The Linear Transfer Function (LTF) Identification Method

An alternative to the "pre-whitening" approach to identifying transfer functions is the linear transfer function (LTF) method. (See Pankratz for a detailed description.)

The pre-whitening approach is difficult to use if there are two or more input variables because there usually exists no filter which simultaneously whitens all the input variables. Also, correlations between the input variables complicate matters. With correlated inputs, the cross-correlation functions may no longer be proportional to the transfer functions.

Suppose you wish to identify the transfer functions in the model:

$$Y_t = C + \frac{B^{b_1}\omega_1(B)}{\delta_1(B)} X_{1,t} + \frac{B^{b_2}\omega_2(B)}{\delta_2(B)} X_{2,t} + N_t$$

Temporarily replace the transfer functions by simple linear transfer functions (also called a free-form distributed lag model):

$$Y_t = C + v_1(B) X_{1,t} + v_2(B) X_{2,t} + N_t$$

where

$$\begin{aligned} v_1(B) &= v_{1,0} + v_{1,1}B + v_{1,2}B^2 + \cdots + v_{1,K}B^K \\ v_2(B) &= v_{2,0} + v_{2,1}B + v_{2,2}B^2 + \cdots + v_{2,K}B^K \end{aligned}$$

The number of lags K should be reasonably large but not too large.

We expect the noise term N_t to be auto-correlated. Pick a simple 'proxy' model for the noise. Say, AR(1) or AR(2) and include a seasonal AR term ($P = 1$) if seasonality is expected. This proxy model is not likely to be correct, but is better than nothing, and will give the model enough validity so that the reported *p*-values will be of some use in making judgements of significance.

Procedure:

- Fit (estimate) the model:

$$Y_t = C + v_1(B)X_{1,t} + v_2(B)X_{2,t} + \frac{1}{(1 - \phi_1 B)(1 - \Phi_1 B^s)}\, a_t$$

- Compute estimated values for the noise process

$$\hat{N}_t = Y_t - \hat{C} - \hat{v}_1(B)X_{1,t} - \hat{v}_2(B)X_{2,t}$$

for all "possible" t.

- Study the series $\{\hat{N}_t\}$. Look at the time series plot, and the sample ACF and PACF.
- If $\{\hat{N}_t\}$ seems stationary, then choose a (tentative) ARMA model (orders p, q) for N_t and choose (tentative) transfer functions (e.g., orders b_1, h_1, r_1, b_2, h_2, r_2) for

$$\frac{B^{b_1}\omega_1(B)}{\delta_1(B)} \quad \text{and} \quad \frac{B^{b_2}\omega_2(B)}{\delta_2(B)}$$

  based on the pattern of the estimated v-weights $\hat{v}_1(B)$ and $\hat{v}_2(B)$. Try this model.
- If $\{\hat{N}_t\}$ is not stationary but has a nonconstant mean, then choose the type of differencing that seems most likely to help ($\nabla = 1 - B$ or $\nabla_s = 1 - B^s$) and assume $n_t = \nabla N_t$ (or $n_t = \nabla_s N_t$) is stationary. Note that

$$n_t = \nabla N_t \;\Rightarrow\; N_t = \frac{n_t}{\nabla}$$

so that our model becomes

$$Y_t = C + v_1(B)X_{1,t} + v_2(B)X_{2,t} + \frac{n_t}{\nabla} \quad \text{or equivalently}$$
$$\nabla Y_t = C' + v_1(B)\nabla X_{1,t} + v_2(B)\nabla X_{2,t} + n_t \,. \qquad (*)$$

- Fit the model $(*)$ with a proxy (say ARIMA$(1,0,0)(1,0,0)_s$) for the noise model

$$\nabla Y_t = C' + v_1(B)\nabla X_{1,t} + v_2(B)\nabla X_{2,t} + \frac{1}{(1-\phi_1 B)(1-\Phi_1 B^s)}\, a_t$$

and compute the estimated values for the noise process

$$\hat{n}_t = \nabla Y_t - \hat{C}' - \hat{v}_1(B)\nabla X_{1,t} - \hat{v}_2(B)\nabla X_{2,t}$$

for all "possible" t.

- Study the series $\{\hat{n}_t\}$. Look at the time series plot, and the sample ACF and PACF. If it seems stationary, identify a tentative ARMA model for n_t (based on the sample ACF/PACF) and tentative forms for the transfer functions (based on the pattern of the estimated v-weights).
- If n_t is not stationary, then (if it seems likely to help) consider additional differencing and follow a similar procedure.