ECO–5282
Financial Economics II: Homework #3
Fall 2005
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1. Consider an economy with a representative consumer with preferences described by $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u(c_t) = \ln(c_t + \gamma)$ where $\gamma \geq 0$ and c_t denotes consumption of the fruit in period t. The sole source of the single good is an everlasting tree that produces d_t units of the consumption good in period t. The dividend process d_t is Markov, with $prob\{d_{t+1} \leq d' | d_t = d\} = F(d', d)$. Assume that the conditional density $f(d', d)$ of F exists. There are competitive markets in the title of trees and in state-contingent claims. Let p_t be the price at t of a title to all future dividends from the tree.

   (a) Prove that the equilibrium price p_t satisfies

   $$p_t = (d_t + \gamma) \sum_{j=1}^{\infty} \beta^t E_t \left(\frac{d_{t+j}}{d_{t+j} + \gamma} \right),$$

   (b) Find a formula for the risk-free one-period interest rate R_{1t}. Prove that in the special case in which $\{d_t\}$ is independently and identically distributed, R_{1t} is given by $R_{1t}^{-1} = \beta k(d_t + \gamma)$, where k is a constant. Give a formula for k.

   (c) Find a formula for the risk-free two-period interest rate R_{2t}. Prove that in the special case in which $\{d_t\}$ is independently and identically distributed, R_{2t} is given by $R_{2t}^{-1} = \beta^2 k(d_t + \gamma)$, where k is the same constant you found in part b.

2. Consider the following version of the Lucas's tree economy. There are two kind of trees. The first kind is ugly and gives no direct utility to consumers, but yields a stream of fruit $\{d_{1t}\}$, where d_{1t} denotes a positive random process obeying a first-order Markov process. The second tree is beautiful and yields utility on itself. This tree also yields a stream of the same kind of fruit $\{d_{2t}\}$, where it happens that $d_{2t} \equiv d_{1t} = (1/2)d_t$ for all t, so that the physical yields of the the two kinds of trees are equal. There is one of each tree for each N individuals in the economy. Trees last forever, but the fruit is not storable. Trees are the only source of fruit.

   Each of the N individuals in the economy has preferences described by

   $$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, s_{2t})$$

   where $u(c_t, s_{2t}) = \ln c_t + \gamma \ln s_{2t}$, where $\gamma \geq 0$, c_t denotes consumption of the fruit in period t and s_{2t} is the stock of beautiful trees owned at the beginning of the period t. The owner of a tree of either kind i at the start of the period receives the fruit d_{it} produced by the tree during that period.

   Let p_{it} be the price of a tree of type i (i=1,2) during period t. Let R_{it} be the gross rate of returns of tree i during that period held from period t to $t + 1$.

   (a) Write down the consumer optimization problem in sequential and recursive form.

   (b) Define a rational expectations equilibrium.

   (c) Find the pricing functions mapping the state of the economy at t unto p_{1t} and p_{2t} (give precise formulas). [Hint: You should be able to directly derive p_{1t} from the example seen in class, then since pricing functions have to be linear you can guess a pricing function $p_{2t} = kd_t$ and solve for k parameter using the Euler equation of the second stock.]

   (d) Prove that if $\gamma > 0$, then $R_{1t} > R_{2t}$ for all t.