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Financial Homework 6.3 (Already turned in 6.1-6.2 in mailbox)

1. 6.3 Verify the following:

$$d_t P(t,T) = P(t,T)[\Sigma(t,T) d_t \widetilde{W}_t + r_t\, d_t] \tag{1}$$

Given the following, will use to verify (1):

$$\begin{aligned}
\widetilde{W}_t &= W_t + \int_t^T \gamma_s ds & (2)\\
\Sigma(t,T) &= -\int_t^T \sigma(t,u) du & (3)\\
Z(t,T) &= \beta_t^{-1} P(t,T) & (4)\\
& \quad Z(t,T)\text{is the SDE of the discounted bond, where} \\
& \quad \beta_t \Rightarrow \text{cash bond and} \\
& \quad P(t,T) \Rightarrow \text{price of the T-maturity bond.} \\
d_t Z(t,T) &= Z(t,T)\Sigma(t,T) d\widetilde{W}_t & (5)\\
& \quad \text{for discounted bond under under } \mathbb{Q} & (6)
\end{aligned}$$

Since β_t is a zero-vaildity process satisfying SDE,

$$d\beta_t = r_t \beta_t dt, \tag{7}$$

Using above, will verify (1) by using the product rule and simplify using above equations.

$$\begin{aligned}
d_t(P(t,T)) &= d_t(\beta_t\, Z(t,T)) & (8)\\
& \quad \text{applying the product rule obtains} & (9)\\
&= \beta_t \underbrace{d_t Z(t,T)}_{(5)} + Z(t,T) \underbrace{d_t \beta_t}_{(7)} & (10)\\
&= \beta_t\, Z(t,T)\Sigma(t,T) d\widetilde{W}_t + Z(t,T) r_t \beta_t dt & (11)\\
&= \beta_t Z(t,T)(\Sigma(t,T) d\widetilde{W}_t + r_t dt) & (12)\\
&= P(t,T)(\Sigma(t,T) d\widetilde{W}_t + r_t dt) & (13)
\end{aligned}$$

$\therefore$