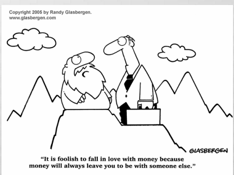
Determination of Forward and Futures Prices

If we can price forward, can we sell backwards?



Let us go back in time….

- **Think about CAPM….**
- **How do we price stocks, i.e, non-derivative assets**
  - $R_i=R_f+B_i(R_m-R_f)$
- **Stocks have an embedded risk premium**
- **Should all instruments have a risk premium?**

- **For example**
  - $F_i=S_i(1+B_i(R_m-R_f))^T$

Foreign Exchange Quotes for GBP June 3, 2003

	Bid	Offer
Spot	**1.6281**	**1.6285**
1-month forward	**1.6248**	**1.6253**
3-month forward	**1.6187**	**1.6192**
6-month forward	**1.6094**	**1.6100**

Forward Price

- **The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)**
- **The forward price may be different for contracts of different maturities**

Example

- **On June 3, 2003 the treasurer of a corporation enters into a LONG forward contract to BUY £1 million in six months at an exchange rate of 1.6100**
- **This obligates the corporation to pay $1,610,000 for £1 million on December 3, 2003**
- **What are the possible outcomes?**

Notation for Valuing Futures and Forward Contracts

S_0: **Spot price today**

F_0: **Futures or forward price today**

T: **Time until delivery date**

r: **Risk-free interest rate for maturity T**

Arbitrage Pricing

- **Let us price a futures contract**

Transaction	Time 0	Time T

Consumption vs Investment Assets

- **Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)**
- **Consumption assets are assets held primarily for consumption (Examples: copper, oil)**

Short Selling

- **Short selling involves selling securities you do not own**
- **Your broker borrows the securities from another client and sells them in the market in the usual way**
- **At some stage you must buy the securities back so they can be replaced in the account of the client**
- **You must pay dividends and other benefits the owner of the securities receives**

1. Gold: An Arbitrage Opportunity?

- **Suppose that:**
  - The spot price of gold is US$390
  - The quoted 1-year forward price of gold is US$425
  - The 1-year US$ interest rate is 5% per annum
  - No income or storage costs for gold
- **Is there an arbitrage opportunity?**
- **And how would you construct the trade?**

2. Gold: Another Arbitrage Opportunity?

- **Suppose that:**
  - The spot price of gold is US$390
  - The quoted 1-year forward price of gold is US$390
  - The 1-year US$ interest rate is 5% per annum
  - No income or storage costs for gold
- **Is there an arbitrage opportunity?**

The Forward Price of Gold

- **If the spot price of gold is *S* and the futures price is for a contract deliverable in *T* years is *F*, then**
  - $F = S\ (1+r\ )^T$

  where *r* is the 1-year (domestic currency) risk-free rate of interest.

- **In our examples, *S*=390, *T*=1, and *r*=0.05 so that**

  $F\ = 390(1+0.05) = 409.50$

When Interest Rates are Measured with Continuous Compounding

$$F_0 = S_0 e^{rT}$$

This equation relates the forward price and the spot price for any investment asset that provides no income and has no storage costs

When an Investment Asset Provides a Known Dollar Income

$$F_0 = (S_0 - I)e^{rT}$$

where I is the present value of the income during life of forward contract

Let us consider a stock with a price of $20, and risk-free rate of 5%, and a $0.65 dividend every quarter. What is the appropriate price on a seven month forward?

When an Investment Asset Provides a Known Yield

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Valuing a Forward Contract

- Suppose that K is delivery price in a forward contract and F_0 is forward price that would apply to the contract today
- The value of a long forward contract, f, is
  - $f = (F_0 - K)e^{-rT}$
- Similarly, the value of a short forward contract is
  - $f = (K - F_0)e^{-rT}$

Intuition Test

- Trader A goes long for 1 million sterling 3 month using a forward contract
- Trader B goes long sterling 16 contracts 3 months using a futures contract
- 1 futures contract=62,500 pounds
- The current exchange rate is 1.6000
- Within minutes the price increases to 1.604
- What is the profit to each trader?

Forward vs Futures Prices

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
- A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
- A strong negative correlation implies the reverse

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Stock Index

- **Can be viewed as an investment asset paying a dividend yield**
- **The futures price and spot price relationship is therefore**

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average dividend yield on the portfolio represented by the index during life of contract

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Stock Index
(continued)

- **For the formula to be true it is important that the index represent an investment asset**
- **In other words, changes in the index must correspond to changes in the value of a tradable portfolio**
- **The Nikkei index viewed as a dollar number does not represent an investment asset**

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Index Arbitrage

- **When $F_0 > S_0e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures**
- **When $F_0 < S_0e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index**

Index Arbitrage

(continued)

- **Index arbitrage involves simultaneous trades in futures and many different stocks**
- **Very often a computer is used to generate the trades**
- **Occasionally (e.g., on Black Monday) simultaneous trades are not possible and the theoretical no-arbitrage relationship between F_0 and S_0 does not hold**

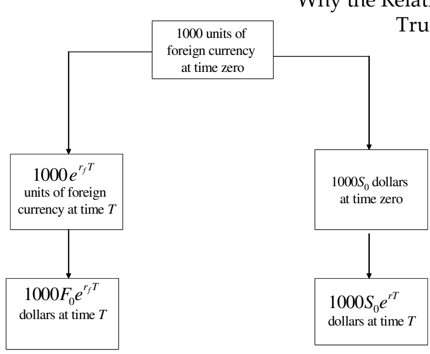
Futures and Forwards on Currencies

- **A foreign currency is analogous to a security providing a dividend yield**
- **The continuous dividend yield is the foreign risk-free interest rate**
- **It follows that if r_f is the foreign risk-free interest rate**

$$F_0 = S_0 e^{(r - r_f)T}$$

- **Suppose the 3 year rate in the UK and the US are 5% and 3%. The spot rate is .63 GBP per USD. What is the forward rate expressed in USD per GBP? How would you arbitrage if the forward rate was 1.64?**

Why the Relation Must Be True



Futures on Consumption Assets

$$F_0 \leq S_0 \, e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U) e^{rT}$$

where U is the present value of the storage costs.

The Cost of Carry

- **The cost of carry, c, is the storage cost plus the interest costs less the income earned**
- **For an investment asset $F_0 = S_0 e^{cT}$**
- **For a consumption asset $F_0 \leq S_0 e^{cT}$**
- **The convenience yield on the consumption asset, y, is defined so that**
  - $F_0 = S_0 \, e^{(c-y)T}$

Futures Prices & Expected Future Spot Prices

- **Suppose k is the expected return required by investors on an asset**
- **We can invest $F_0 e^{-rT}$ at the risk-free rate and enter into a long futures contract so that there is a cash inflow of S_T at maturity**
- **This shows that**

$$(F_0 e^{-rT}) e^{kT} = E(S_T)$$

or

$$F_0 = E(S_T) e^{(r-k)T}$$

Futures Prices & Future Spot Prices (continued)

- **If the asset has**
  - no systematic risk, then $k = r$ and F_0 is an unbiased estimate of S_T
  - positive systematic risk, then $k > r$ and $F_0 < E(S_T)$
  - negative systematic risk, then $k < r$ and $F_0 > E(S_T)$