ECO–5282
Financial Economics II: Homework #4
Fall 2005
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1. **(Equilibrium with Exogenous Borrowing Constraints)** Consider an economy with a continuum $[0,1]$ of consumers of symmetric typed who live forever. Consumers have utility

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i.$$

Consumers of type 1 have an endowment stream of the single good in each period $(w_0^1, w_1^1, w_2^1, ...) = (8, 1, 8, 1, ...)$ while consumers of type 2 have $(w_0^2, w_1^2, w_2^2, ...) = (1, 8, 1, 8, ...)$. In addition there is one unit of trees that produces $d = 1$ units of good at each period. Each consumer of type i owns $\overline{s}_0^i$ of such a trees in period $t = 0$, $\overline{s}_0^i > 0$, $\overline{s}_0^1 + \overline{s}_0^2 = 1$. Trees do not grow or decay over time.

   (a) Define an Arrow-Debreu equilibrium. Given the endowment, find the initial asset holdings $\overline{s}_0^1$ and $\overline{s}_0^2$ such that in equilibrium, $\widehat{c}_t^1 = \widehat{c}_t^2$ for all t. Compute the equilibrium prices. [**Hint:** You have to be careful when writing down the consolidated budget constraint for the individuals.]

   (b) Suppose that consumers cannot borrow or lend. Define an equilibrium for this liquidity constraint economy.

   (c) Compute the stochastic discount factor for this economy and compare it with the complete markets solution.

   (d) Consider the function

$$F^L(c^g) = u'(c^g)(c^g - 8) + \beta u'(10 - c^g)(9 - c^g)$$

   Show that when $\beta = 0.9$, $F^L(c^g) > 0$ and that $\widehat{c}_t^i = 5$ satisfies all of the equilibrium conditions for the liquidity constraint economy for the right choice of $\overline{s}_0^i$. Show that when $\beta = 0.2$, the solution of $F^L(c^g) = 0$ and $c^g \in [5, 8]$ is such that

$$\widehat{c}_t^i = \begin{cases} c^g & \text{if } w_t^i = 8 \\ 10 - c^g & \text{if } w_t^i = 1 \end{cases}$$

   is an equilibrium allocation for the right choice of $\overline{s}_0^i$. (Hint: you can find the solution of $F^L(c^g) = 0$ by solving a quadratic equation).

2. **(Equilibrium with Endogenous Borrowing Constraints)** Consider the economy from the previous question. However, we assume that consumers face borrowing constraints of the form

$$\sum_{j=0}^{\infty} \beta^i \log c_{t+j}^i \geq \sum_{j=0}^{\infty} \beta^i \log w_{t+j}^i \qquad \forall t, i$$

but otherwise the definition of equilibrium is the same as in part a).

(a) Provide a motivation for this environment and explain the economic intuition of the constraint.

(b) Define an equilibrium for this debt constraint economy.

(c) Consider the function

$$F^D(c^g) = u(c^g) - u(8) + \beta[u(10 - c^g) - u(1)]$$

Show that when $\beta = 0.9$, $F^D(c^g) > 0$ and that $\widehat{c}_t^i = 5$ satisfies all of the equilibrium conditions for the debt constraint economy for the right choice of $\overline{s}_0^i$. Show that when $\beta = 0.2$, the solution of $F^D(c^g) = 0$ and $c^g \in [5, 8]$ is such that

$$\widehat{c}_t^i = \begin{cases} c^g & \text{if } w_t^i = 8 \\ 10 - c^g & \text{if } w_t^i = 1 \end{cases}$$

is an equilibrium allocation for the right choice of $\overline{s}_0^i$. (Hint: if $\beta = 0.2$, the solution to $F^D(c^g) = 0$ and $c^g \in [5, 8]$ is approximately $c^g = 6.09076$).

(d) Compute the stochastic discount factor for this economy and compare it with the liquidity constraint model. Do you think these two economies can predict the same pattern for asset price movements?

(e) Unfortunately, the value of $\overline{s}_0^i$ that you calculated in the previous section when $\beta = 0.2$ is negative. Can you think of another way to make the proposed steady state an equilibrium? Explain.

(f) Write down the endogenous borrowing constraint if the agent is allowed to save in an storage technology with a return $R > 0$, following a default period.

(g) Bankruptcy law only preclude individuals from trading just for a finite number of periods. How would you modify this constraint to accommodate this institutional feature.

3. **(Equilibrium with Endogenous Borrowing Constraints and uncertainty)** We want to take the model one step further and include uncertainty along two dimensions. First, we assume that income shocks are persistent, and evolve according to a symmetric Markov process with a probability distribution

$$\Pi_{w'/w} = \begin{bmatrix} \pi_{gg} & \pi_{bg} \\ \pi_{gb} & \pi_{bb} \end{bmatrix} = \begin{bmatrix} 1-\pi & \pi \\ \pi & 1-\pi \end{bmatrix},$$

Second, we assume i.i.d. dividend shock where γ is the probability of receiving a good dividend shock $d_g = 3$, and $1 - \gamma$ is the probability of receiving a bad dividend or aggregate shock, $d_b = 1$. Since we now have aggregate uncertainty we can still solve for a symmetric equilibrium, but consumption across states of nature will differ due to aggregate uncertainty.

(a) Write down the market equilibrium in the endogenous borrowing constraint economy. [**Hint:** You should be very careful when writing the endogenous borrowing constraint, since it could depend on aggregate uncertainty.]

(b) Solve for the equilibrium allocations when the borrowing constraint binds, and calculate the stochastic discount factor in the presence of aggregate shocks. Will the borrowing constraint always bind? [**Hint:** You can use the social planner problem to solve for efficient allocations.]