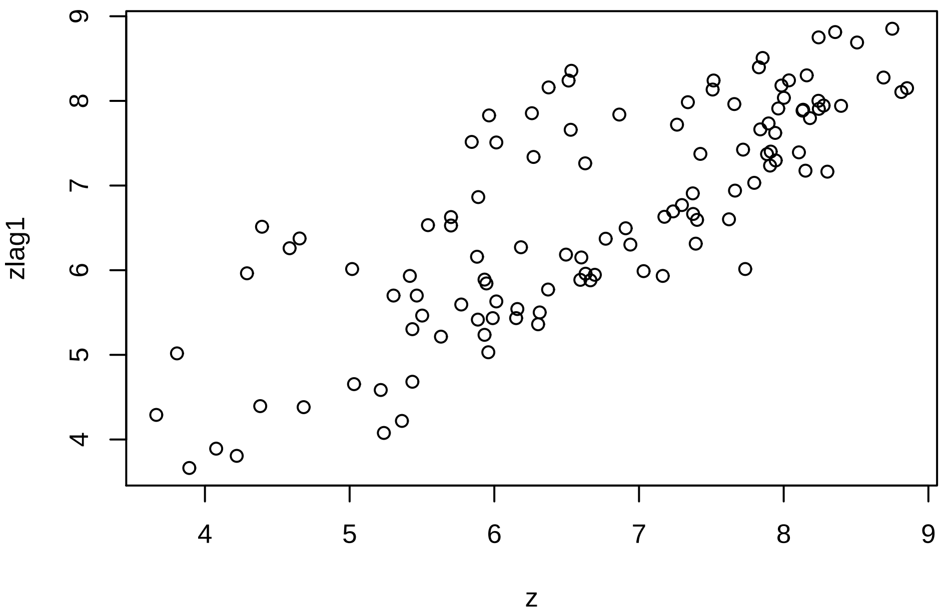
First column is time (years numbered 1 − 114)

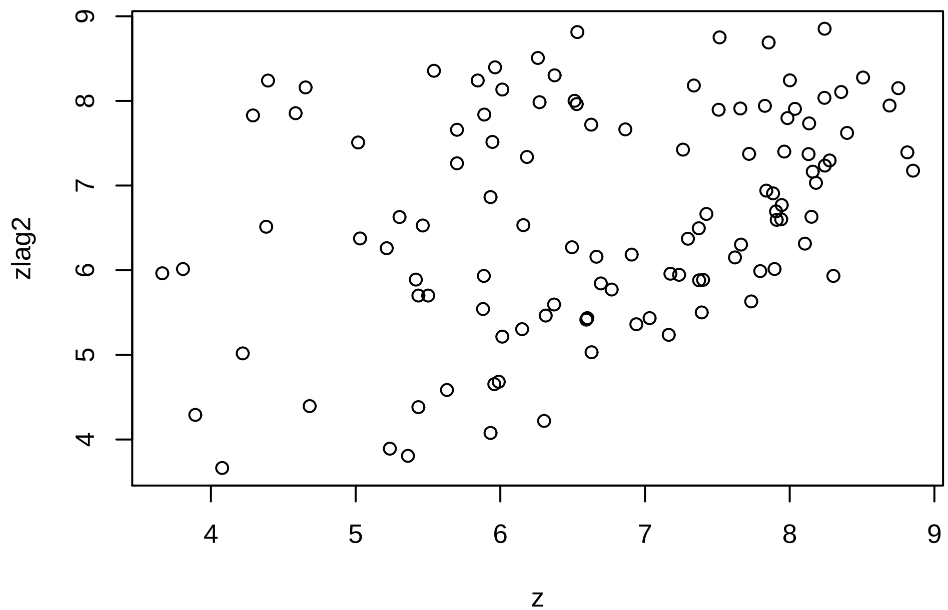
Columns 3 − 10 are obtained by "lagging" the lynx data.

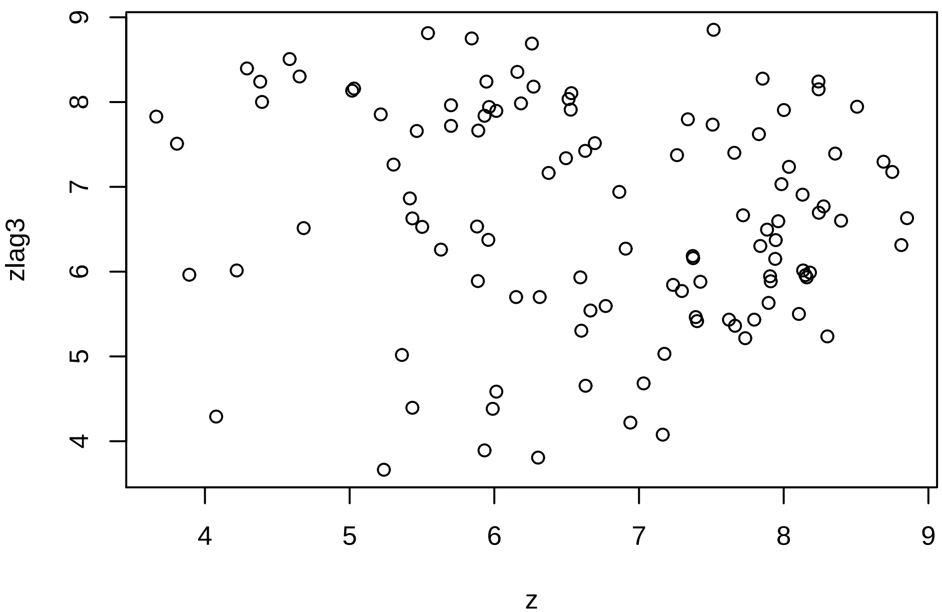
	z	zlag1	zlag2	zlag3	zlag4	zlag5	zlag6	zlag7	zlag8
1	5.59	.	.	.	.	.	.	.	.
2	5.77	5.59	.	.	.	.	.	.	.
3	6.37	5.77	5.59	.	.	.	.	.	.
4	6.77	6.37	5.77	5.59	.	.	.	.	.
5	7.30	6.77	6.37	5.77	5.59	.	.	.	.
6	7.94	7.30	6.77	6.37	5.77	5.59	.	.	.
7	8.28	7.94	7.30	6.77	6.37	5.77	5.59	.	.
8	8.69	8.28	7.94	7.30	6.77	6.37	5.77	5.59	.
9	8.51	8.69	8.28	7.94	7.30	6.77	6.37	5.77	5.59
10	7.85	8.51	8.69	8.28	7.94	7.30	6.77	6.37	5.77
11	6.26	7.85	8.51	8.69	8.28	7.94	7.30	6.77	6.37
12	4.58	6.26	7.85	8.51	8.69	8.28	7.94	7.30	6.77

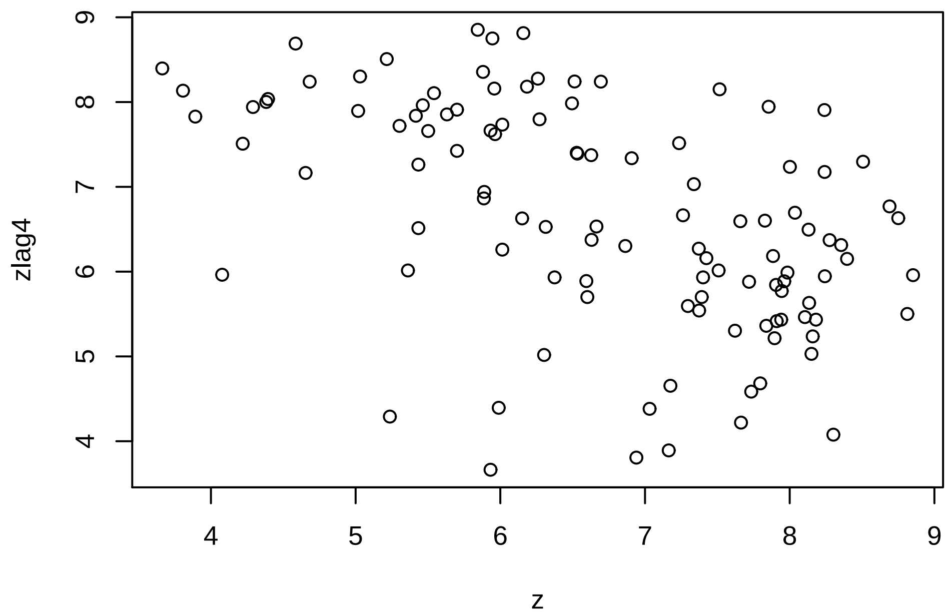
(Rows 13 through 108 are omitted)

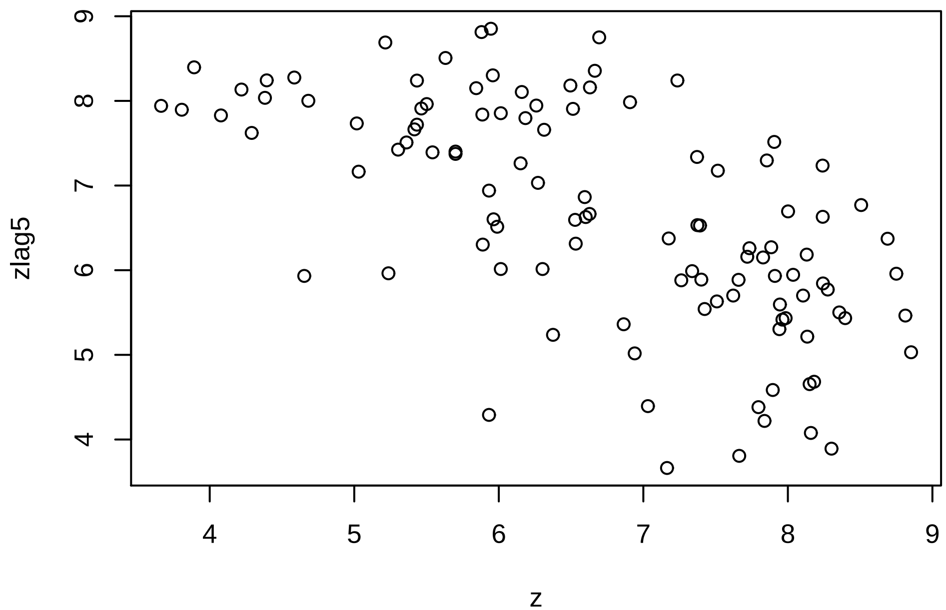
109	6.18	6.27	7.34	7.98	8.18	7.80	7.03	5.99	5.43
110	6.50	6.18	6.27	7.34	7.98	8.18	7.80	7.03	5.99
111	6.91	6.50	6.18	6.27	7.34	7.98	8.18	7.80	7.03
112	7.37	6.91	6.50	6.18	6.27	7.34	7.98	8.18	7.80
113	7.88	7.37	6.91	6.50	6.18	6.27	7.34	7.98	8.18
114	8.13	7.88	7.37	6.91	6.50	6.18	6.27	7.34	7.98

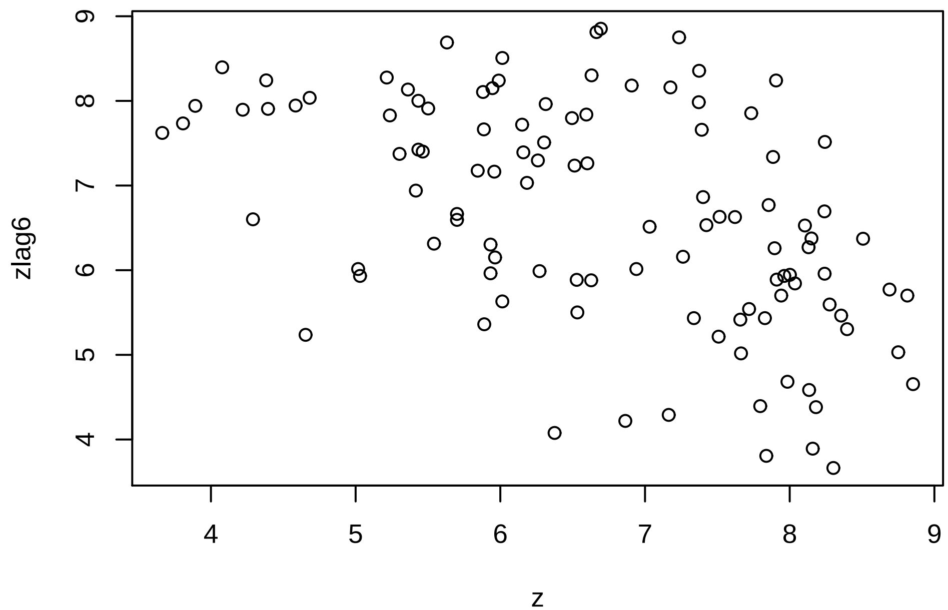

zlag1
z
4
5
6
7
8
9

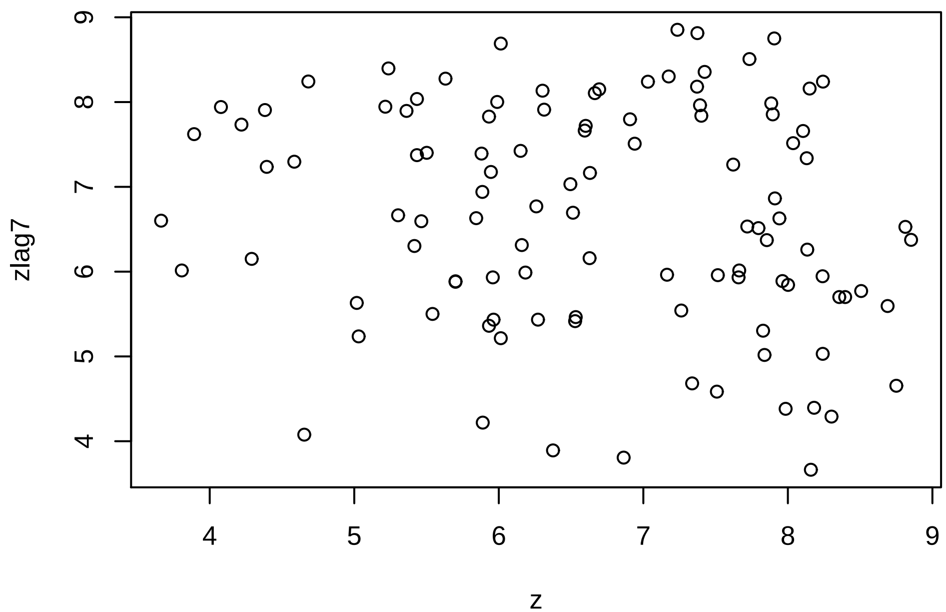

zlag2
z
4
5
6
7
8
9

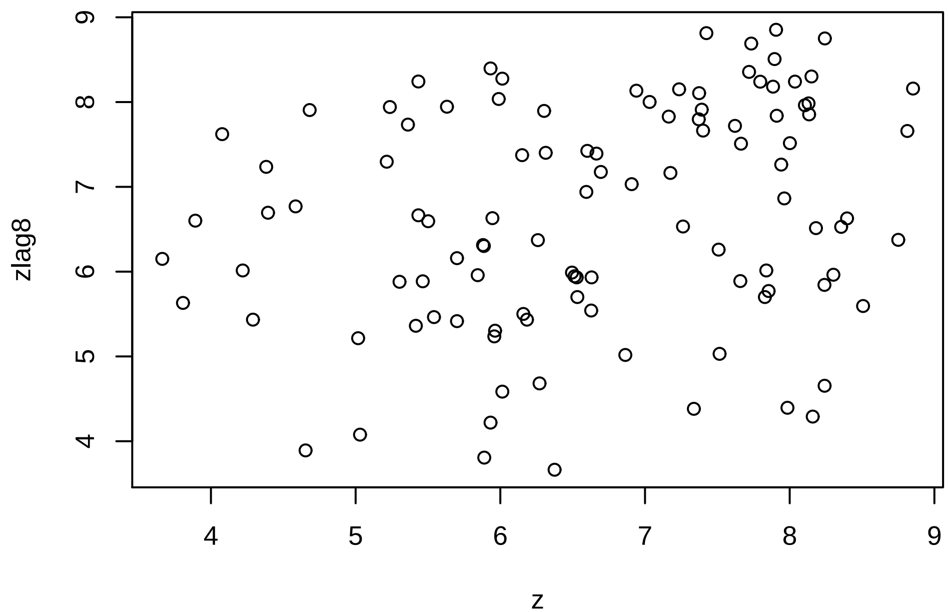

zlag3
z
4
5
6
7
8
9

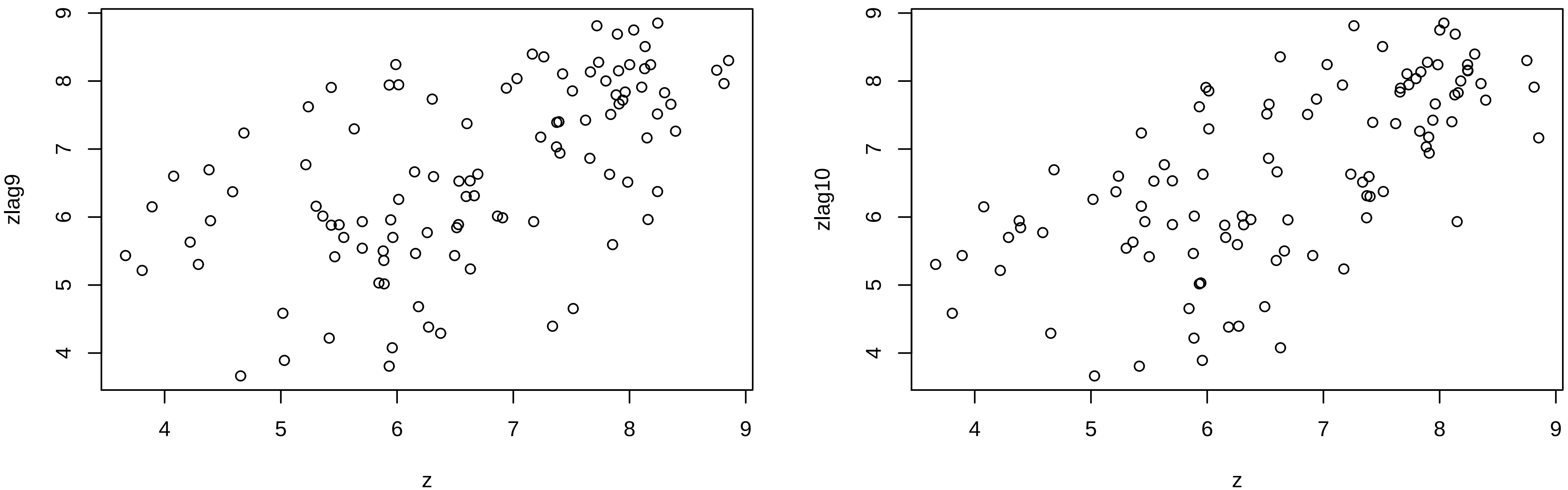

zlag4
z
4
5
6
7
8
9


zlag5
z
4
5
6
7
8
9


zlag6
z
4
5
6
7
8
9


zlag7
z
4
5
6
7
8
9

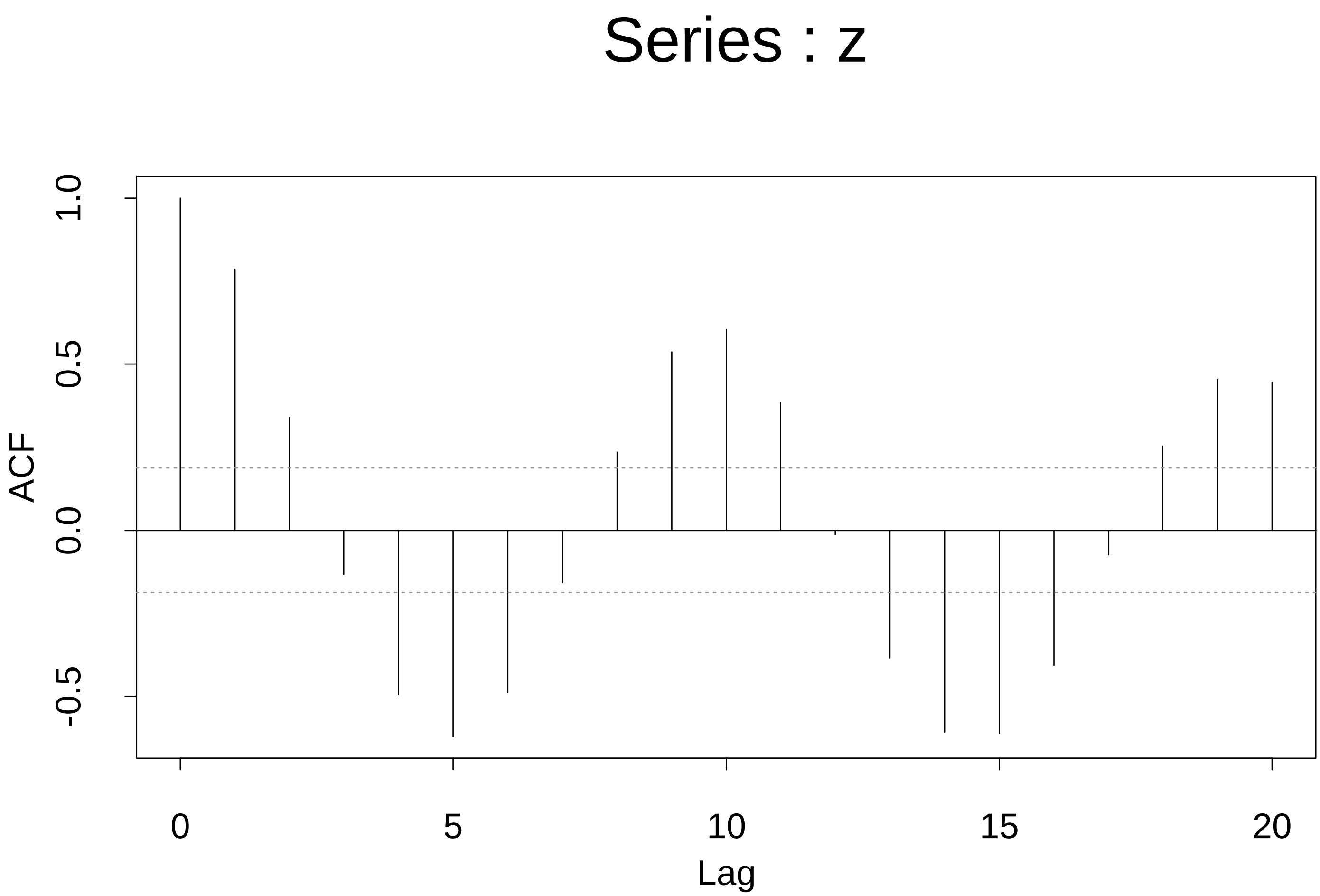

zlag8
z
4
5
6
7
8
9

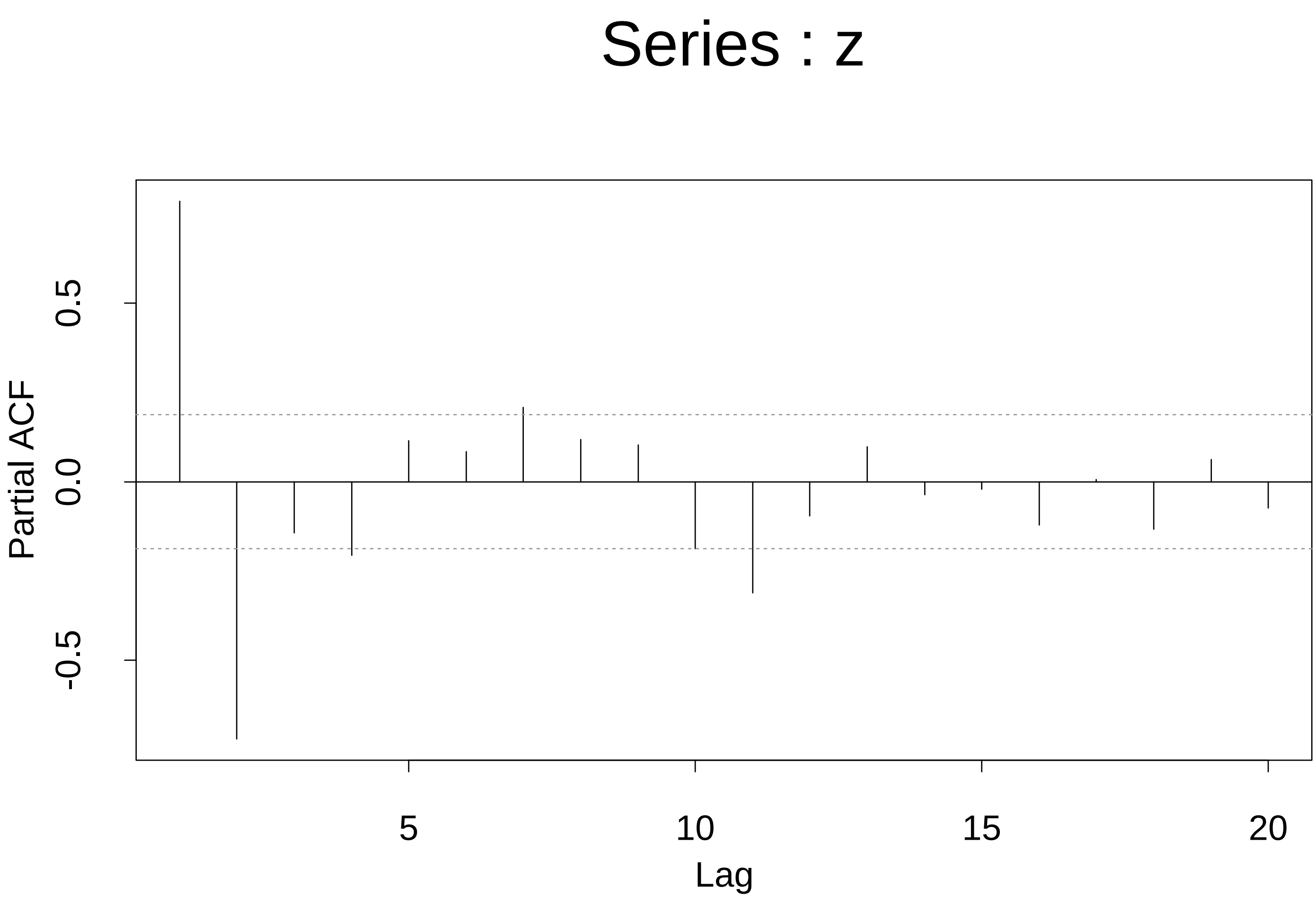

zlag9
z
zlag10
z
4
5
6
7
8
9

ACF and PACF

The Autocorrelation Function and Partial Autocorrelation Function of z = log(lynx) for the first 20 lags.

LAG	ACF	PACF
0	1.0000	.
1	0.7851	0.7851
2	0.3402	-0.7200
3	-0.1323	-0.1431
4	-0.4939	-0.2062
5	-0.6205	0.1152
6	-0.4879	0.0846
7	-0.1578	0.2077
8	0.2349	0.1184
9	0.5372	0.1028
10	0.6055	-0.1869
11	0.3829	-0.3110
12	-0.0123	-0.0955
13	-0.3848	0.0969
14	-0.6073	-0.0359
15	-0.6102	-0.0220
16	-0.4069	-0.1208
17	-0.0727	0.0057
18	0.2532	-0.1328
19	0.4550	0.0624
20	0.4465	-0.0740


Series : z
1.0
0.5
0.0
-0.5
ACF
0
5
10
15
20
Lag


Series : z
0.5
0.0
-0.5
Partial ACF
5
10
15
20
Lag

Interpretation of sample ACF and PACF

ACF

The autocorrelation at lag k is (almost) the correlation in the scatter plot of z versus z lagged by k.

> Note: If the original time series has length n, the series lagged by k will have k missing values so that the lagged scatterplot has only $n-k$ points on it (points with missing values are omitted).

Compare the following with the ACF given earlier:

```
corr. between z and zlag1 =  0.7922  (113 data points)
corr. between z and zlag2 =  0.3457  (112 data points)
corr. between z and zlag3 = -0.1342  (111 data points)
corr. between z and zlag4 = -0.5020  (110 data points)
etc.
```

PACF

The partial autocorrelation at lag k is (almost) the estimated regression coefficient for zlagk when doing the standard regression (OLS) of z on the variables zlag1, zlag2, ..., zlagk.

Compare the following with the PACF given earlier:

```
------------------------------------------
Regression of z on zlag1 gives

Coefficients:
 (Intercept)     zlag1
    1.396133 0.7941462

------------------------------------------
Regression of z on zlag1 and zlag2 gives

Coefficients:
 (Intercept)    zlag1      zlag2
    2.435215 1.384238 -0.7477757

---------------------------------------------
Regression of z on zlag1, zlag2 and zlag3 gives

Coefficients:
 (Intercept)    zlag1      zlag2    zlag3
    2.719594 1.295248 -0.5819785 -0.11964

----------------------------------------------
etc.
```