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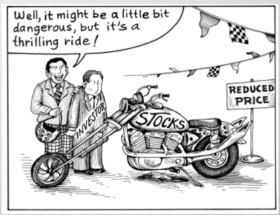

Value at Risk

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The Question Being Asked in VaR

"What loss level is such that we are *X*% confident it will not be exceeded in *N* business days?"

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VaR and Regulatory Capital

- **Regulators base the capital they require banks to keep on VaR**
- **The market-risk capital is *k* times the 10-day 99% VaR where *k* is at least 3.0**

VaR vs. C-VaR

- **VaR is the loss level that will not be exceeded with a specified probability**
- **C-VaR (or expected shortfall) is the expected loss given that the loss is greater than the VaR level**
- **Although C-VaR is theoretically more appealing, it is not widely used**

Advantages of VaR

- **It captures an important aspect of risk in a single number**
- **It is easy to understand**
- **It asks the simple question: "How bad can things get?"**

Time Horizon

- **Instead of calculating the 10-day, 99% VaR directly analysts usually calculate a 1-day 99% VaR and assume**

$$10\text{-day VaR} = \sqrt{10} \times 1\text{-day VaR}$$

- **This is exactly true when portfolio changes on successive days come from independent identically distributed normal distributions**

Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day and so on

Historical Simulation continued

- Suppose we use *m* days of historical data
- Let v_i be the value of a variable on day *i*
- There are *m*-1 simulation trials
- The *i*th trial assumes that the value of the market variable tomorrow (i.e., on day *m*+1) is

$$v_m \frac{v_i}{v_{i-1}}$$

The Model-Building Approach

- The main alternative to historical simulation is to make assumptions about the probability distributions of return on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically
- This is known as the model building approach or the variance-covariance approach

Daily Volatilities

- In option pricing we measure volatility "per year"
- In VaR calculations we measure volatility "per day"

$$\sigma_{day} = \frac{\sigma_{year}}{\sqrt{252}}$$

Daily Volatility continued

- Strictly speaking we should define σ_{day} as the standard deviation of the continuously compounded return in one day
- In practice we assume that it is the standard deviation of the percentage change in one day

Microsoft Example

- We have a position worth $10 million in Microsoft shares
- The volatility of Microsoft is 2% per day (about 32% per year)
- We use N=10 and X=99

Microsoft Example continued

- **The standard deviation of the change in the portfolio in 1 day is $200,000**
- **The standard deviation of the change in 10 days is**

$$200{,}000\sqrt{10} = \$632{,}456$$

Microsoft Example continued

- **We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)**
- **We assume that the change in the value of the portfolio is normally distributed**
- **Since *N*(–2.33)=0.01, the VaR is**

$$2.33 \times 632{,}456 = \$1{,}473{,}621$$

AT&T Example

- **Consider a position of $5 million in AT&T**
- **The daily volatility of AT&T is 1% (approx 16% per year)**
- **The S.D per 10 days is**

$$50{,}000\sqrt{10} = \$158{,}144$$

- **The VaR is**

$$158{,}114 \times 2.33 = \$368{,}405$$

Portfolio

- **Now consider a portfolio consisting of both Microsoft and AT&T**
- **Suppose that the correlation between the returns is 0.3**

S.D. of Portfolio

- **A standard result in statistics states that**

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

- **In this case σ_X = 200,000 and σ_Y = 50,000 and ρ = 0.3. The standard deviation of the change in the portfolio value in one day is therefore 220,227**

VaR for Portfolio

- **The 10-day 99% VaR for the portfolio is**

$$220{,}227 \times \sqrt{10} \times 2.33 = \$1{,}622{,}657$$

- **The benefits of diversification are**

(1,473,621+368,405)–1,622,657=$219,369

- **What is the incremental effect of the AT&T holding on VaR?**

The Linear Model

We assume

- **The daily change in the value of a portfolio is linearly related to the daily returns from market variables**
- **The returns from the market variables are normally distributed**

The General Linear Model continued

$$\Delta P = \sum_{i=1}^{n} \alpha_i \Delta x_i$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2\sum_{i<j} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

where σ_i is the volatility of variable i

and σ_P is the portfolio's standard deviation

Monte Carlo Simulation

To calculate VaR using M.C. simulation we

- **Value portfolio today**
- **Sample once from the multivariate distributions of the Δx_i**
- **Use the Δx_i to determine market variables at end of one day**
- **Revalue the portfolio at the end of day**

Monte Carlo Simulation

- Calculate ΔP
- Repeat many times to build up a probability distribution for ΔP
- VaR is the appropriate fractile of the distribution times square root of N
- For example, with 1,000 trial the 1 percentile is the 10th worst case.

Comparison of Approaches

- Model building approach assumes normal distributions for market variables. It tends to give poor results for low delta portfolios
- Historical simulation lets historical data determine distributions, but is computationally slower

Stress Testing

- This involves testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years

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Back-Testing

- **Tests how well VaR estimates would have performed in the past**
- **We could ask the question: How often was the actual 10-day loss greater than the 99%/10 day VaR?**

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Principal Components Analysis for Interest Rates

- **The first factor is a roughly parallel shift**
  - (83.1% of variation explained)
- **The second factor is a twist**
  - (10% of variation explained)
- **The third factor is a bowing**
  - (2.8% of variation explained)

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Using PCA to calculate VaR

Example: Sensitivity of portfolio to rates ($m)

1 yr	2 yr	3 yr	4 yr	5 yr
+10	+4	-8	-7	+2

Sensitivity to first factor is from Table 18.3:

$10\times0.32 + 4\times0.35 - 8\times0.36 - 7\times0.36 + 2\times0.36 = -0.08$

Similarly sensitivity to second factor = -4.40

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Using PCA to calculate VaR continued

- **As an approximation**

$$\Delta P = -0.08 f_1 - 4.40 f_2$$

- **The f_1 and f_2 are independent**
- **The standard deviation of ΔP is**

$$\sqrt{0.08^2 \times 17.49^2 + 4.40^2 \times 6.05^2} = 26.66$$

- **The 1 day 99% VaR is 26.66 × 2.33 = 62.12**