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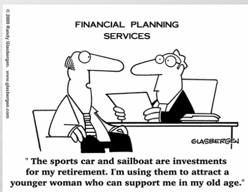
Interest Rates

High rates, higher returns, but for whom?



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Types of Rates

- **Treasury rates**
- **LIBOR rates**
- **Repo rates**

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Measuring Interest Rates

- **The compounding frequency used for an interest rate is the unit of measurement**
- **The difference between quarterly and annual compounding is analogous to the difference between miles and kilometers**

Continuous Compounding

- **In the limit as we compound more and more frequently we obtain continuously compounded interest rates**
- **\$100 grows to \$100e^{RT} when invested at a continuously compounded rate *R* for time *T***
- **\$100 received at time *T* discounts to \$100e^{-RT} at time zero when the continuously compounded discount rate is *R***

Conversion Formulas

Define

R_c : continuously compounded rate

R_m: same rate with compounding *m* times per year

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

Zero Rates

A zero rate (or spot rate), for maturity *T* is the rate of interest earned on an investment that provides a payoff only at time *T*

Maturity (years)	Zero Rate (% cont comp)
0.5	4.5
1.0	4.8
1.5	5.4
2.0	5.7

Bond Pricing

- **To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate**
- **In our example, the theoretical price of a two-year bond providing a 8% coupon semiannually is**
- **Price this bond:**

Bond Yield

- **The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond**

Par Yield

- **The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.**

Par Yield continued

In general if *m* is the number of coupon payments per year, *P* is the present value of $1 received at maturity and *A* is the present value of an annuity of $1 on each coupon date

$$c = \frac{(100 - 100P)m}{A}$$

Sample Data

Bond Principal (dollars)	Time to Maturity (years)	Annual Coupon (dollars)	Bond Cash Price (dollars)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

The Bootstrap Method

- **An amount 2.5 can be earned on 97.5 during 3 months.**
- **The 3-month rate is 4 times 2.5/97.5 or 10.256% with quarterly compounding**
  - This is a discrete payment

The Bootstrap Method continued

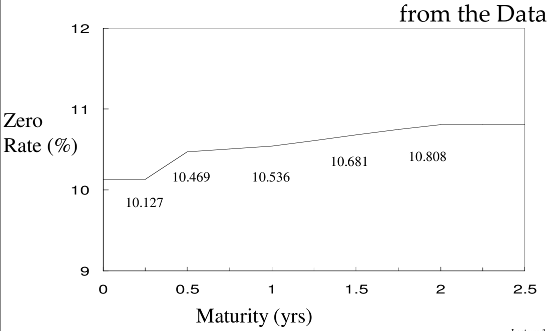
- **To calculate the 1.5 year rate we solve**

$$4e^{-0.10469\times 0.5} + 4e^{-0.10536\times 1.0} + 104e^{-R\times 1.5} = 96$$

to get *R* = 0.10681 or 10.681%

- **Similarly the two-year rate is?**

Zero Curve Calculated from the Data



Forward Rates

The forward rate is the future zero rate implied by today’s term structure of interest rates

Calculation of Forward Rates

Year (n)	Zero Rate for an n-year Investment (% per annum)	Forward Rate for n th Year (% per annum)
1	3.0	
2	4.0	5.0
3	4.6	5.8
4	5.0	6.2
5	5.3	6.5

Formula for Forward Rates

- **Suppose that the zero rates for time periods T_1 and T_2 are R_1 and R_2 with both rates continuously compounded.**
- **The forward rate for the period between times T_1 and T_2 is** $$\frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$
- **Is the forward rate greater than the zero rate if the yield curve is upward sloping?**

Instantaneous Forward Rate

- **The instantaneous forward rate for a maturity T is the forward rate that applies for a very short time period starting at T. It is**

$$R + T\frac{\partial R}{\partial T}$$

where R is the T-year rate

- **Why is this useful?**

Forward Rate Agreement

- **A forward rate agreement (FRA) is an agreement that a certain rate will apply to a certain principal during a certain future time period**

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Forward Rate Agreement continued

- **An FRA is equivalent to an agreement where interest at a predetermined rate, R_K is exchanged for interest at the market rate**
- **An FRA can be valued by assuming that the forward interest rate is certain to be realized**
- **Who is this good for?**

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Valuation Formulas

- **Value of FRA where a fixed rate R_K will be received on a principal L between times T_1 and T_2 is**
- **Value of FRA where a fixed rate is paid is**

$$L(R_K - R_F)(T_2 - T_1)e^{-R_2T_2}$$

- **R_F is the forward rate for the period and R_2 is the zero rate for maturity T_2**

$$L(R_F - R_K)(T_2 - T_1)e^{-R_2T_2}$$

- **What compounding frequencies are used in these formulas for R_K, R_M, and R_2?**

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FRA Example

- **Assume the yield curve has 1 year rate equal to 6% and a 2 year rate equal to 6.5%**
- **You have entered into a FRA earning 7% annually between year 1 and 2 for $1,000,000**
- **Is this a good deal?**
- **What is the FRA worth?**

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Duration

- **Duration of a bond that provides cash flow c_i at time t_i is**

$$\sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{B} \right]$$

where B is its price and y is its yield (continuously compounded)

- **This leads to**

$$\frac{\Delta B}{B} = -D\,\Delta y$$

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Duration Continued

- **When the yield y is expressed with compounding m times per year**

$$\Delta B = -\frac{BD\Delta y}{1 + y/m}$$

- **The expression**

$$\frac{D}{1 + y/m}$$

is referred to as the "modified duration"

Duration based Hedging

- **You have invested 10 million in Govt. Bonds and you are concerned that interest rates are going to be volatile**
  - What risks are you exposed to?
- **You are going to hedge using a T-Bond future using 94-07. T-Bonds are quoted in 32nd and each contract is worth $100,000**
- **Let us assume that the duration of the bond portfolio is 4.8 and the duration of the hedge is 6.7 years.**
  - How many contracts do you use to hedge?

Convexity

The convexity of a bond is defined as

$$C = \frac{1}{B}\frac{\partial^2 B}{\partial y^2} = \frac{\sum_{i=1}^{n} c_i t_i^2 e^{-yt_i}}{B}$$

so that

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

Theories of the Term Structure

- **Expectations Theory: forward rates equal expected future zero rates**
- **Market Segmentation: short, medium and long rates determined independently of each other**
- **Liquidity Preference Theory: forward rates higher than expected future zero rates**