Intervention Analysis

Time Series are often affected by external events such as holidays, strikes, sales, promotions, and other policy changes. These external events are called **Interventions**.

Outliers in time series are often caused by interventions.

Additive Outliers: A simple model for a one-period "impulse" event, an event affecting the time series only at one time, is given by

$$Y_t = \omega_0 X_t + N_t \qquad (*)$$

where N_t is an ARIMA process, and

$$X_t = \begin{cases} 1 & \text{if } t = t_{\text{event}} \\ 0 & \text{if } t \neq t_{\text{event}} . \end{cases}$$

We will call this series X_t an "impulse" function.

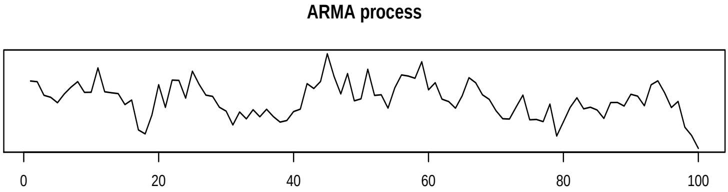
Level Shift (LS): Step interventions. A simple model for an event which moves a time series to a new level is

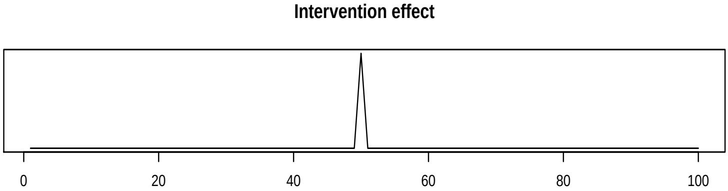
$$Y_t = \omega_0 X_t + N_t \qquad (*)$$

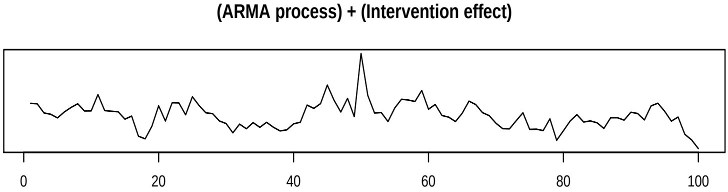
with X_t replaced by a "step" function:

$$X_t = \begin{cases} 1 & \text{if } t \geq t_{\text{event}} \\ 0 & \text{if } t < t_{\text{event}} \,. \end{cases}$$

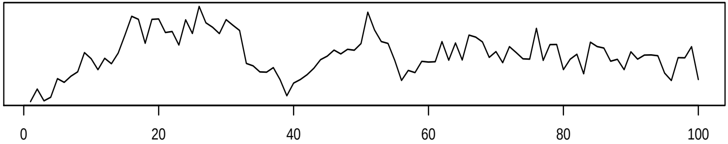
Examples of simple **Impulse** and **Step** interventions are given on the next two pages.


ARMA process
0
20
40
60
80
100

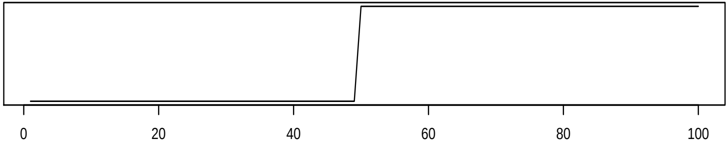

Intervention effect
0
20
40
60
80
100


(ARMA process) + (Intervention effect)
0
20
40
60
80
100

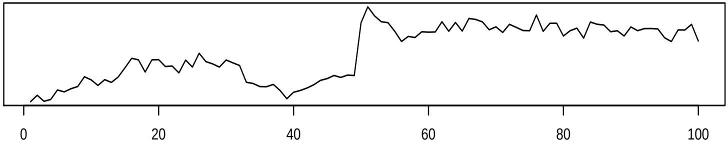
ARMA process



Intervention effect



(ARMA process) + (Intervention effect)



If an event affects the series at more than one point of time, but the effects are short-term, we might use a transfer function to model the effects of the event:

$$Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + N_t \qquad (\dagger)$$

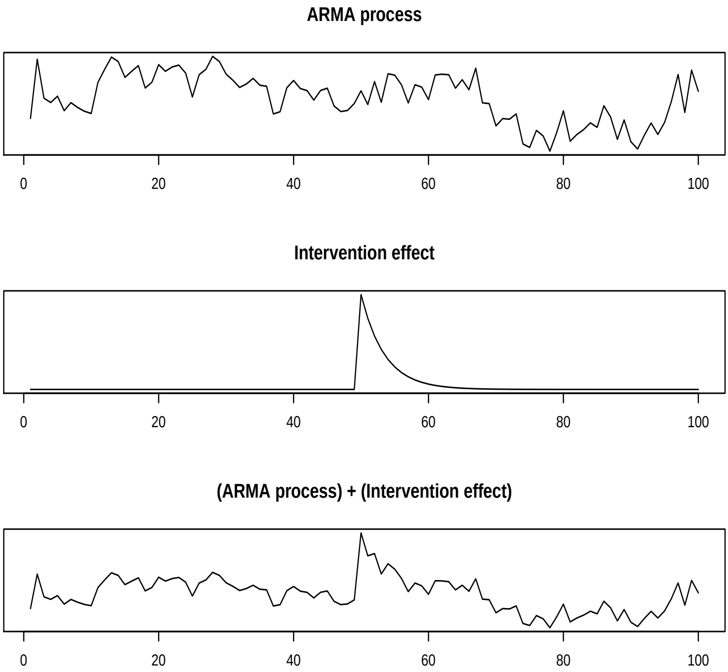
where X_t is an impulse function.

Similarly, if an event permanently moves a series to a new level, but this change is not immediate, but occurs over several time periods, we might use ($\dagger$) with a step function for X_t.

The next two pages give illustrations of intervention effects having the special form

$$Y_t = \frac{1}{1 - \delta_1 B} X_t + N_t$$

where X_t is an **impulse** (page 1) or a **step** (page 2) function.


ARMA process
0
20
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80
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Intervention effect
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(ARMA process) + (Intervention effect)
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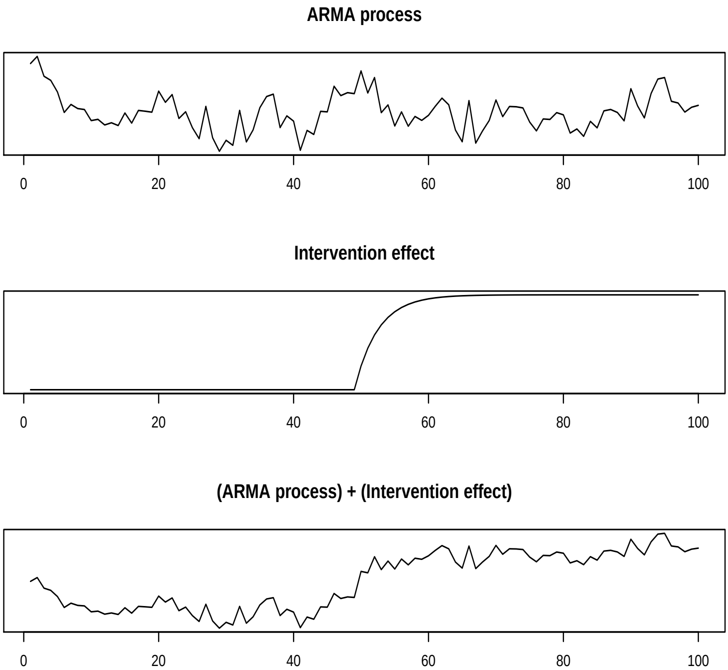
ARMA process
0
20
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80
100
Intervention effect
0
20
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60
80
100
(ARMA process) + (Intervention effect)
0
20
40
60
80
100

Identifying Transfer Functions for Intervention Analysis

The transfer function is chosen by observing the pattern of change in the time series and comparing this with known patterns (see later discussion). (Or perhaps it is chosen by intuition concerning the likely changes an event will produce.)

It is **not** chosen by studying cross-correlations between Y_t and X_t, which are generally useless for deterministic input series (such as impulse or step functions, or trend functions).

Write the transfer function model as

$$Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + N_t$$

$$\text{or} \quad Y_t = v(B) X_t + N_t \qquad \text{where} \quad v(B) = \frac{B^b \omega(B)}{\delta(B)}$$

$$\text{or} \quad Y_t = C_t + N_t \qquad \text{where } C_t = v(B) X_t \, .$$

Here C_t is the change in Y_t at time t due to the intervention. (C_t is the intervention effect at time t.)

For an Impulse Intervention: If X_t is an **impulse** at time t_{event}, then C_t is the same as the v-weights shifted (to the right) by t_{event}.

So we compare the pattern of change in the time series with the pattern of v-weights for various transfer functions, and pick one that is similar.

For Step Interventions: If X_t is a **step** at time t_{event}, then C_t is the same as the **cumulative sums** of the v-weights shifted (to the right) by t_{event}.

This means

$$C_t = \begin{cases} 0 & \text{for } t < t_{\text{event}} \\ v_0 & \text{for } t = t_{\text{event}} \\ v_0 + v_1 & \text{for } t = t_{\text{event}} + 1 \\ v_0 + v_1 + v_2 & \text{for } t = t_{\text{event}} + 2 \\ v_0 + v_1 + v_2 + v_3 & \text{for } t = t_{\text{event}} + 3 \\ \text{etc.} & \end{cases}$$

So we compare the pattern of change in the time series with the patterns of the cumulative sums of the v-weights for various transfer functions, and pick one that is similar.

We will look at various possibilities later.

Models with non-stationary noise processes N_t: If N_t is a non-stationary ARIMA process, SAS PROC ARIMA requires us to write (†) in an equivalent form involving differencing of Y_t and X_t.

For example, if N_t is ARIMA(1,1,1), then

$$N_t = \frac{1 - \theta_1 B}{(1 - B)(1 - \phi_1 B)}\, a_t$$

and (†) becomes

$$Y_t = \frac{B^b \omega(B)}{\delta(B)} X_t + \frac{1 - \theta_1 B}{(1 - B)(1 - \phi_1 B)}\, a_t$$

which is equivalent to (using $\nabla = 1 - B$)

$$\nabla Y_t = \frac{B^b \omega(B)}{\delta(B)} \nabla X_t + \frac{1 - \theta_1 B}{(1 - \phi_1 B)}\, a_t$$

which is the form PROC ARIMA requires.