Dynamic Analysis of Addiction: Impatience and Heterogenous Habits

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Abstract

This study develops a dynamic analysis of rational addiction suggesting a theoretical model that places special emphasis on the effects of impatience and heterogenous habits. The model describes how heterogenous habits affect the consumption paths via a subjective rate of time preference varying the rate of habit adjustment and the patience-dependence trade off. The intertemporal structure of preferences incorporating impatience and heterogenous habits explains how an increase in the rate of return to savings implies a decrease in the rate of time preference and an increase in the elasticity of intertemporal substitution. This makes a forward-looking agent more patient than a myopic one.

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The usual assumption in economics is that discount rates on future utilities are constant and fixed to each person, although they may differ between persons. This assumption is a good initial simplification, but it cannot explain why discount rates differ by age, income, education and other personal characteristics or why they change over time for the same individual, as when a person matures from being a child to being an adult.

(Becker G., "Accounting for Tastes," 1996)

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1 Introduction

The habit formation process is affected by economic variables and other exogenous factors such as demographic characteristics and the psychological state of consumers. A habit is formed when past and current consumption are linked by a positive relation. The higher is previous consumption, the larger the habit, and the higher should the current consumption level be to deliver the same utility. It follows that the derived utility depends on the difference between current consumption and a weighted sum of the quantities consumed in the past. Utility reaches a peak after consumption rose to a permanently higher level. Then it declines over time as the person becomes accustomed to that level. Similarly, utility reaches a minimum just after consumption fell to a permanently lower level. Comparisons with past consumption can be so effective that past consumption can be weighted more heavily than present consumption. When the habit formation process is sustained, then a consumer may turn a habit into a state of addiction. A habit may evolve into addiction by being exposed to habit itself. Becker and Murphy [4] define a person addicted to some goods when an increase in current consumption increases future consumption.

Consumption of the addictive good is not equally harmful to all individuals. For example, many people can drink regularly without becoming alcoholists. Addiction involves an interaction between people and goods. Each individual possesses a subjective belief structure concerning his potential to become addicted. People of comparable wealth and education but with different past experiences do not share the same risk to become addicted. In essence, people have different rates of time preference. Part of this heterogeneity may be explained by personal differences in past experiences, demographic characteristics, genetic patrimony, and other exogenous factors. The rate of time preference is a subjective indicator of impatience representing the desire of an agent to anticipate and enjoy the benefits stemming from higher current consumption. A high rate of time preference lowers the propensity towards future utility in determining current consumption choices. The main objective of the present paper is to treat the rate of time preference as endogenous and dependent upon demographic characteristics in order to capture heterogeneity in the addiction formation process.

We believe that by introducing heterogeneity in our model, we can develop a useful tool to evidence which policies can effectively reduce alcoholism without strongly penalizing people who enjoy a moderate consumption of alcoholic beverages. To most people, current consumption of alcohol in moderation provides enjoyment without serious side effects. To others the same pattern of consumption may lead to a state of dependence and eventually addiction[1]. This developement path is critically affected by personal characteristics. Analysing the causes for the low marriage rate, Akerlof [1] observes that there are noticeable differences in the lifestyle of married and unmarried men. Married men stay longer in the labour force, are less inclined to substance abuse, commit less crime and are less likely to be victims of crime. Moreover, they have better health and are less accident prone. A simple explanation is advanced: low marriage rates, or, in general, solitude, will lead to increases in some social pathologies such as crime, drugs and alcohol addiction.

As shown in Perali et al. [16] and [17], the gender of household head is a crucial characteristic in determining alcohol consumption, with female headed households which

[1] In general, addiction creates physical abstinence or withdrawal symptoms, when the use of the substance is discontinued, and generates tolerance, which is a physiological phenomenon requiring the individual to use more and more of the substance ([11, Kennedy, 1987] and [21, Stein et al., 1988]). Tolerance for a substance may be independent of the drug ability to produce physical dependence which manifests itself by the symptoms of abstinence when the drug is withdrawn.

consume signifinicantly less than male headed ones. Other variables (like marital status, education, the presence of children, etc.) affect alcohol consumption and have to be taken in consideration, but to keep things simple we focus on male and female singles to bring evidence of how heterogeneity is important for an effective welfare policy. Our idea is that it can be possible to think about a gender-specific policy, taking into account that in general women are considered more forward looking then men, and, possibly, to develop gender-specific instruments for the particular situations in wich addiction to alcohol seems to be more likely to rise. In fact, even if alcohol abuse is commonly considered a social bad, moderate cunsumption, expecially of wine, is seen as part of the Italian culture and so auspicable.

Given that the data seem to support our working hypothesis, we proceed by assuming an endogenous discount rate depending on past consumption as in Shi and Epstein [20], and by parameterizing the rate of time preference to incorporate heterogenous habits nesting as special cases both the Ramsey model, characterized by a constant rate of time preference, and the Uzawa [22] or Obstfeld [15] models, that assume an endogenous discount rate depending on current consumption. The results derived from the dynamic comparative analysis developed in this study are in line with those formulated by the theory of rational addiction. With respect to the analysis of Becker and Murphy [4] and Becker and Mulligan [2], we extend the model in order to study the impact of habits on intertemporal consumption paths by the rate of time preference and elasticity of substitution varying the rate of habit adjustment. This variation describes the heterogeneity of consumers who differ for a set of demographic characteristics. The study simulates the behaviour of two types of agents, a myopic and a forward-looking, where the myopic with respect to the forward-looking reveals a potential habit to alcohol for a set of characteristics that reveal a predisposition of the myopic to alcohol. This approach may be relevant to understand the political economy dimension of addiction. An effective policy would have the short run objective to find the set of actions that makes every person discount the future more highly and the long run objective to increase the frequency of forward-looking people within a population.

The study is structured as follows. Section 2 develops a rewriting of the basic dynamic optimization problem proposed by Shi and Epstein [20] where the consumer maximizes an istantaneous utility function discounted by an endogenous rate defined with respect to an index of past consumption, trying to put in evidence the possible extensions. Section 3 extends the basic model and develops the analytical properties of the extended models. The last section presents the conclusions.

2 The Basic Model with an Endogenous Discount Rate: a Unified Notation

The seminal works of Blanchard and Fisher [5], Deaton [7] and Romer [18] have criticized the assumption of a constant rate of time preference as suggested more by convenience than economic rationales. Most of the economic literature represents the preference structure in a dynamic context using the Ramsey model (Table 1) through functionals in which an additive utility function is discounted by a constant rate. Additivity implies that the marginal rate of substitution between the consumption at time t and $t+1$ is independent of consumption for each t different from t and $t+1$. Situations such as habit formation in alcohol consumption (but also in drug use or sigarette smoking), or the existence of goods as holidays and works of art whose benefits continue over the consumption act, cannot be described properly by an additive preference structure[2]. A formulation that involves non separability of preferences is suggested by Ryder and Heal [19] who introduce the notion of adjacent complementarity. An increase or decrease in consumption at $t-1$ can induce a variation of the marginal rate of substitution of current and future consumption at $t+1$. The complementarity is represented by a utility function that depends on both current consumption, c_t, and an index of past consumption

$$z_t = \sigma \int_{-\infty}^{t} c_\tau e^{\sigma(\tau - t)} d\tau \qquad with \quad \sigma \geq 0, \tag{2.1}$$

which is a weighted average of past consumption levels. The weights decline exponentially in the past at exogenous adjustment rate σ which is a measure of permanence of physical and mental effects of past consumption in present consumption c_t (Table 1). As σ gets larger, less weight is given to past consumption in determining z_t. Therefore, the degree of addiction is more intense for a lower σ.

Non separable preferences can be adopted assuming that the consumer discounts felicity by an endogenous discount rate depending on

1. current consumption, $\rho(c_t)$, according to a model with impatience developed by Obstfeld [15] (Table 1);
2. the index of past consumption z_t, $\rho(z_t)$, according to a model with impatience and habits developed by Shi and Epstein [20] (Table 1).

In the economic literature there is an open discussion if the endogenous discount rate must be considered increasing or decreasing with respect to current consumption c_t. Koopmans [12] suggests a decreasing rate of impatience, while Lucas and Stokey [14] observe that an increasing rate of impatience is necessary to obtain a single, stable, non degenerable equilibrium point into wealth distribution in a deterministic horizon with a finite number of agents. According to Blanchard and Fisher [5], the assumption of an increasing rate of impatience is difficult to defend *ex ante*. On the other side Epstein [8],[9] argues that the more a person consumes, the more discounts the future. In line with Epstein, we assume that the endogenous discount rate, $\rho(z_t)$, is strictly increasing with respect to current consumption c_t. This condition is necessary for ensuring the stability of the long-run optimal consumption plan, because it guarantees that consumptions in different dates are substitutes. In this case as wealth and consumption rise, the marginal private return to further savings, which depends on the marginal utility of future consumption, falls. If $\rho'(z_t) < 0$, consumption in different rates are complements, and a rise in present consumption rises the marginal utility of future consumption. Such an assumption is plausible in a model with habit formation, but it does not seem much coherent when we consider consumption in general. This is a further argument in favor of the assumption that the subjective discounting of future utility rises with consumption.

The implication of a discount rate $\rho(z_t)$ strictly increasing with respect to present consumption c_t, is that a higher consumption level at time t increases the discount rate applied to utility at t and after t. An increase in current consumption in t induces an increase in the rate of time preference: the consumer's desire to anticipate effects of future consumption is picked up by more current consumption at $t+1$. An increase in current

[2] In general, addiction creates physical abstinence or withdrawal symptoms, when the use of the drug is discontinued, and generates tolerance, which is physiological phenomenon requiring the individual to use more and more of the substance. Tolerance for a drug may be independent of the drug ability to produce physical dependence which manifests itself by the symptoms of abstinence when the drug is withdrawn.

consumption at $t+1$ rises the stock of habits at $t+2$ inducing a further increase in the discount rate: the higher is previous consumption, the larger the habit, and the higher must be the current level of consumption to deliver the same effect. An increase in the discount rate rises the degree of adjacent complementarity and hence strengthens the commitment to all habits.

2.1 The Basic Model

Consider an agent who can have access to a potentially harmful good at each instant of an infinite horizon. The consumption level of the t^{th} period, corresponding to the life cycle path C, is denoted c_t, while the intertemporal utility at time 0, $U(C_0^\infty)$, is delivered from the weighted sum of all future flows of utility, $u(c_t)$. The felicity function, $u(c_t)$, satisfies the Inada conditions and, in line with Shi and Epstein [20], we assume that the discount rate is linear ($\rho''(z_t)=0$), positive ($\rho(z_t)>0$) and increasing ($\rho'(z_t)>0$). Over the relevant time interval from $t_0=0$ to $t_1=\infty$, the actual level of welfare, U, derived from the consumption trajectory $\{c_t\}$, is obtained integrating all future flows of utility $u(c_t)$ discounted by the discount factor $e^{-\Theta_t}$

$$U(C_0^\infty)=\int_0^\infty u(c_t)\,e^{-\Theta_t}e^{-rt}dt \tag{2.1.1}$$

where

$$\Theta_t=\int_0^t[\rho(z_s)-r]\,ds, \tag{2.1.2}$$

subject to the following set of equations of motion

$$\dot{a}_t=ra_t-c_t \tag{2.1.3.a}$$
$$\dot{\Theta}_t=\rho(z_t)-r \tag{2.1.3.b}$$
$$\dot{z}_t=\sigma(c_t-z_t)\,. \tag{2.1.3.c}$$

Expression (2.1.2), denoted as the cumulative discount rate, is an indicator of accumulated impatience obtained by the difference between the discount rate $\rho(z_t)$, depending on consumer preferences with respect to the type of good and varying from agent to agent, and the rate of return to savings r, an opportunity variable equal for everyone and at every t. The introduction of the cumulative discount rate allows us to obtain a significant simplification in the problem solving, without any effect on the optimal consumption path.

The single control variable is the per-capita consumption c_t and the real assets per person, a_t, the cumulative subjective discount rate, Θ_t, and the stock of habits, z_t, are the state variables. We assume a constant rate of habits adjustment (σ) as well not depending on the characteristics of the individual in this section.

The control problem (2.1.1),(2.1.3.a),(2.1.3.b) and (2.1.3.c) is solved according to the Maximum Principle. The current value hamiltonian function, $H_d=e^{rt}H$ is

$$\begin{aligned}H_d\left\{c_t,a_t,\Theta_t,z_t;\widetilde{q}_t,\widetilde{\varphi}_t,\widetilde{\Psi}_t\right\}&=u(c_t)\,e^{-\Theta_t}+\widetilde{q}_t[ra_t-c_t]\\&\quad-\widetilde{\varphi}_t[\rho(z_t)-r]+\widetilde{\Psi}_t[\sigma(c_t-z_t)]\end{aligned} \tag{2.1.4}$$

where $\widetilde{q}_t=e^{rt}\widehat{q}_t$, $\widetilde{\varphi}_t=e^{rt}\widehat{\varphi}_t$ and $\widetilde{\Psi}_t=e^{rt}\widehat{\Psi}_t$ are the discounted costate variables. The necessary first-order conditions of the current value Hamiltonian function (2.1.4) for an interior maximum are

$$\frac{\partial H_d}{\partial c_t}=0\longrightarrow\widetilde{q}_t=u'(c_t)\,e^{-\Theta_t}+\widetilde{\Psi}_t\sigma \tag{2.1.5}$$

and

$$\frac{\partial H_d}{\partial a_t} = r\widetilde{q}_t - \dot{\widetilde{q}}_t \quad \longrightarrow \quad \dot{\widetilde{q}}_t = r\widetilde{q}_t - r\widetilde{q}_t = 0 \tag{2.1.6.a}$$

$$\frac{\partial H_d}{\partial \Theta_t} = r\widetilde{\varphi}_t - \dot{\widetilde{\varphi}}_t \quad \longrightarrow \quad \dot{\widetilde{\varphi}}_t = r\widetilde{\varphi}_t - u\left(c_t\right)e^{-\Theta_t} \tag{2.1.6.b}$$

$$\frac{\partial H_d}{\partial z_t} = r\widetilde{\Psi}_t - \dot{\widetilde{\Psi}}_t \quad \longrightarrow \quad \dot{\widetilde{\Psi}}_t = \left(r+\sigma\right)\widetilde{\Psi}_t + \widetilde{\varphi}_t\rho'\left(z_t\right). \tag{2.1.6.c}$$

It is convenient to rescale the costate variables in order to eliminate Θ_t. Let $q_t = \widetilde{q}_t e^{\Theta_t}$, $\varphi_t = \widetilde{\varphi}_t e^{\Theta_t}$ and $\Psi_t = \widetilde{\Psi}_t e^{\Theta_t}$. Then, the first-order necessary conditions take the form

$$q_t = u'\left(c_t\right) + \Psi_t\sigma. \tag{2.1.7}$$

and, given that $q_t = \widetilde{q}_t e^{\Theta_t}$,

$$\dot{q}_t = \dot{\widetilde{q}}_t e^{\Theta_t} + \widetilde{q}_t e^{\Theta_t}\dot{\Theta}_t = 0 + q_t\dot{\Theta}_t, \tag{2.1.8}$$

the other first order conditions are

$$\dot{q}_t = q_t\left(\rho\left(z_t\right) - r\right) \tag{2.1.9.a}$$

$$\dot{\varphi}_t = \varphi_t\rho\left(z_t\right) - u\left(c_t\right) \tag{2.1.9.b}$$

$$\dot{\Psi}_t = \left(\rho\left(z_t\right)+\sigma\right)\Psi_t + \varphi_t\rho'\left(z_t\right), \tag{2.1.9.c}$$

and differentiating equation (2.1.7) with respect to time we obtain

$$\dot{q}_t = u''(c_t)\dot{c}_t + \dot{\Psi}_t\sigma. \tag{2.1.10}$$

The differential equation 2.1.9.b gives a continuos-time specification of the recursive structure of consumer preferences for every feasible consumption path C. If we solve the differential equation[3] (2.1.9.b) we obtain

$$\varphi_t = \int_t^\infty u\left(c_v\right)e^{-\int_t^v \rho(z_s)ds}dv, \tag{2.1.11}$$

which is the present value of future utilities at time t and corresponds to the shadow price of the accumulated impatience rate Θ_t.

By equating the two equations we have for $\dot{q}_t$ (2.1.9.a and 2.1.10), we can solve for $\dot{c}_t$ and find the Euler Equation

$$u''(c_t)\dot{c}_t = q_t\left(\rho\left(z_t\right) - r\right) - \dot{\Psi}_t\sigma \qquad \Longrightarrow$$

$$\frac{\dot{c}_t}{c_t} = \frac{\left(u'\left(c_t\right)+\Psi_t\sigma\right)}{u''(c_t)c_t}\left[\rho\left(z_t\right) - \frac{\sigma\left(\rho\left(z_t\right)+\sigma\right)\Psi_t + \varphi_t\rho'\left(z_t\right)}{\left(u'\left(c_t\right)+\Psi_t\sigma\right)} - r\right]. \tag{2.1.12}$$

Rewriting expression (2.1.12) in terms of rate of time preference and elasticity of intertemporal substitution, the Euler Equation becomes

$$r = \theta(c_t, z_t, \varphi_t, \Psi_t) - \frac{1}{\eta(c_t,\Psi_t)}\frac{\dot{c}_t}{c_t}, \tag{2.1.13}$$

[3] Recall that the solution for a differential equation with no constant coefficients as $\dot{y}_t + P_t y_t = Q_t$ is $y_t = e^{-\int Pdt}\int Q_t e^{\int Pdt}dt + ce^{-\int Pdt}$. The value that the solution approaches is reffered to as the steady state so the limit for $t\rightarrow\infty$ of the solution is $y_t = \int Q_t e^{\int Pdt}dt$.

where

$$\theta(c_t, z_t, \varphi_t, \Psi_t) = \rho(z_t) - \frac{\sigma(\rho(z_t) + \sigma)\Psi_t + \varphi_t \rho'(z_t)}{(u'(c_t) + \Psi_t \sigma)} \tag{2.1.14}$$

is the rate of time preference, and

$$\eta(c_t, \Psi_t) = \frac{(u'(c_t) + \Psi_t \sigma)}{u''(c_t)c_t} \tag{2.1.15}$$

is the elasticity of intertemporal substitution.

3 Extension to the Basic Model

3.1 Impatience and Heterogeneous Habits

This section extend the basic model taking advantage of the hypotesis of linearity of the discount rate proposed by Shi and Epstein [20] in order to obtain a relatively simpler Euler Equation where the role of the rate of habits adjustment (σ) is widened by allowing for differences among consumers, due to the heterogeneity of preferences. The parameter σ strongly influences consumer behavior. A proper modeling of the role of heterogeneity in the process of habit formation and in distinguishing different rate of time preference will also be crucial for a correct specification of econometric models.

The endogenous rate of time preference represents a subjective indicator of impatience (i.e. the desire to anticipate future consumption) and can depend upon demographic variables, not only on past consumption path. In this section we propose an extension to the basic model in order to take into account the subjective degree of impatience and, indirectly capture an important component of heterogeneity.

We can obtain a reformulation of the first order condition of the hamiltonian function (2.1.7) as the first derivative of generating function with respect to current consumption c_t. Consider the first-order condition of the Hamiltonian function (2.1.7)

$$q_t = u'(c_t) + \Psi_t \sigma.$$

The costate variable, Ψ_t, is the shadow price of the stock of habits, z_t, and is defined as

$$\Psi_t = \frac{\partial U(C_t^\infty)}{\partial z_t} = -\left[\int_t^\infty u(c_v)\, e^{-\int_t^v \rho(z_s)ds} \left(\int_t^v \rho'(z_t) e^{\sigma(t-s)} ds\right) dv\right] > 0 \tag{3.1.1}$$

where

$$U(C_t^\infty) = \int_t^\infty u(c_v)\, e^{-\int_t^v \rho(z_s)ds} dv$$

states the present value of future utilities valued at t[4] and corresponds to φ_t, as shown in the previous section. The preceding expression can be rewritten as

$$\Psi_t = -\varphi_t \rho'(z_t) \int_t^v e^{\sigma(t-s)} ds \tag{3.1.2}$$

[4] Expression (3.1.1) is derived integrating by parts expression (2.1.11) for $t < v$ $z_t = \sigma e^{-\sigma v} \int_{-\infty}^t e^{\sigma s} c_s ds = c_t e^{\sigma(t-v)}$ and considering that $\dot{c}_t = 0$ along a locally constant path consumption.

given the condition of linearity of the discount rate[5], where φ_t is the shadow price of the rate of the accumulated impatience. Expression (3.1.2) is analytically different from the one formulated in the Shi and Epstein model [20]. According to our model, the first-order condition (2.1.7) is reformulated as

$$q_t = u'(c_t) - \left[\varphi_t \rho'(z_t) \int_t^v e^{\sigma(t-s)} ds\right] \sigma = u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma \tag{3.1.3}$$

with

$$\xi(\sigma) = \int_t^v e^{\sigma(t-s)} ds = \frac{1}{\sigma} - \frac{e^{\sigma(t-v)}}{\sigma}$$

where, to simplify the mathematical treatment, we assume $\xi(\sigma)$ as a constant whose value is calculated numerically by varying the rate of habits adjustment, σ, and assuming two values for the lower, t, and the upper, v, extreme of integration. This reformulation is useful to characterize the behavioural properties of the model as it will be explained in the next sections.

The Euler equation is derived from the first-order condition (3.1.3). By differentiating expression (3.1.3) with respect to time and considering that $\rho''(z_t) = 0$,

$$\dot{q}_t = u''(c_t) \dot{c}_t - \rho'(z_t) \dot{\varphi}_t \xi(\sigma) \sigma \tag{3.1.4}$$

Equating equations (3.1.4) and (2.1.9.a) and replacing expression (2.1.9.c), a differential equation giving in every time t the time rate of change of the control variable c_t is obtained

$$\begin{aligned} \dot{c}_t = & \left[\frac{u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma}{u''(c_t)}\right] \cdot \\ & \left\{1 + \left[\frac{\varphi_t - u(c_t) / \rho(z_t)}{u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma}\right] \rho'(z_t) \xi(\sigma) \sigma - r\right\}. \end{aligned} \tag{3.1.5}$$

Divide both sides by current consumption c_t and define as the endogenous rate of time preference

$$\theta(c_t, z_t, \varphi_t, \sigma) = 1 + \left[\frac{\varphi_t - u(c_t) / \rho(z_t)}{u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma}\right] \rho'(z_t) \xi(\sigma) \sigma \tag{3.1.6}$$

as the endogenous elasticity of intertemporal substitution

$$\eta(c_t, \varphi_t, \sigma) = -\frac{u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma}{u''(c_t) c_t}, \tag{3.1.7}$$

and as the Euler equation with respect to the rate of return r

$$r = \theta(c_t, z_t, \varphi_t, \sigma) - \frac{1}{\eta(c_t, \varphi_t, \sigma)} \frac{\dot{c}_t}{c_t}. \tag{3.1.8}$$

The model dynamics are described by equations (2.1.9.b), (3.1.5), (2.1.3.a), (2.1.3.b) and (2.1.3.c). Convergence to the steady state is derived equating the system of equations

[5] A linear function is a homogeous function of first degree so $f(kx) = kf(x)$ where k is constant and x is the independent variable.

to zero and the interaction of the differential equations defines the unique optimum

$$\dot{z}_t = \sigma\left(c_t - z_t\right) = 0 \longrightarrow z^* = c^* \tag{3.1.9.a}$$

$$\dot{\varphi}_t = \varphi_t \rho\left(z_t\right) - u\left(c_t\right) = 0 \longrightarrow \varphi^* = \frac{u\left(c^*\right)}{r} \tag{3.1.9.b}$$

$$\dot{c}_t = \left[\frac{u'\left(c_t\right) - \varphi_t \rho'(z_t)\xi\left(\sigma\right)\sigma}{u''\left(c_t\right)}\right]\left[\theta\left(c_t, z_t, \varphi_t, \sigma\right) - r\right] = 0 \longrightarrow \theta\left(c^*, z^*\right) = r \tag{3.1.9.c}$$

$$\dot{a}_t = r a_t - c_t = 0 \longrightarrow a^* = \frac{c^*}{r} \tag{3.1.9.d}$$

$$\dot{\Theta}_t = \rho\left(z_t\right) - r = 0 \longrightarrow \rho\left(z_t\right) = r. \tag{3.1.9.e}$$

System (3.1.9.a), (3.1.9.b), (3.1.9.c), (3.1.9.d) and (3.1.9.e) presents dynamic properties similar to the Shi and Epstein model [20] and similar roots of the characteristic equation of the matrix whose coefficients are delivered by a first-order Taylor expansion of the system. In this analysis, the characteristic roots depend on the rate of habit adjustment, σ, and the interest rate, r, assumed constant for convenience. Different rates of habit adjustment allow us to examine the local stability of the system:

1. $\sigma > \sigma_1$ the roots are two real unstable and two real stable. The equilibrium point is a saddle point;

2. $\sigma < \sigma_1$ the roots are two real unstable and two complex with negative real parts and so stable.

The convergence to the steady state is cyclical. According to Shi and Epstein [20] "*... the cycles are local and they dampen towards the steady state. Cycles are more likely if habits adjust slowly or the steady state rate of time preference is more sensitive to the level of consumption or the desire to smooth consumption is weaker*" as it is the case of a myopic agent in response of a rise in interest rates. On the other hand, cyclical behaviour is impossible, when the rate of habits adjustment approaches $\sigma \to \infty$, and as expected, when the model reduces to the Ramsey model for $\sigma \to 0$.

3.2 Behavioral Analysis

The definition of an endogenous rate of time preference permits to identify a crucial feature of heterogeneity, separate from a generic habit formation process.

In this section we describe and analyze the main properties and features of our model, taking into account differences with models presented in the literature. Entering a bit into details, we first analize the behaviour induced by our specification of the endogenous rate of intertemporal substitution, then we describe some characteristics of the endogenous elasticity of intertemporal substitution, and finally we put the elements togheter to explain our specification of the Euler equation.

In line with the evidence of Italian household budget data, we consider the case of a myopic agent, a middle-aged woman who becomes jobless in a certain moment of his life and has dependent children. Since being jobless, she spends her time home and when children go to school, she drinks wine in small doses. As time passes, the myopic become used to alcohol consumption revealing habits with respect to alcohol. Becoming addicted requires the accumulation of a stock of past consumption beyond some critical level. Once this level is reached, consumption follows an unstable accumulation path and addiction results. Then we analyze behaviour of the myopic agent joined with a forward-looking agent, a forty-year-old single, satisfied with his employment, who does not disdain half a litre of wine per meal but he is a health friend and a sport-loving. The likelihood that the myopic agent reveals addiction with respect to wine since being jobless is picked up by a rate of habits adjustment $\sigma(d_m)$ higher than the rate of the forward-looking agent, $\sigma(d_f)$. The larger is $\sigma(\cdot)$, the more weight is given to past consumption in determining z_t. Therefore, the degree of addiction is more intense for an increasing $\sigma(\cdot)$. Heterogeneity is described by the two rates of habits adjustment $\sigma(d_m)$ and $\sigma(d_f)$ whose values depend on demographic characteristics of the two agents[6].

In doing so, we perform a simulation analysis imposing some arbitrary coherent values to parameters. In figure 3 we present the phase diagran[7] along with the simulated policy function for the model[8]. This figure serves only to confirm that the model is well-behaves around the equilibrium point and that it is stable.

Figure 4 shows time paths of consumption for the forward looking and myopic agents. This graph represent the system's speed of convergence to the steady state. The comparison of the two pats put in evidence a significantly different behavior: the myopic agent have a higher steady state level of consumption and reach it much faster than a forward looking. Moreover, for the myopic individual future utility is discounted more heavily since the habits stock adjust more rapidly and toward a higher level, and hence also the discount rate, This leads to a higher degree of impatience which explain the different behavior of the two curves.

3.2.1 The Endogenous Rate of Time Preference

In the literature the rate of time preference is associated with the slope of the indifference curves along the 45° line in the plane $[c_t, c_{t+1}]$, where c_t is current consumption and c_{t+1} future consumption. This rate can be obtained differentiating with respect to time the natural logarithm of the first-order condition of the control problem considered with negative sign. An analytical expression of the rate of time preference can be delivered for the considered problem too.

The first-order condition of the Hamiltonian function measures a variation of life-cycle utility $U(C)$ with respect to an infinitesimal small increment of current consumption along a constant path consumption, as well as the rate of decrease of marginal utility, and at times near t

$$q_t = u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma.$$

The rate of change, denoted $U_t(C)$, is discounted by a rate $-\Theta_t - rt$

$$U_t(C) = q_t e^{(-\Theta_t - rt)} = \left[u'(c_t) - \varphi_t \rho'(z_t) \xi(\sigma) \sigma\right] e^{(-\Theta_t - rt)}. \tag{3.2.1}$$

A feature of U is its implicit rate of time preference $\theta(\cdot)$, a real valued function, that is calculated along a locally constant consumption path by expression (3.2.1) replacing expression (2.1.9.b) (Table 2-D3)

[6] Note that $\sigma(d_m)$ is referred to a myopic agent, while $\sigma(d_f)$ is referred to a forward looking one.

[7] Note that this is the phase diagram of a reduced system. Around the steady state we plausibly assume that $z_t = c_t$ and hence the system is reduced by one variable.

[8] In all simulation we use a logarithmic utility function ($u(c_t) = \alpha \log c_t$), with consumption bounded between 0 and 1, the discount rate takes the linear form $\rho(z_t) = \gamma + \kappa z_t$, and parameters are chosen to be: $\alpha = 0.2$, $\gamma = 0.02$, $\kappa = 0.05$, $\xi(\sigma) = 200$, $r = 0.05$. The rate of habits adjustment takes the values of 0.02 for a forward looking agent ($\sigma(d_f)$) and of 0.04 for a myopic one ($\sigma(d_m)$).

$$
\begin{aligned}
\theta\left(c_t, z_t, \varphi_t, \sigma\right) = & -\frac{\partial}{\partial t} \log\left[q_t e^{(-\Theta_t - rt)}\right]\Big|_{\dot{c}_t=0} = -\frac{\partial}{\partial t}\log q_t + \frac{\partial}{\partial t}\left(\Theta_t + rt\right)\Big|_{\dot{c}_t=0} \\
= & -\frac{1}{q_t}\left[u''\left(c_t\right)\dot{c}_t - \rho'(z_t)\dot{\varphi}_t\xi\left(\sigma\right)\sigma\right] + \rho\left(z_t\right)\Big|_{\dot{c}_t=0} \\
= & \; 1 + \left[\frac{\varphi_t - u\left(c_t\right)/\left(\rho\left(z_t\right)\right)}{u'\left(c_t\right) - \varphi_t\rho'(z_t)\xi\left(\sigma\right)\sigma}\right]\rho'(z_t)\xi\left(\sigma\right)\sigma. \qquad (3.2.2)
\end{aligned}
$$

The same expression of the rate of time preference (3.1.6) in the Euler equation is derived.

Our rate of time preference describes a subjective preference structure that links the past, present and future consumption. The rate of time preference in (3.2.2) incorporates the following behavioral assumptions:

1. the memory of past events by the rate of habits adjustment, σ;
2. the perception of present events by the current consumption level, c_t;
3. the anticipation of future events by the present-value of future utilities, Ψ_t.

Consumer behaviour is non separable along time, revealing complementarity. Though, the importance of the individual is clearest in the rate of time preference in determining whether there is adjacent complementarity. Present consumption, c_t, and future consumption, by the present value of future utilities φ_t, depend on past consumption, by the rate of habits adjustment, σ, and need not be valued equally along a locally constant consumption path. The rate of time preference expresses the propensity that a person reveals towards future utility in determining current choices. An agent is more or less oriented to the future with respect to the present value of future utilities. This depends on the capability of anticipating benefits of future consumption and so physical and mental future consequences of present and past consumption effects.

In line with Shi and Epstein [20], the rate of time preference is characterized by the same analytical properties. The rate of time preference is strictly increasing with respect to the present value of future utilities φ_t. An increase in t indicates an increase in future consumption and the response is to give more weight to the present, discounting more the felicity $u\left(c_t\right)$.

Proposition 2 *The rate of time preference $\theta\left(c_t, z_t, \varphi_t, \sigma\right)$ is strictly increasing with respect to the present value of future stream of utilities φ_t, holding current consumption and the rate of habits adjustment constant.*

$$
\frac{\partial\theta\left(\overline{c_t}, \overline{z_t}, \varphi_t, \overline{\sigma}\right)}{\partial\varphi_t} > 0 \qquad (3.2.3)
$$

The rate of time preference in (3.2.2) is decreasing with respect to current consumption c_t and indicates that the more an agent consumes, the less is concerned with tomorrow rather than today. In such cases there may be no need to "*to save against a rainy day*". The rate of time preference approaches the greatest values when current consumption and the present value of the future utility, therefore future consumption, are greatest.

Proposition 3 *The rate of time preference $\theta\left(c_t, z_t, \varphi_t, \sigma\right)$ is strictly decreasing with respect to current consumption c_t, holding the rate of habits adjustment and present value of future stream of utilities constant in a region around the steady state.*

$$
\frac{\partial\theta\left(c_t, \overline{z_t}, \overline{\varphi_t}, \overline{\sigma}\right)}{\partial c_t} < 0 \qquad (3.2.4)
$$

As σ measures the declining marginal utility with respect to time, $U_t(C)$, an increase in the rate of habits adjustmet should imply that the marginal utility declines more rapidly. An increase in dangerous substances as drugs, alcohol and smoking tends to give more weight to current felicity at the expense of future felicity that is more discounted. As a result drug addicts and alcoholics tend to be present oriented. A decline of future felicity reduces the benefits delivered from a low discount rate and induces an increasingly higher rate of time preference.

Proposition 4 *The rate of time preference $\theta(c_t, z_t, \varphi_t, \sigma)$ is increasing with respect to the rate of habits adjustment σ, holding current consumption and the present value of future streamof utilities constant in a region around the steady state.*

$$\frac{\partial\theta\left(\overline{c_t}, \overline{z_t}, \overline{\varphi_t}, \sigma\right)}{\partial\sigma} > 0 \tag{3.2.5}$$

The analytical and behavioural properties of the rate of time preference (Propositions 3 and 4) allow us to describe the dynamic evolution of an agent from a condition of potential habit to a state of addiction[9]. Reconsider the case of the myopic and forward-looking agent introduced in Section 2.

From fig.1 note the degree of impatience of the myopic agent is higher than the degree of the forward-looking because higher is the degree of habits. The propensity to exchange current for future consumption becomes less and less considerable. The myopic agent reveals an increasing impatience since his stock of habits with respect to alcohol is higher. The subjective rate of time preference of the myopic agent encloses reinforcement and tolerance, two behavioural factors that are closely related to the concept of adjacent complementarity. Reinforcement means that greater current consumption of a good rises its future consumption in accordance while tolerance means that given levels of consumption are less satisfying when past consumption has been greater. On the other hand, the forward-looking agent is patient, since has greater capability to anticipate the future consequences of present and past consumption.

The analysis clearly reveals a patience-dependence tradeoff. A patient person has a lower stock of habits than an impatient person, since the desire to anticipate future consumption is lower. It is not surprising that addiction is more likely for people who discount the future heavily since they pay less attention to the adverse consequences. Becker, Grossman and Murphy [3] suggested that poorer and younger persons discount the future more heavily while Chaloupka [6] found that less educated persons may have higher rates of time preference. Capability of anticipating the consequences of present and past consumption depends on income, education, rank and degree of awareness of dangers.

According to Becker and Mulligan[2] "*... the analysis of endogenous discount rates implies that even fully rational utility-maximizing individuals who become addicted to drugs and other harmful substances or behaviour are induced to place less weight on the future, even if the addiction itself does not affect the discount rate.*" In the Becker and Mulligan's analysis [2], addiction affects the discount rate through the rate of habits adjustment. The degree of impatience is higher for lower values of the discount rate and so the likelihood that the consumer reveals addiction to a good is greater.

3.2.2 The Endogenous Elasticity of Intertemporal Substitution

If the rate of time preference is associated to the slope of the indifference curve along a 45° line in the plane $[c_t, c_{t+1}]$, the elasticity of intertemporal substitution gives the

[9] For proofs of these propositions see appendix A1, for a graphical evidence, see Figures 5 and 6.

proportionate change in the magnitude of the slope in response to a proportionate change in the ratio c_t/c_{t+1}, where c_t is current consumption and c_{t+1} future consumption.(Table 2-D4)

$$\eta\left(c_t, \varphi_t, \sigma\right) = -\left[\frac{u'\left(c_t\right) - \varphi_t \rho'(z_t)\xi\left(\sigma\right)\sigma}{u''\left(c_t\right)}\right]\frac{1}{c_t}. \tag{3.2.6}$$

The expression allows us to predict more intertemporal substitution of consumption relative to a constant elasticity. The elasticity η assumes higher values than the constant elasticity and approaches to the last one at the highest present value of future utilities. An increase in the present value of future utilities, φ_t, induces an increase in future consumption and the response is to give more weight to the present. An increase in future consumption makes agent less available for abstaining from current consumption at t in favour of future at $t+1$, making intertemporal substitution less considerable. The elasticity declines with respect to current consumption c_t, holding the present value of future utilities constant. This property of the curve could have an empirical correspondence with a consumption analysis of alcohol. The more an agent consumes, the least the agent is willing to sacrifice current for future consumption.

The elasticity assumes different curvatures varying the rate of habits adjustment σ (Figure 2). This allows us to analyse the effects induced by habits in intertemporal substitution and consider again the myopic and forward-looking agents. The forward-looking agent is more available for changing his path consumption than the myopic one to pick the intertemporal incentives, given an equal consumption and future utility level: the elasticity of the forward-looking approaches higher values than the elasticity of the myopic that incorporates a greater stock of habits with respect to alcohol than the forward-looking type for $\sigma\left(d_m\right) > \sigma\left(d_f\right)$. The curve of the elasticity of the myopic is flat because she/he does not like to exchange current for future consumption: the elasticity assumes the same values at each φ_t. The myopic does not anticipate dangers of an excessive alcohol consumption and is not willing to exchange alcohol consumption today, at t, for more consumption tomorrow, at $t+1$, $t+2$ and so on.

3.2.3 The Euler Equation

After having analysed the analytical properties and behavioural contents of the rate of time preference and the elasticity of intertemporal substitution, we consider now the Euler equation

$$r = \theta\left(c_t, \varphi_t, \sigma\right) - \frac{1}{\eta\left(c_t, \varphi_t, \sigma\right)}\frac{\dot{c}_t}{c_t}, \tag{3.2.7}$$

where $\theta\left(c_t, z_t, \varphi_t, \sigma\right)$ is the endogenous rate of time preference (equation 2.2.6) and $\eta\left(c_t, \varphi_t, \sigma\right)$ is the endogenous elasticity of intertemporal substitution.

The Euler equation is different from the canonical expression (Table 2 - A2), because it comprehends the complementarity between past consumption, σ, current consumption, c_t, and future consumption by the present value of future utilities, φ_t, by the endogenous rate of time preference and the endogenous elasticity of intertemporal substitution. The complementarity allows us to explain why an increased rate of return to savings, r, tends to induce more patience in consumers. First consider the simple case where an increased rate of return is compensated holding the marginal utility q_t constant. All future consumption rises since the rate of return is higher and current consumption is unchanged by marginal utility assumption holding the growth rate of consumption constant. The effect is picked up by an increase in the elasticity of intertemporal substitution: the agent is more in favour to the intertemporal substitution between future and current consumption, given the increased rate of return to savings. The problem can be more complicated. We do not consider a constant marginal utility and so an increased rate of return can lower the rate of time preference inducing more patience on the agent. The growth rate of consumption declines thus increasing savings. Therefore future consumption and the simultaneous effect on the growth rate of consumption is picked up by an increase in elasticity to allow the model to approach to an another consumption level equilibrium point. The impact of an increased rate of return to savings on the rate of time preference and the elasticity of intertemporal substitution and indeed the growth rate of consumption changes from person to person because people are not equally patient because of the heterogenous structure of preferences. The analysis of the effects induced by habits on consumption paths reveals how habits can influence the reaction of an agent with respect to an increased rate of return.

Does an increase in the rate of return lower the rate of time preference of the myopic and forward-looking agents making them more patient?An increase in the rate of return should have a more considerable impact on the rate of time preference of the forward-looking than the myopic agent. This depends on the stock of habits held by the two agents. The myopic agent is addicted to alcohol and his rate of habit adjustment, $\sigma(d_m)$, approaches to zero. This implies a flat elasticity of intertemporal substitution for the myopic agent: at all the values of future utility the elasticity assumes the same values given a current consumption level. The myopic's capability to abstain from current consumption in favour of savings is reduced and the agent does not react to an increase of the interest return.

4 Conclusions

Traditionally, the economic literature represents the structure of preferences in a dynamic context through functionals where a utility function is discounted by a constant discount rate. This choice, often adopted for the sake of of mathematical tractability, does not allow to explain why the discount rate differs by income, education, occupational standing and sex or changes over time for the same individual.

Assuming an endogenous discount rate depending on past consumption as adopted in the Shi and Epstein [20], the study develops analytically a new formulation of the rate of time preference that can be reduced with respect to the extreme values of the rate of habits adjustment (0 and ∞) to the constant rate of time preference according to the Ramsey model or to the rate of time preference obtained by an endogenous discount rate with respect to current consumption according to the Obstfeld model [15]. The rate of time preference supports a subjective structure of preferences that comprehends the memory of past events, the perception of present events and the anticipation of future events revealing adjacent complementarity. The behavioural contents delivered by the dynamic comparative analysis are in line with the results of the theory of rational addiction. As regards Becker and Murphy [4] and Becker and Mulligan [2], the extension of the model allows us to verify the impact of habits produces on intertemporal consumption paths by the rate of time preference varying the rate of habits adjustment that describes the heterogeneity among agents.

The dynamic analysis of addiction proposed by the model is characterized by the following behavioural properties:

1. an increase in the stock of habits induces an increase in the degree of impatience. The higher is previous consumption, the larger the habit, and the higher must be the

current level of consumption to deliver the same effect. The behavioural dynamics of an agent who evolves from habits to a state of addiction are delivered, deriving a *patience-dependence tradeoff*. A patient agent reveals himself forward-looking valuing the future more than a myopic agent, whose level of habits is higher than the first one, because he is less worried about the consequences of an excessive current consumption.

2. the higher is the incidence of past consumption on current consumption choices, the lower are the values assumed by the endogenous elasticity of intertemporal substitution as well the lower is the agent's propensity to exchange current for future consumption.

3. the heterogenous structure of habits allows us to explain how an increase in the rate of return to savings tends to induce more patience in the forward-looking than myopic consumer.

This means that this model is potentially useful in applications with micro-data, specially if one can use panel-data. Observing individual behaviour over time, together with demographic informations, may help to identify the parameters in this highly nonlinear model. We believe that an endogenous habit formation process could play an important role in explaining part of unobserved heterogeneity in individual data. The task of a well specified and identified econometric model will be objective of a further work.

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A Appendices

A.1 Proofs of propositions

Proposition 2. *The rate of time preference $\theta(c_t, z_t, \varphi_t, \sigma)$ is strictly increasing with respect to the present value of future stream of utilities φ_t, holding current consumption and the rate of habits adjustment constant.*

$$\frac{\partial\theta\left(\overline{c_t}, \overline{z_t}, \varphi_t, \overline{\sigma}\right)}{\partial\varphi_t} > 0 :$$

Assuming that the discount rate is linear and increasing in z_t, that the utility function takes the logarithmic form $(\alpha \log c_t)$ and that consumption lies between 0 and 1, the derivative of the rate of time preference $\theta(\cdot)$ with respect to the index of impatience φ_t is

$$\frac{\sigma^2\xi(\sigma)^2\rho'(z_t)^2\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2} + \frac{\sigma\xi(\sigma)\rho'(z_t)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)} \tag{A1}$$

which, after summing up and collecting $\sigma\xi(\sigma)\rho'(z_t)$ become

$$\frac{\sigma\xi(\sigma)\rho'(z_t)\left[\rho(z_t)u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)u(c_t)\right]}{\rho(z_t)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}. \tag{A2}$$

We know that $\xi(\sigma)$ is positive along with parameter σ. We also know by assumption that $u'(c_t) > 0$, $\rho'(z_t) > 0$, $\rho(z_t) > 0$. Because of consumption bounds and of assumption about preferences form, instantaneous utility is always negative $u(c_t)$. The Denominator is for sure positive and so is the numerator, since $u(c_t)$ is negative the term $(\rho(z_t)u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)u(c_t))$ is positive, so the whole expression is always positive[10]. ■

Proposition 3. *The rate of time preference $\theta(c_t, z_t, \varphi_t, \sigma)$ is strictly decreasing with respect to current consumption c_t, holding the rate of habits adjustment and present value of future stream of utilities constant in a region around the steady state.*

$$\frac{\partial\theta\left(c_t, \overline{z_t}, \overline{\varphi_t}, \overline{\sigma}\right)}{\partial c_t} < 0 :$$

Under the same assumption of the previous proof, the derivative of the rate of time preference $\theta(\cdot)$ with respect to current consumption c_t is

$$-\frac{\sigma\xi(\sigma)\rho'(z_t)u'(c_t)}{\rho(z_t)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)} - \frac{\sigma\xi(\sigma)\rho'(z_t)u''(c_t)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}. \tag{A3}$$

Summing up and expandig terms we obtaing

$$\begin{aligned}&\frac{\sigma\xi(\sigma)\rho'(z_t)u'(c_t)^2 - \sigma^2\xi(\sigma)^2\rho'(z_t)^2\varphi_t u'(c_t)}{\rho(z_t)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}\\ &- \frac{\rho(z_t)\sigma\xi(\sigma)\rho'(z_t)u''(c_t)\varphi_t + \sigma\xi(\sigma)\rho'(z_t)u''(c_t)u(c_t)}{\rho(z_t)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2},\end{aligned} \tag{A4}$$

and finally, collecting $\sigma\varepsilon(\sigma)\rho'(z_t)$

$$-\frac{\sigma\xi(\sigma)\rho'(z)\left[u'(c_t)^2 - \sigma\xi(\sigma)\rho'(z_t)u'(c_t)\varphi_t + u''(c_t)\left(\rho(z_t)\varphi_t - u(c_t)\right)\right]}{\rho(z_t)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}. \tag{A5}$$

[10] It is possible to proove that this proposition is true also if utility function takes the form of a power function $u(c_t) = \frac{c^{1-\alpha}}{1-\alpha}$, provided that $\frac{\sigma\xi(\sigma)}{1-\alpha} < 1$, without any upper bound for consumption.

The denominator and the term outside the square brackets are for sure positive under current assumptions. As regards the square bracket, we analyze each single component. $u'(c_t)$ is positive by assumption, $-\sigma\xi(\sigma)\rho'(z_t)\varphi_t$ is positive since φ_t is negative and all other terms are positive. The last term $u''(c_t)\left(\rho(z_t)\varphi_t - u(c_t)\right)$ is determinant for the sign. If $(\rho(z_t)\varphi_t - u(c_t))$ is negative it implies that certainly the whole expression is negative. If not one must look to the entire square bracketed term. In our simulations, we find that this proposition is always true, except for a quite narrow region which correspond to very low level of consumption and high level of future streams of utility (see figure 5). ■

Proposition 4. *The rate of time preference $\theta\left(c_t, z_t, \varphi_t, \sigma\right)$ is increasing with respect to the rate of habits adjustment σ, holding current consumption and the present value of future streamof utilities constant in a region around the steady state.*

$$\frac{\partial\theta\left(\overline{c_t}, \overline{z_t}, \overline{\varphi_t}, \sigma\right)}{\partial\sigma} > 0 :$$

Under the same assumptions of previous proofs, the derivative of the rate of time preference $\theta(\cdot)$ with respet to the rate of habits adjustment σ is

$$\frac{\rho'(z_t)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)\left(\xi(\sigma) + \sigma\xi'(\sigma)\right)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)} - \frac{\sigma\xi(\sigma)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)\rho'(z_t)\left(-\varphi_t\xi(\sigma)\rho'(z_t) - \sigma\varphi_t\xi'(\sigma)\rho'(z_t)\right)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}. \tag{A6}$$

Summing up and collecting $\rho'(z_t)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)$ we obtain

$$\frac{\rho'(z_t)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2} \cdot \frac{\left[\left(\xi(\sigma) + \sigma\xi'(\sigma)\right)\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right) + \sigma\xi(\sigma)\left(\varphi_t\xi(\sigma)\rho'(z_t) + \sigma\varphi_t\xi'(\sigma)\rho'(z_t)\right)\right]}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}. \tag{A7}$$

Expanding and recollecting the square bracketed terms leads to

$$\frac{\rho'(z_t)\left(\varphi_t - \frac{u(c_t)}{\rho(z_t)}\right)\left[u'(c_t)\left(\xi(\sigma) + \sigma\xi'(\sigma)\right)\right]}{\left(u'(c_t) - \sigma\xi(\sigma)\rho'(z_t)\varphi_t\right)^2}, \tag{A8}$$

which, in turn can be written as

$$\frac{u'(c_t)\rho'(z_t)\left(\xi(\sigma) + \sigma\xi'(\sigma)\right)\left(\rho(z_t)\varphi_t - u(c_t)\right)}{\rho(z_t)\left(u'(c_t) - \sigma\varphi_t\xi(\sigma)\rho'(z_t)\right)^2}. \tag{A9}$$

The denominator is positive, and so are terms $u'(c_t)$ and $\rho'(z_t)$. $\xi(\sigma)$ is an always positive and growing function with respect to σ and then also $\left(\xi(\sigma) + \sigma\xi'(\sigma)\right)$ is positive. Again the key term is $(\rho(z_t)\varphi_t - u(c_t))$. Figure 6 evidences that the derivative of the rate of time preference with respect to the rate of habits adjustment is slightly negative in a quite wide region for high values of consumption and low values of future stream of utility, but moving towards the opposite situation it assumes relevant positive values. Around the steady state the derivative is slightly positive. ■

A.2 Tables and Figures

Table 1: Intertemporal Utility Functions

Intertemporal utility $U\left(C\right)$	Constant discount rate*	Endogenous discount rate*
Preference independent of past consumption	$\int_0^\infty u\left(c_t\right)e^{-\theta t}dt$ Ramsey Model	$\int_0^\infty u\left(c_t\right)e^{-\int_0^t \theta(c_\tau)d\tau}dt$ Obstfeld Model (1990)
Preference dependent on past consumption	$\int_0^\infty u\left(c_t, z_t\right)e^{-\theta t}dt$ Ryder and Heal Model	$\int_0^\infty u\left(c_t\right)e^{-\int_0^t \theta(z_\tau)d\tau}dt$ Shi and Epstein Model (1993)

*Along constant paths

Table2: Euler Equation, Rate of Time Preference and Elasticity of Intertemporal Substitution

	Intertemporal utility $U(C)$
Ramsey Model	$\int_0^\infty u(c_t)e^{-\theta t}dt$
Obstfeld Model (1990)	$\int_0^\infty u(c_t)e^{-\int_0^t \theta(c_\tau)d\tau}dt$
Shi and Epstein (1993)	$\int_0^\infty u(c_t)e^{-\int_0^t \theta(z_\tau)d\tau}dt$
Present paper	$\int_0^\infty u(c_t)e^{-\int_0^t \theta(z_\tau)d\tau}dt$
	Euler Equation
Ramsey Model	$r = \theta - \frac{1}{\eta}\frac{c_t}{c_t}$
Obstfeld Model (1990)	$r = \theta(c_t,\varphi_t) - \frac{1}{\eta(c_t,\varphi_t)}\frac{c_t}{c_t}$
Shi and Epstein (1993)	$r = \theta(c_t,z_t,\varphi_t,\Psi_t) - \frac{1}{\eta(c_t,\Psi_t)}\frac{c_t}{c_t}$
Present paper	$r = \theta(c_t,z_t,\varphi_t,\sigma) - \frac{1}{\eta(c_t,\varphi_t,\sigma)}\frac{c_t}{c_t}$
	Rate of Time Preference
Ramsey Model	ρ
Obstfeld Model (1990)	$\theta(c_t,\varphi_t) = \rho(c_t)\left\{1 + \left[\frac{\varphi_t - u(c_t)/\rho(c_t)}{u'(c_t) - \varphi_t\rho'(c_t)}\right]\rho'(c_t)\right\}$
Shi and Epstein (1993)	$\theta(c_t,z_t,\varphi_t,\Psi_t) = \rho(z_t) - \frac{\sigma[(\rho(z_t)+\sigma)\Psi + \varphi\rho'(z_t)]}{u'(c_t)+\sigma\Psi_t}$
Present paper	$\theta(c_t,z_t,\varphi_t,\sigma) = 1 + \left[\frac{\varphi_t - u(c_t)/(\rho(z_t))}{u'(c_t) - \varphi_t\rho'(z_t)\xi(\sigma)\sigma}\right]\rho'(z_t)\xi(\sigma)\sigma$
	Elasticity of Interemporal Substitution
Ramsey Model	$\eta = -\frac{u'(c_t)}{u''(c_t)c_t}$
Obstfeld Model (1990)	$\eta(c_t,\varphi_t) = -\frac{u'(c_t) - \rho'(c_t)\varphi_t}{[u''(c_t) - \rho''(c_t)\varphi_t]c_t}$
Shi and Epstein (1993)	$\eta(c_t,\varphi_t) = -\frac{u'(c_t) - \rho'(z_t)\varphi_t}{[u''(c_t) - \rho''(z_t)\varphi_t]c_t}$
Present paper	$\eta(c_t,\varphi_t,\sigma) = -\frac{u'(c_t) - \varphi_t\rho'(z_t)\xi(\sigma)\sigma}{u''(c_t)c_t}$

Figure 1 - Rate of Time Preference of a Forward-looking and Myopic Agent

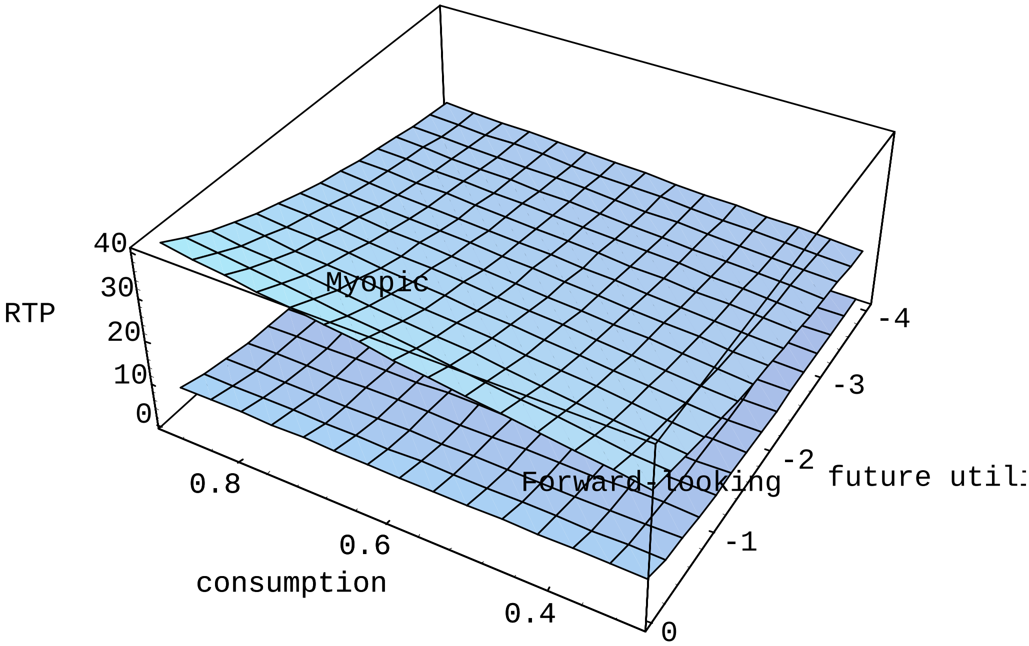

Figure 2 - Intertemporal Elasticity of a Forward-looking and Myopic Agent

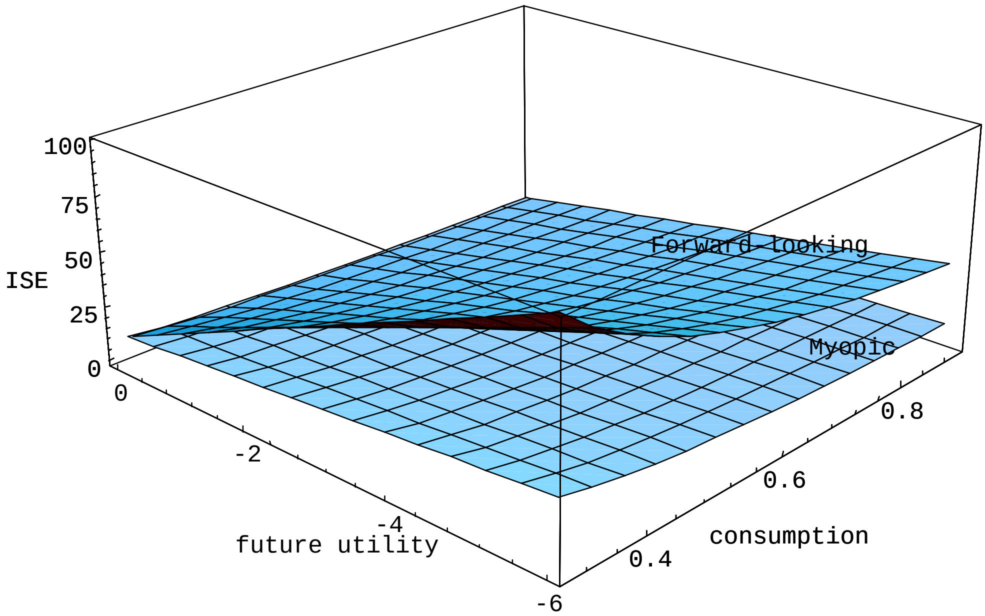

Figure 3: Phase Diagram and Policy Function

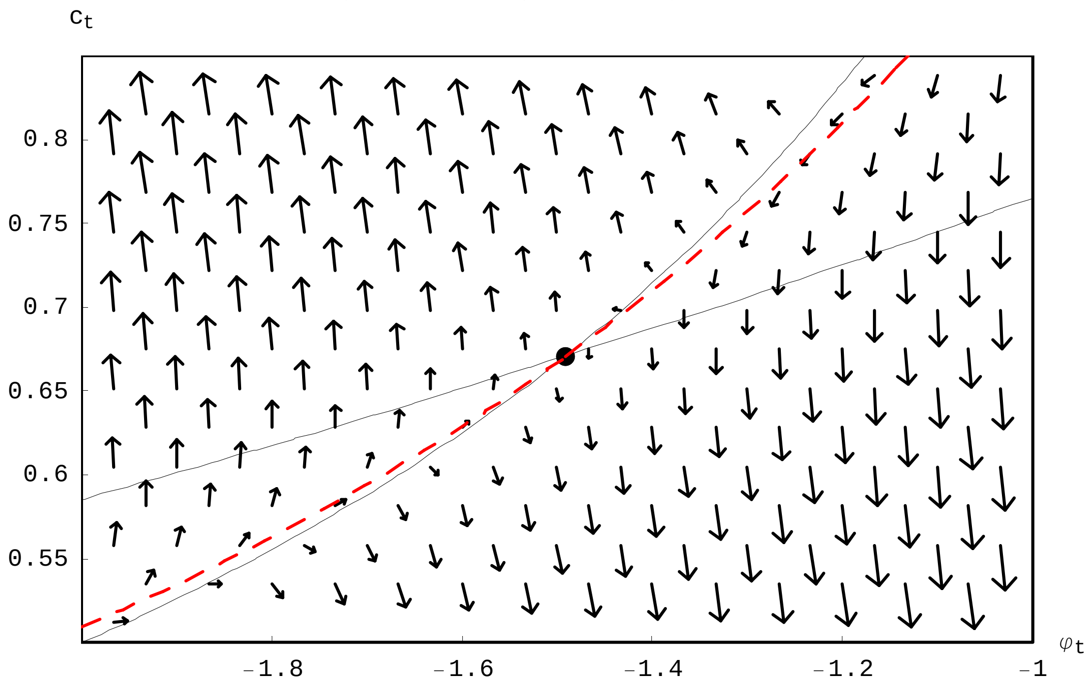

Figure 4: Consumption Time Path for a Myopic and a Forward Looking Agent

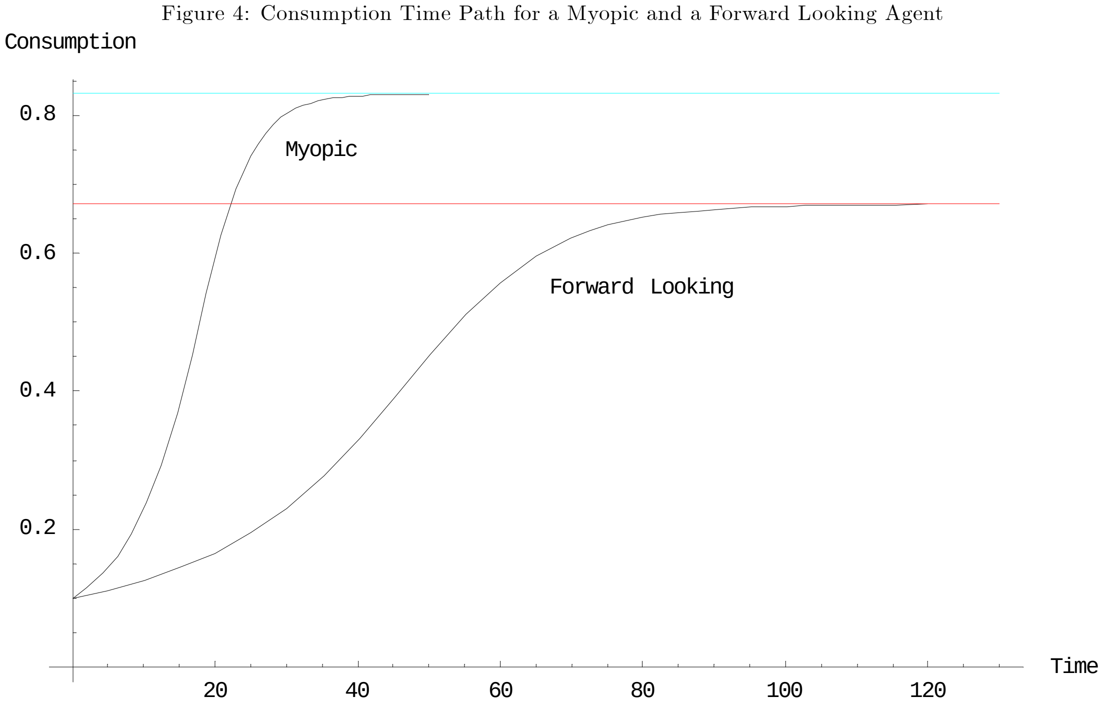

Figure 5: Derrivative of $\theta(\cdot)$ with respect to c_t

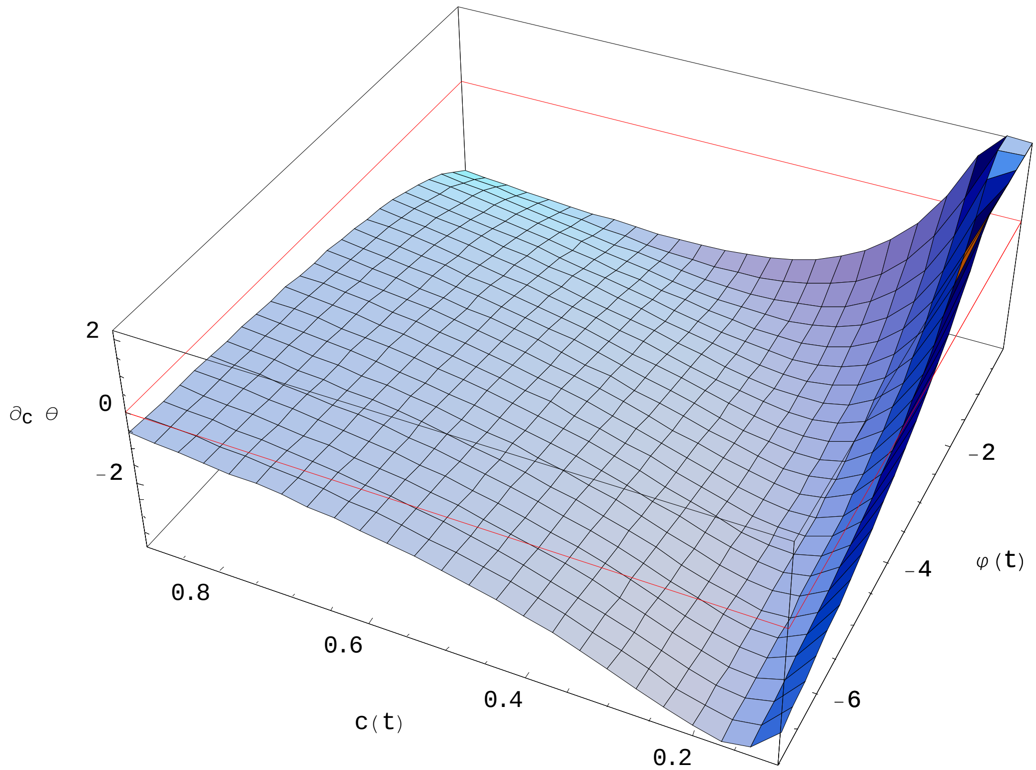

Figure 6: Derrivative of $\theta(\cdot)$ with respect to σ