ECO–5282
Financial Economics II: Homework #2
Fall 2005
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1. An economy consists of two infinitely lived consumers named $i = 1, 2$. There is one nonstorable consumption good. Consumer i consumes c^i at time t. Consumer i ranks consumption streams by $\sum_{t=0}^{\infty} \beta^t u(c_t^i)$ where $\beta \in (0,1)$ and $u(c)$ is strictly increasing, concave, and twice continuously differentiable. Consumer 1 is endowed with a stream of the consumption of good $y_t^1 = 1, 0, 0, 1, 0, 0, 1, \dots$ Consumer 2 is endowed with a stream of consumption good $y_t^2 = 0, 1, 1, 0, 1, 1, 0, \dots$ Assume that markets are complete with time-0 trading.

   (a) Define a competitive equilibrium.

   (b) Compute a competitive equilibrium.

   (c) Suppose that one of the consumers markets a derivative asset that promises to pay .05 units of consumption each period. What would the price of that asset be?

2. Consider an economy with a single consumer. There is one good in the economy, which arrives in the form of an exogenous endowment obeying $y_{t+1} = \lambda_{t+1} y_t$, where y_t is the endowment at time t and $\{\lambda_{t+1}\}$ is governed by a two-state Markov chain with transition matrix $P = \begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22} \end{bmatrix}$,and initial distribution $\pi_\lambda = [\pi_0, 1-\pi_0]$. The value of λ_t is given by $\overline{\lambda}_1 = 0.98$ in state 1 and $\overline{\lambda}_2 = 1.03$ in state 2. Assume that the history of y_s, λ_s, up to t is observed at time t. The consumer has endownment process $\{y_t\}$ and has preferences over consumption streams that are ordered by $E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$ where $\beta \in (0,1)$ and $\gamma \geq 1$.

   (a) Define a competitive equilibrium, being careful to name all of the objects of which it consists.

   (b) Tell how to compute a competitive equilibrium. For the remainder of this problem, suppose that $p_{11} = 0.8$, $p_{22} = 0.85$, $\pi_0 = 0.5$, $\beta = .96$, and $\gamma = 2$. Suppose that the economy begins with $\lambda_0 = .98$ and $y_0 = 1$.

   (c) Compute the (unconditional) average growth rate of consumption, computed before having observed λ_0.

   (d) Compute the time-0 prices of three risk-free discount bonds, in particular, those promising to pay one unit of time-j consumption for $j = 0, 1, 2$, respectively.

   (e) Compute the time-0 prices of three bonds, in particular, ones promising to pay one unit of time-j consumption contingent on $\lambda_j = \overline{\lambda}_1$ for $j = 0, 1, 2$, respectively.

   (f) Compute the time-0 prices of three bonds, in particular, ones promising to pay one unit of time-j consumption contingent on $\lambda_j = \overline{\lambda}_2$ for $j = 0, 1, 2$, respectively.

   (g) Compare the prices that you computed in parts d, e, and f.

3. An economy consists of two consumers, named $i = 1, 2$. The economy exists in discrete time for periods $t \geq 0$. There is one good in the economy, which is not storable and arrives in the form of an endowment stream owned by each consumer. The endowments to consumers $i = 1, 2$. are $y_t^1 = s_t$ and $y_t^2 = 1$, where s_t is a random variable governed by a two-state Markov chain with values $s_t = \overline{s}_1 = 0$ or $s_t = \overline{s}_2 = 1$. The Markov chain has time-invariant transition probabilities denoted by $\pi(s_{t+1} = s' | s_t = s) = \pi(s'|s)$, and the probability distribution over the initial state is $\pi_0(s)$. The aggregate endowment at t is $Y(s_t) = y_t^1 + y_t^2$.

Let c^i denote the stochastic process of consumption for agent i. Household i orders consumption streams according to

$$U(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \ln[c_t^i(s^t)],$$

where $\pi(s^t)$ is the probability of the history $s^t = (s_0, s_1, ..., s_t)$.

(a) Give a formula for $\pi(s^t)$.

(b) Let $\theta \in (0,1)$ be a Pareto weight on household 1. Consider the planning problem

$$\max_{\{c_t^1(s^t), c_t^2(s^t)\}} \{\theta \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \ln[c_t^1(s^t)] + (1-\theta) \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \ln[c_t^1(s^t)]\}$$

where the maximization problem is subject to

$$c_t^1(s^t) + c_t^2(s^t) = Y(s^t).$$

Solve the Pareto problem, taking θ as a parameter.

(c) Define a competitive equilibrium with history-dependent Arrow-Debreu securities traded once and for all at time 0. Be careful to define all of the objects that compose a competitive equilibrium.

(d) Compute the competitive equilibrium price system (i.e., find the prices of all of the Arrow-Debreu securities).

(e) Tell the relationship between the solutions (indexed by θ) of the Pareto problem and the competitive equilibrium allocation. If you wish, refer to the two welfare theorems.

(f) Briefly tell how you can compute the competitive equilibrium price system before you have figured out the competitive equilibrium allocation.

(g) Now defne a recursive competitive equilibrium with trading every period in one-period Arrow securities only. Describe all of the objects of which such an equilibrium is composed. (Please denominate the prices of one-period time-t+1 state-contingent Arrow securities in units of time-t consumption.) Define the "natural borrowing limits" for each consumer in each state. Tell how to compute these natural borrowing limits.

(h) Tell how to compute the prices of one-period Arrow securities. How many prices are there (i.e., how many numbers do you have to compute)? Compute all of these prices in the special case that $\beta = 0.95$ and $\pi(s_j|s_i) = P_{ij}$ where $P = \begin{bmatrix} .8 & .2 \\ .3 & .7 \end{bmatrix}$.

(i) Within the one-period Arrow securities economy, a new asset is introduced. One of the households decides to market a one-period-ahead riskless claim to one unit of consumption (a one-period real bill). Compute the equilibrium prices of this security when $s_t = 0$ and when $s_t = 1$. Justify your formula for these prices in terms of first principles.

(j) Within the one-period Arrow securities equilibrium, a new asset is introduced. One of the households decides to market a two-period-ahead riskless claim to one unit consumption (a two-period real bill). Compute the equilibrium prices of this security when $s_t = 0$ and when $s_t = 1$.

(k) Within the one-period Arrow securities equilibrium, a new asset is introduced. One of the households decides at time t to market five-period-ahead claims to consumption at t + 5 contingent on the value of s_{t+5}. Compute the equilibrium prices of these securities when $s_t = 0$ and $s_t = 1$ and $s_{t+5} = 0$ and $s_{t+5} = 1$.