

We begin by defining a competitive equilibrium in this class of economies.

Definition: A competitive equilibrium in the stochastic economy is an contingent consumption allocation $\{\{c_t^i\}_{t=0}^\infty\}_{i=1}^2$ and state contingent prices $\{p_t\}_{t=0}^\infty$, st.

- Consumers solve

$$\max(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i)$$

$$s.t. \quad E_0 \sum_{t=0}^{\infty} p_t c_t^i \leq E_0 \sum_{t=0}^{\infty} p_t (w_t^i + \theta_0^i d) \quad \forall t$$

$$(1 - \beta)E_t \sum_{j=0}^{\infty} \beta^{j-1} u(c_{t+j}) \geq (1 - \beta)E_t \sum_{j=0}^{\infty} \beta^{j-1} u(w_{t+j}) \quad \forall t \geq 0$$

- Markets clear

$$c_t^1 + c_t^2 = \omega^g + \omega^b + d = \omega \quad \forall t$$

As in the previous case, we want to focus the attention on the steady state of both economies. We want to compute the decision rules for both shocks. Now its agent is going to face the good shock with a certain probability. For simplicity we drop the time index and all the notation is contingent the shock. In a symmetric steady state

$$c^i(s) = \begin{cases} c^g & \text{if } w^i(s) = \omega^g \\ c^b & \text{if } w^i(s) = \omega^b \end{cases}$$

The stochastic steady state is like the deterministic case. We lower c^g from the individual with the good productivity shock until, either the symmetric first-best $c^g = \omega/2$ is achieved or the participation constraint binds. For the stochastic case, we can also compute the expected utility associated to the symmetric steady state, where π denotes the probability of continue in the same state, and $1 - \pi$ denotes the probability of reversal.

$$E_t \sum_{j=0}^{\infty} \beta^{j-1} u(c_{t+j}) = u(c^g) + \beta[\pi u(c^g) + (1 - \pi)u(c^b)] + \dots$$

$$\beta^2[\pi^2 u(c^g) + \pi(1 - \pi)u(c^g) + \pi(1 - \pi)u(c^g) + (1 - \pi)^2 u(c^b)] + \dots$$