The v-weights for Some Transfer Functions

Recall that: $v(B) = \sum_{i=0}^{\infty} v_i B^i = \frac{B^b \omega(B)}{\delta(B)}$ with

$\omega(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_h B^h$,

$\delta(B) = 1 - \delta_1 B - \delta_2 B^2 - \cdots - \omega_r B^r$.

Beware the sign convention used by SAS!

For example, if $b = 0$ and $r = 0$, then:

$$v(B) = \omega_0 - \omega_1 B - \omega_2 B^2 - \cdots - \omega_h B^h$$

means that $v_0 = \omega_0, v_1 = -\omega_1, \ldots, v_h = -\omega_h$ (and $v_i = 0$ for $i > h$).

Comments:

• The lag of the first nonzero v-weight is b. (always)

• A nonzero value of b shifts the pattern of v-weights by b lags. (always)

• If $r = 0$, the lag of the last nonzero v-weight is $b + h$. There is a cutoff after lag $b + h$.

Some cases with $r = 0$ (no denominator)

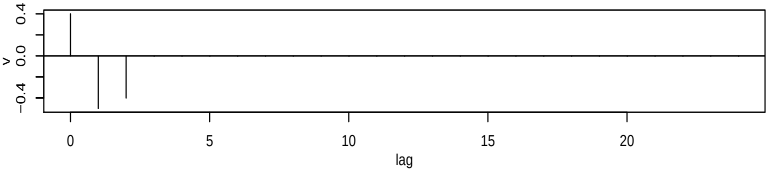
On the next page are plots of:

$v(B) = 0.4 - 0.5B - 0.4B^2$

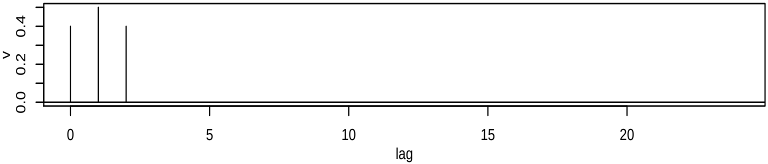
$v(B) = 0.4 - (-0.5B) - (-0.4B^2)$

$v(B) = B^3\left(0.4 - (-0.5B) - (-0.4B^2)\right)$

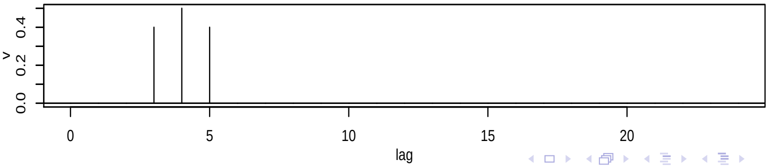
Transfer function for omegas = 0.4, 0.5, 0.4



Transfer function for omegas = 0.4, −0.5, −0.4



Transfer function for omegas = 0.4, −0.5, −0.4, b = 3



Some cases with $r = 1$ and $h = 0$

Recall: $\frac{\omega_0}{1 - \delta B} = \omega_0 + \omega_0 \delta_1 B + \omega_0 \delta_1^2 B^2 + \omega_0 \delta_1^3 B^3 + \cdots$

If $r = 1$, the v-weights decay exponentially (with alternating decay if $\delta_1 < 0$) starting with lag b.

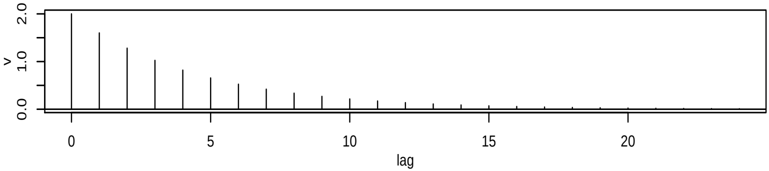
On the next page are plots of:

$$v(B) = \frac{2}{1 - 0.8B}$$

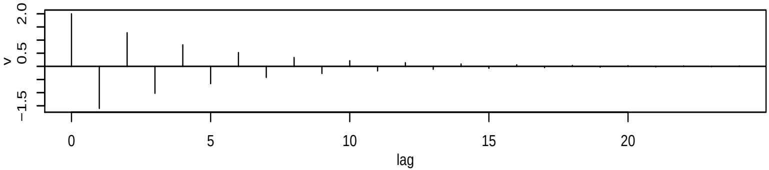
$$v(B) = \frac{2}{1 - (-0.8)B}$$

$$v(B) = \frac{2B^3}{1 - 0.8B}$$

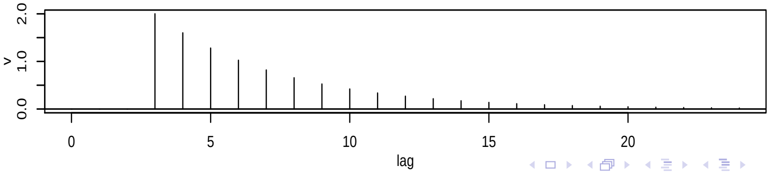
Transfer function for omegas = 2, deltas = 0.8



Transfer function for omegas = 2, deltas = −0.8



Transfer function for omegas = 2, deltas = 0.8, b = 3



Some Cases with $r = 1$ and $h > 0$

When $r = 1$, the v-weights decay exponentially starting with lag $b + h$.

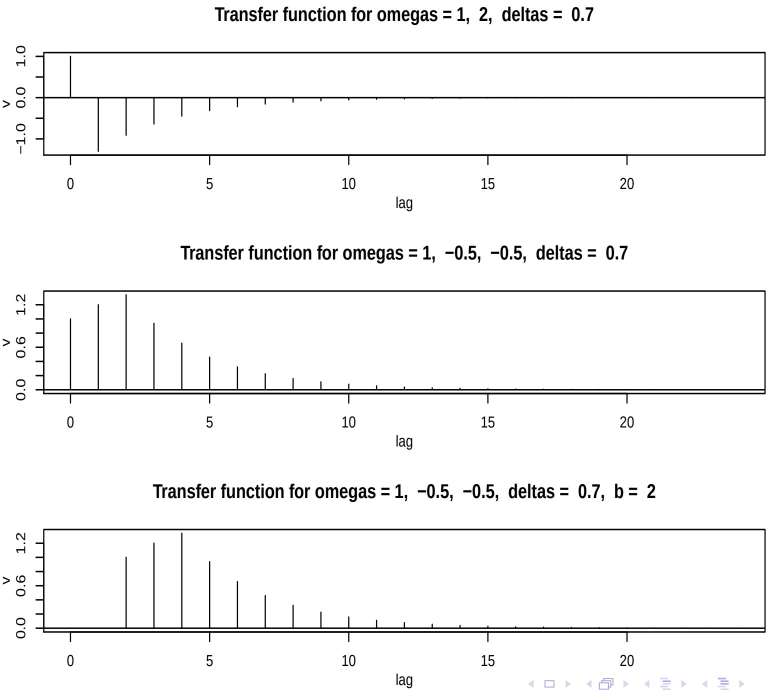
The v-weights $v_b, v_{b+1}, \ldots, v_{b+h}$ can assume arbitrary values depending on $\omega_0, \omega_1, \ldots, \omega_h$.

On the next page are plots of:

$$v(B) = \frac{1 - 2B}{1 - 0.7B} \quad (h = 1)$$

$$v(B) = \frac{1 + 0.5B + 0.5B^2}{1 - 0.7B} \quad (h = 2)$$

$$v(B) = \frac{B^2(1 + 0.5B + 0.5B^2)}{1 - 0.7B} \quad (h = 2)$$


Transfer function for omegas = 1, 2, deltas = 0.7
v
lag
Transfer function for omegas = 1, −0.5, −0.5, deltas = 0.7
v
lag
Transfer function for omegas = 1, −0.5, −0.5, deltas = 0.7, b = 2
v
lag

$r = 2$ can give Sinusoidal Decay

On the next page are plots of:

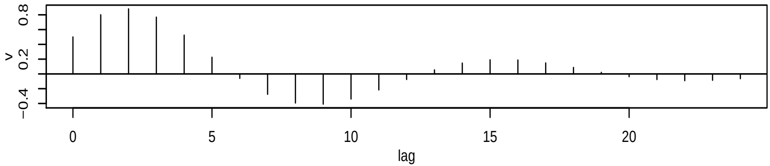
$$v(B) = \frac{0.5}{1 - 1.6B - (-0.8)B^2} \quad (h = 0)$$

$$v(B) = \frac{0.5 - 0.3B}{1 - 1.6B - (-0.8)B^2} \quad (h = 1)$$

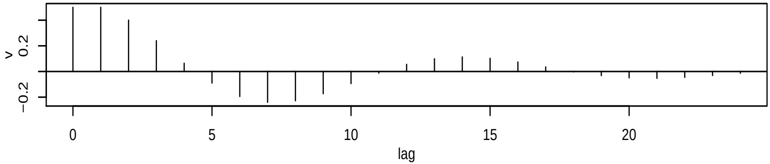
$$v(B) = \frac{B^2(0.5 - 0.3B)}{1 - 1.6B - (-0.8)B^2} \quad (h = 1)$$

The v-weights $v_b, v_{b+1}, \ldots, v_{b+h}$ can assume arbitrary values depending on $\omega_0, \omega_1, \ldots, \omega_h$. Then sinusoidal decay starts with lag $b + h$.

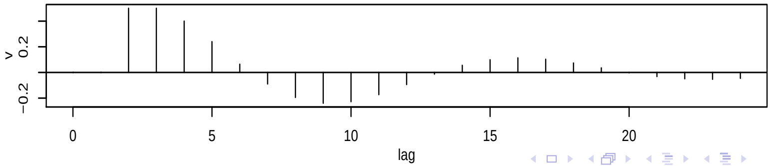
Transfer function for omegas = 0.5, deltas = 1.6, −0.8



Transfer function for omegas = 0.5, 0.3, deltas = 1.6, −0.8



Transfer function for omegas = 0.5, 0.3, deltas = 1.6, −0.8, b = 2



But $r = 2$ can also give other kinds of behavior

It doesn't have to be sinusoidal decay; it could also be a weighted sum (or difference) of two different exponential decay patterns.

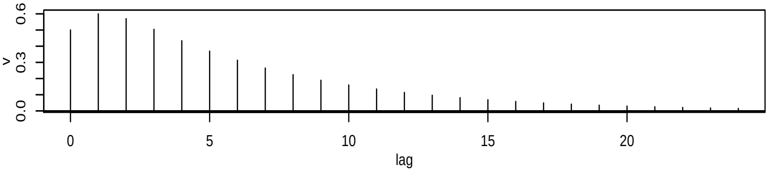
On the next page are plots of:

$$v(B) = \frac{0.5}{1 - 1.2B - (-0.3)B^2}$$

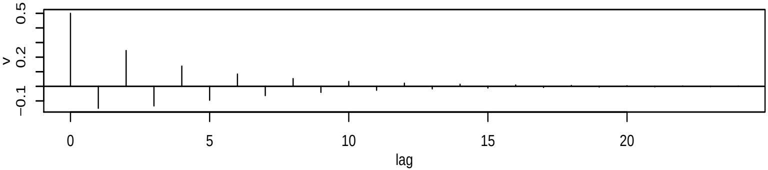
$$v(B) = \frac{0.5}{1 - (-0.3)B - 0.4B^2}$$

$$v(B) = \frac{0.5}{1 - (-1.3)B - (-0.4)B^2}$$

Transfer function for omegas = 0.5, deltas = 1.2, −0.3



Transfer function for omegas = 0.5, deltas = −0.3, 0.4



Transfer function for omegas = 0.5, deltas = −1.3, −0.4