ECO–5282
Financial Economics II: Homework #1
Fall 2005
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1. **(Cake eating problem I)** Consider the standard cake eating problem with a modification of the transition equation for the cake

$$W' = RW - c,$$

if $R > 1$, the cake yields a positive return, whereas if $R \in (0,1)$ the cake depreciates.

   (a) If preferences are of the form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, calculate the optimal sequence of consumption. What happens when $\sigma = 1$? and when $\sigma = 0$?

   (b) Show that we can reduce the cake eating problem to a system of nonlinear equations of the form $F(W) = 0$. Write down code to solve this problem for an arbitrary number of parameters, but make sure your routine is stable to solve the problem for high values of σ.

   (c) Modify your code to incorporate a new law of motion

$$W' = y + RW - c,$$

   that includes a constant term $y \geq 0$ at every period. You can interpret this value as period income or the size of an additional cake given at each period

2. **(Cake eating problem II)** Consider the previous version of the cake eating problem with a law of motion of the form $W' = y + \delta W - c$, where $\delta \in (0, R)$ is the return/depreciation of the cake, and y is income.

   (a) Formulate the Bellman equation of the infinite horizon problem.

   (b) If preferences are of the form $u(c) = \log(c)$ and $y = 0$, compute the optimal value function and the implied decision rules. Does the size of δ matters for the optimal values?

   (c) Write down code to calculate the optimal decision rule and value function.

3. **(Habit formation)** Consider an instantaneous utility function that depends on previous consumption $u(c_{t-1}, c_t)$. Previous consumption affects present consumption, so consumers have to take into account that today's consumption choice will also affect future utility besides the size of the cake.

   (a) Solve the $T-$period cake eating problem using this preferences. Provide some economic intuition of the implied first-order conditions.

   (b) Formulate the Bellman equation for the infinite horizon version of this problem.

   (c) Assume that $u(c_{t-1}, c_t) = \log c_{t-1} + \gamma \log c_t$, compute the optimal value function, and the imply policy function if we assume that $v(W, c_{-1}) = A + B \log W + C \log c_{-1}$.