Modeling 30 Year Fixed Mortgage Rates and the Effects on Mortgage-Backed Securities.

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Contents

Chapter 1

1.1 Abstract

The focus of this paper is to determine a model for 30 year fixed rate mortgage processes using various United States treasury yields. Various statistical methods were used, and two are highlighted. The application of this model will be used to in the valuation of mortgage-backed securities. In this type of security, one of the main concerns is the prepayment risk. The model presented in this paper should be used in a dynamic prepayment model, where forecasts of future mortgage rates are required.

A discussion on the approaches to forecasting mortgage rates as a function of Treasury rates is outlined. This paper presents several statistical models, which forecasts will be analyzed and the relative performance of using a single Treasury rate or a combination of short and long-term rates to forecast mortgage rates.

The paper also gives a brief introduction to the mortgage market and how the market has developed into what it is today. In order to understand the effects of prepayment on MBS, illustrations of cash flow are presented. To the investor of a MBS, it is important to determine a model for the prepayment risk. The results of these models can be incorporated into prepayments models, as well as, analysis of possible trends in markets.

The models in this paper are based on using statistical regression. Although there are advantages,

there are also disadvantages. One should take much consideration in the forecasts of any regression model. The behavior of these models to exhibit the trends in the mortgage market, one still should consider other factors in future models.

1.2 Introduction

In a growing market, estimated in the trillions[1], valuation methods for mortgage-backed securities are gaining interest. One of the factors that are considered in the valuation procedures is the prepayment risk. This is a critical component that puts the valuation methods at an imbalance during high times of refinancing where mortgage rates are low. The key is to find a process of rates that can be used in an adequate prepayment model to price or hedge mortgage-backed securities.

The derivation of a mortgage rate model can have many implications in pricing methods of the securities that are affected by changes in the mortgage market. Once a model is substantiated, trends and forecasts can predict possible refinancing booms. The data will can also be used in prepayment models that use mortgages as inputs to predict trends and effects on cash flows of a pool of MBS.

I will focus much of the discussion of the paper detailing MBS, by giving an introduction as well as, focusing in on the effects of mortgage rates forecasts from models in the prepayment risk of MBS.

[1]Reports from the Securities Industry and Financial Markets Association (www.bondsmarkets.com), state that there is approximately $6.1 trillion outstanding in U.S. MBS at the end of the first quart of 2006. In comparison with the total outstanding asset-backed securities, as well as the US Treasury notes with an estimated $4.2 trillion in volume, MBS overtook the market in value.

Chapter 2

Mortgage Market

2.1 Evolution of Market

The structure of a mortgage in the mortgage market has seen much change and has evolved from its beginnings. Due to the demand of the changing environment as well as the demand for regulation, the market restructured the foundation.

Before the Great Depression, mortgages were primarily a type of balloon loan, a mortgage that has one payment due upon maturity. However, the sound of the loan may appear beneficial, it was problematic. The borrower may use the available money for other uses, yet had to be very stringent upon the final large payment at the end of the loan. The lender did not have the contractual payments or principal fully, or even partially amortized, yet, depending on the lender, were able to request repayment of loan at any given time. With more borrowers wanting to enter the market, the government sought out ways of providing new instruments to stabilize and promote the development of the market.[1]

The new type of mortgage was designed as a fixed-rate, level payment, fully amortized, and otherwise known as a "level-payment mortgage." This instrument would become insured by the then newly created Federal Housing Administration, FHA. In this mortgage, the mortgagee pays interest and repays the principal in equal installments over the term of the loan. Each payment consists of the

[1] [2] pg 10

interest on the principal and a portion of the principal. The payment is the same, therefore, at each time step, the more and more of the principal is paid off. [2] Thus, illustrating the advantage that the borrower has equal payments, but the interest rate is generally higher than the adjustable rate or interest only loans. [3]

Still very prevalent today, the level-payment mortgage is not the only option for people seeking a mortgage. Other loan structures available are, but not limited to the following: traditional structures as tiered-payment (popular in 1980s), balloon mortgages, revolving lines of credit, and reverse mortgages. Many more have been developed to match more unique individuals entering the primary market. In this paper, the focus will be on the level-payment mortgage type.

2.2 Cash Flow Characteristics of Mortgages

Mortgages are highly liquefiable assets because the real estate property is used as collateral to ensure payment of the loan for the purchase of the property. If contracted payments are not made, the mortgagee has the right to foreclose and seize the property to ensure the obligation is fulfilled. In the contract, the interest rate, when and how many payments are to be made, and the final date of the loan.[4]

Similar to a callable bond, where the cash flow is unknown, in mortgages, the mortgagee, depending how stimulated in the contract, allows the mortgager holder to prepay or make a partial prepayment (hence a curtailment) of the outstanding balance of the mortgage. For bonds, the issuer has the right to redeem or call the issue. Paying off the loan can be seen as exercising a call option on a bond with an exercise price equal to the remaining balance. The lower the rate, the higher the option, and therefore "more-in-the money is the prepayment option[5]." To fully see the effect of prepayment on cash flows of a mortgage, an example will be discussed at a later point.

The importance for the illustration on cash flows is given because of the effects on mortgage-

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[2] [2] pg 11

[3] Lenders generally charge lower initial interest rates for ARMs than for fixed-rate mortgages. Data from the federal reserve illustrates these trends.

[4] [1] pg. 303

[5] [3] pg. 5

backed securities. "Mortgages are the underlying collateral for mortgage backed securities[6]" An introductory example of cash flows from a level-payment mortgage with no prepayment is given with one borrower for illustration of cash flows. Table 1[7] illustrates how the difference between the monthly mortgage payment and a portion of the payment that represents interest equals the amount that is applied to reduce the outstanding mortgage balance. Each monthly payment, M_n, on this type of mortgage consists of a divided portion of the interest of annual interest rate time the amount of the outstanding balance at the beginning of the previous month. The other portion of the payment, consists of a repayment of the outstanding principal, P_n. [8]

2.3 Cash Flows and Servicing Fee

Scheduled stream of payments for an investor in a mortgage-backed security is a monthly mortgage payment of each mortgage in a pool. In table 1[9], an illustration of cash flow is used for one mortgage holder with no servicing fees. To incorporate fees, as well as prepayments in a pool of mortgages, the next few paragraphs will discuss how these items are developed and calculated. An investor in a pool of mortgage-backed securities will analyze all the components that play a role in the cash flow of the mortgages.

The origination of service fees play a role due to the fact that a mortgage payment schedule needs to be monitored to check that the mortgagee is complying with the terms of the contract. The investor and borrower must both be supplied with information concerning the mortgage. These processes take time and resources, which are captured by servicing fees of the mortgage. The mortgage rate stimulated in the contract includes the cost of servicing[10]. The fee is a fixed percentage of each month's outstanding balance.

Table 2[11], illustrates the monthly cash flow of a single mortgage with servicing fees included, but no prepayments. Therefore, the monthly cash flow for an investor in a single mortgage can be split

[6] [1] pg. 304
[7] Appendix B
[8] Appendix B: Table 2
[9] Appendix B
[10] See Appendix A for listings of formulas
[11] Appendix B

into three sections[12]:

1. Interest payment net servicing fee

2. Amount to servicing fee, (S)

3. Scheduled principal repayment, (MB_n)

2.4 Cash Flows Characteristics of Mortgage-Backed Securities

Table 2[13], illustrated an example of a level-payment fixed rate mortgage of a single homeowner. In this example, it was assumed that the homeowner did not curtail or repay the original amount at any given time during the length of the loan. In reality, this is not the case. A certain percentage of homeowners do pay some or all of his/her mortgage at any given time. These excess payments, prepayments, are problematic for an investor depending on the stream of cash flows for an MBS. Thus an investor would wish to know what factors may cause prepayments and how a cash flow may be affected by prepayments.

2.5 Factors Affecting Valuation of MBS

2.5.1 Prepayment

Prepayments occur for several reasons that either controllable or inevitable by the homeowner. When homeowners sell their home, they make one large prepayment. The selling of a home may be due to a multitude of factors, such as a need to relocate because of job or family, "trading up" (moving into a bigger home), or settlements due to termination of marriages[14].

[12] Appendix A contains a listing of formulas
[13] Appendix B
[14] [2] pg 35

2.5.2 Housing Turnover

A mortgage rate model will benefit those who wish to refinance when rates fall below the homeowners current rate. If able to, the mortgagee will refinance to obtain a new loan under a lower interest rate. Booms of refinancing occur when interest rates are low and homeowners have built up equity in their homes. The equity enables the homeowner to have more choices, even though being issued a low credit rating. Obtaining lower interest rates will help to reduce debts by raising additional funds[15]. Graph [16] illustrates the high negative correlation with amount of homeowners who refinance with the movements of mortgage rates.

Analyzing refinance trends, usually some time goes by before the mortgagee will chose the option to refinance. From this trend, prepayment rates increase from a low level at the beginning of the MBS, and after 12 to 60 months, tend to level off.[17] Refinancing trends are highly exhibit seasonal trends that can correlate with mortgage rates.

For an aged MBS, prepayments rates tend to stabilize. This may be due to as a mortgage reaches maturity, more of the principal is paid, thus a cut in the rates may produce no saving for the mortgagee. Also, the pool may be exposed to a "burnout effect," mortgagees remaining in the aged MBS who did not take advantage of refinancing will be less likely to take advantage of other rate cuts. [18]

2.5.3 Default

When homeowners cannot meet the terms of the contract, default occurs. The lender forecloses on the property, a service incorporated in the fees, and the sold where the funds are used to for repayment. Lastly, there exists homeowner who cannot make payments on the property because there is no property to pay for because of loss due to natural disasters. Insurances, when willing and able, will pay off the mortgage loan[19].

Mobility and refinancing are negative events for investors in MBS. There is a curtailment or fully

[15][1] pg 323
[16]Appendix C
[17] [2] pg 36
[18] [2] pg 37
[19] [1] pg 324

prepayment in the cash flow of the MBS. Therefore, an investor will research any potential way of measuring the possibility of prepayments. The need for ability to forecast prepayments have developed benchmarks for investors to use for prepayment modeling.

2.6 Measurements Used in Prepayment Discussion

To derive valuation methods of MBS, conventions and benchmarks have been setup to discuss how to measure prepayments and the risk involved. A major benchmark of how to measure the MBS's prepayment risk is the pools average underlying mortgage rate, pool "WAC." "The further below current mortgage rates a pool's WAC is, the slower the ppol is expected to prepay. Likewise, prepayments are faster for pools with above the markter WACs [20]"

Due to advancements in the market in data collection and calculations, the market is now able to form views and models or prepayment behavior to make predictions. There are several measures used in modeling prepayments, the conditional prepayment rate, a single mortgage mortalitiy, and the Public Securities Association (PSA) prepayment standard[21]

2.6.1 CPR

In table 2[22], an illustration is provided to where the annual CPR of the pool is assumed to be 6%. This rate is the percentage of the balance which is expected to still owed at the end of each year. In table 2, at the end of the first year, the expected balance will be (1-CPR), 94%. Therefore, the balance at the end of year 1, is \$940000$(1,000,000(1-0.06))$. There is an added multiplication because there still exists monthly payments. The main understanding of a 6% CPR, is that, given an initial balance of the pool of MBS at the end of the year will be (1-CPR) = 94% of the balance that would have existed if no prepayments had been exercised.

[20] [2] pg 36

[21] See formulas and graphs for each in Appendix A/B/C.

[22] Appendix B

2.6.2 SMM

In table 2, as with many illustrations in fixed income textbooks, [23] the CPR, an annual percentage, needs to be illustrated as a monthly rate so that it may be used in streams of monthly cash flows. The single monthly mortality rate, SMM, is the CPR converted to a monthly measure[24]. A SMM of $x\%$, represents the percentage of remaining mortgage balance at the beginning of the month after subtracting the principal payment that have made a full prepayment that month.

Therefore, one can see the amount of the prepayment each month[25].

$$\text{Monthly Prepayment} = SMM \times (\text{Beginning mortgage balance} - \text{scheduled principal})$$

Therefore, in month 1, from table 2, the investor can expect \$166.74 prepayment for the MBS.

2.6.3 PSA

The PSA standard benchmark for prepayments is based on a monthly series of annual CPRs. This is not a model for prepayments, but a measurement of the series. At the start of the pool age, the first month begins at 0.2% and increases by 0.2% in each successive month. When the age of the pool reaches month 30, the series levels off at the annual CPR until end of loan, say 6%, from previous example[26]. Thus, "prepayments are measured as a simple linear multiples of this schedule[27]. This "ramp" effect is illustrated in a graph[28] of the example from table 2.

PSA is used as a benchmark because it serves many advantages. The series illustrates as the pool ages, the normal increase displayed in the CPR. The PSA does not reflect the speed at which the pool or mortgages are prepaid. Fabozzi stress that it is only a the PSA is only a "market convention.[29]." Avoiding analysis of the CPR, thereby relaying only on the PSA prepayment assumptions, can result in improper valuation of MBS. In the Handbook of MBS, Fabozzi states that an investor

[23] [1] pg 305.

[24] see formula and derivation in appendix A

[25] Used in creation of table 2. Note, example assumes the 100% PSA benchmark

[26] Formulas for PSA are found in the Appendix A

[27] [2] pg. 40

[28] Appendix B: Following dervation of PSA

[29] [1] pg. 348

should not use the PSA standard for securities backed by balloon loans[30].

2.7 Example: Table 2

From the discussion of the prepayment conventions used in analysis of MBS prepayment, as well as the derivation of formulas[31], it is time to illustrate cash flows of a MBS with prepayment[32].

The projected cash flow is illustrated in table 2. It is derived from the adding projected monthly interest for the month, projected monthly principal payment, projected principal prepayment, and subtracting servicing fees.

[30] [2] pg. 42
[31] Appendix A
[32] Table 2 for example.

Chapter 3

Modeling the Rates

3.1 Treasury Yield Curve and Mortgage Rates

Mortgage rates are the interest charged on a mortgage by a lender. There are many different products available in the market when it comes to analyzing the rate. A future homeowner seeking a loan will seek out a product that is beneficial to him/her. One benchmark [1], in which this paper is focuses on, the 30-year fixed mortgage, is a loan that has an interest rate that stays the same for the 30-year term of the loan.

Analyzing for trends in the market illustrate the given standard as a "rule of thumb" is to use the Treasury notes to help predict the behavior of the thirty-year fixed mortgage rates. Yields on 10-year and 30-year Treasury securities are typically used to set long-term mortgage rates. This can be seen for example, when bond yields decrease, usually, conventional mortgage rates illustrate the same behavior. It can be seen in the opposite direction as well. The reason may be due to the fact that when a mortgagee is choosing to sell the mortgage loan to an investor, the essentials of an MBS, he/she will use likely use the Treasury yields as reference. See figure[2] to see the correlation between the T-bonds and the FRM.

[1] This product is commonly used as a reference point and comparison in the market.

[2] Appendix B

3.2 Multiple Regression Model

There were many different parameters to choose from when I wanted to estimate the 30 year fixed mortgage rate from January 1998-November 2006. I did not take all the data up today, because I will use the forecasted data as a comparison to the historical data. My first approach I used multiple and multivariate regression techniques. Although there are advantages and disadvantages to using these methods, much of what is not discussed in this paper. I only chose to discuss the major disadvantage to the model, as a precaution to other modeling data.

3.2.1 Determining the variables

It has been common to use linear regression or compare behavior with 10 year Treasury as a predictor for 30 year FRM rate. However, current trends and graphs of 10 year and 30 year mortgage rate[3] illustrate a nonlinear trend. I chose to run multiple regression and multivariate regression to predict the 30 year mortgage rates using a combination of predictors, the treasury yields. I wanted to see if there was more correlation between using short term rates and long terms rates to predict mortgage rates, instead of solely depending on the long term rate.

Correlations

I ran correlations as a preliminary analysis to check for any possible combination of predictors as well as any multicolinearity effects[4]. I found that almost all the yields were highly correlated. This may due to the fact that the long term yields and short term yields are affected by similar factors. There may exist other reasons out of the scope of this paper. The goal of this analysis was to check for behavior between variables for multivariate regression analysis.

Note the high correlations between many yields as well as no correlation between the CPI. The correlation between 30 year FRM rate between each possible predictor decrease as time of the yield decreases.

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[3] Appendix C

[4] Appendix B

3.2.2 Models

There were several possible multiple and multivate regression models to choose from. Table 3 illustrates each model with the adjusted R^2, for each model. Each standard error for the parameter estimate is given.

An analysis of the graphs, using the parameters estimated from each model is illustrated[5]. From the graphs, one can observe that each model does behavior in a similar pattern as the 30 year FRM rate. However, a closer analysis can of the graphs illustrate a possible error in regression[6]. Although there seems to correlation in the trend of the predicted values from each models and the actual 30 year FRM, each model is overestimating the rate. A post-hoc analysis was to eliminate any upward/downward shift in the data, by removing or modifying the intercept term. No model was improved by removal of y-intercept.

In conclusion, although there was a high adjusted R^2 for many of the models, this statistic should be analyzed further. Depending only on this unit-less parameter, the R^2 may lead to many errors in statistical analysis and modeling. Even though it is common practice to use historical data, because of ease or availability, further analysis may need a more term-structure model.

3.3 Nonlinear Regression Model

There were many functions to choose from to do a post-hoc analysis of the 30 year FRM rate process. Due to the shape of the scatter plot between 10 year Treasury and mortgage rate[7], it was suggested to try to model a a cubic function as well as inverse trigonometric function.

Influenced by Dr. Yengeny Goncharov, I ran regression analysis predicting mortgage rates using 10 year Treasury, each β_i is an estimated parameter:

$$y_i = \arctan(\beta_1 X_i + \beta_2) + \beta_3 \tag{3.1}$$

[5] Appendix B
[6] Appendix C
[7] Appendix C

I needed the Taylor expansion for arctangent because of the requirement for the first three derivatives in regression analysis[8]. This analysis was post-hoc and suggested by a professor who has worked extensively in this area. His current project is deriving a modeling using an computer algorithm that does not depend on iterations[9].

After running many regression models using this inverse trigonometric function, no model that fit the data was able to be obtain from this paper. It may be possible to fit the data according to this function, given that the shape of the function does correlate with the trend found in the data. However, no model was able to be obtained in this paper.

3.4 Nonparametric Regression Model

Nonparametric simple regression, a type of scatter plot smoothing[10], was used to analyze the data. This method is often used when trying to trace a smooth curve through given some data. As in nonlinear regression, it is standard to assume that the error, ϵ_i $N(0, \sigma^2)$ i.i.d[11]. Using statistical software, a regression model was created using this method[12]. However, this model was not used, due to the fact that it did not exhibit the slight curvature found in the center of the data.

3.5 Applications of Models: Prepayment Modeling

The purpose of this paper was to forecast possible mortgage rate information to use parameters of prepayment models. MBS are highly affected by prepayment risk. Table 2 illustrated how cash flows are greatly impacted by prepayments in the pool of mortgages. As stated before, an investor must consider more that the rates when analyzing behavior in the market to much assumptions on the prepayment risk.

[8] See Appendix A for derivation.

[9] [4]

[10] [5] pg. 2

[11] [5] pg 2

[12] Appendix C illustrates graph obtained

3.6 Conclusion

Although this paper illustrates many statistical models that exhibit similar trends in the mortgage market, I found that more analysis much be made to predict behavior in the market. Many preliminary statistical regression are highly dependent on the search for an adjusted R^2, however, as I found in this paper, much more must be considered to interpret the data. Although, even though the models are dependent on historical information[13], one can examine the trends as a preliminary procedure to make further structured models in the future.

[13] A common error in financial analysis, yet still a common practice because of availability

Appendix A

Formulas

Below is a listing of formulas[1] used in tables 1 and 2.

A.1 Mortgage Cash Flow Without Prepayments

Monthly Payment: Constant monthly payment.

$$M_n = B_0 \frac{[i(1+i)^N]}{[(1+i)^N - 1]} \tag{A.1}$$

where

$$
\begin{aligned}
M_n &= \text{Monthly payment for month } n \\
B_0 &= \text{Original mortgage balance(\$)} \\
N &= \text{Number of months} \\
i &= \text{Simple monthly interest rate (annual interest rate/12)}
\end{aligned}
$$

[1] All formulas listed in Appendix are found in [1]

Remaining Balance: Remaining balance after n months

$$B_n = B_0 \frac{[(1+i)^N - (1+i)^n]}{[(1+i)^N - 1]} \tag{A.2}$$

where

$$B_n = \text{Remaining balance at the end of month} n \tag{A.3}$$

Principal Payment: Amount of principal paid in month n

$$P_n = B_0 \frac{[i(1+i)^{n-1}]}{[(1+i)^N - 1]} \tag{A.4}$$

where

$$P_n = \text{Principal paid in month} n \tag{A.5}$$

Interest Payment: Amount of interest paid in month n

$$\begin{aligned} I_n &= B_0 \frac{\left[i\left[(1+i)^N - (1+i)^{n-1}\right]\right]}{[(1+i)^N - 1]} \\ &= iB_{n-1} \end{aligned} \tag{A.6}$$

where

$$I_n = \text{Interest paid in month} n$$

To incorporate servicing fees, the annual interest rate is rewritten to include:

$$i = S + C \tag{A.7}$$

where

$$S = \text{Service fee}(\%)$$
$$C = \text{Security coupon rate}(\%)$$

Servicing Amount:

$$\text{Servicing amount} = \left(\frac{S}{C+S}\right) I_n \tag{A.8}$$

Cash Flow: Holder of Security will hold CF_n in month n

$$\begin{aligned} CF_n &= P_n + I_n - \text{Servicing Amt} \\ &= P_n + \left(\frac{C}{C+S}\right) I_n \end{aligned} \tag{A.9}$$

A.2 Prepayments Formulas

On a pool of mortgages, use the following for notation:

$$B_n = \text{Remaining principal balance at end of month } n \text{ if no prepayments}$$
$$C_n = \text{Pool balance, actual remaining balance at end of month } n.$$
$$Q_n = \text{Represent the percentange of mortgages remaining at end of month } n$$

Let $Q_n = \frac{C_n}{B_n}$. Note: When analyzing a pool of mortgages, each that can terminate separately. Therefore, the precentage of intial balance that has been paid is equal to $1 - Q_n$.

Single Monthly Mortality

$$SMM = \text{Proportion of mortgages outstanding at the beginning of the month that are prepaid during the month, thus can be seen as}$$

$$\frac{Q_{n-1} - Q - n}{Q_{n-1}} = 1 - \frac{Q - n}{Q_{n-1}} \tag{A.10}$$

Conditional Prepayment Rate

$$CPR = \text{the SMM expressed as an annual rate, thus can be seen as}$$

$$CPR = 1 - (1 - SMM)^{12} \tag{A.11}$$

$$SMM = 1 - (1 - CPR)^{\frac{1}{12}} \tag{A.12}$$

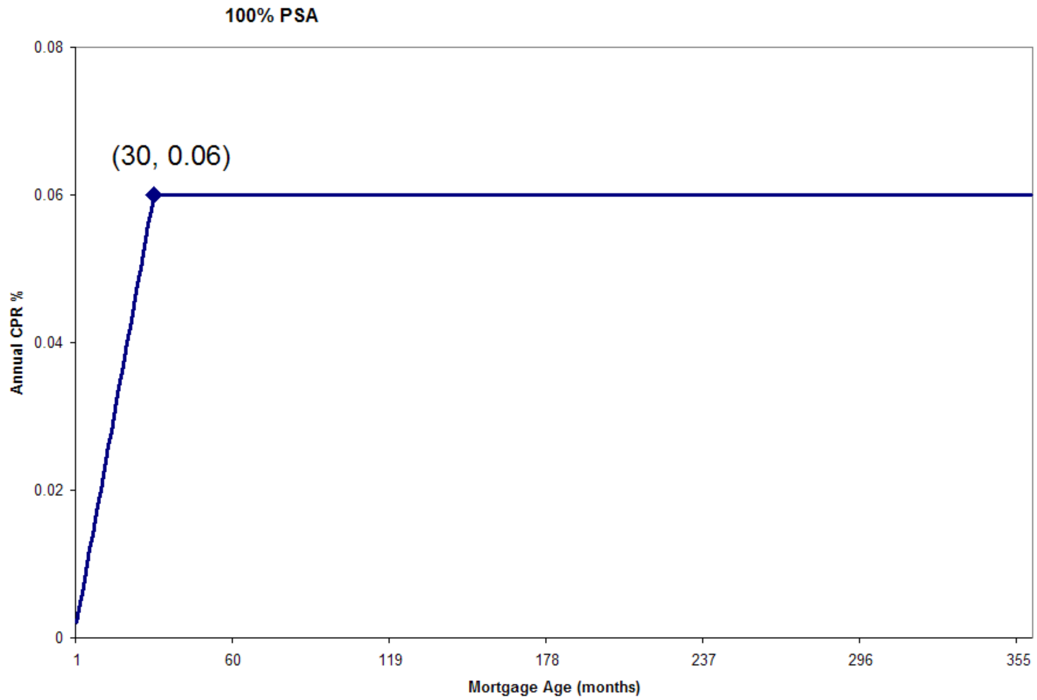
Percentage of PSA

If the mortgage prepays of 100% PSA, then the CPR for month of a mortgage that is n months old

$$\begin{aligned} CPR &= \begin{cases} 6\% \times \frac{n}{30} & \text{if } n \leq 30 \\ 6\% & \text{if } n > 30 \end{cases} \\ &= 6\% \times \min\left(1, \frac{n}{30}\right) \text{ for any } n \end{aligned} \tag{A.13}$$

If the mortgage prepays of $x\%$ PSA, then the CPR for month of a mortgage that is n months old

$$\begin{aligned} CPR &= \begin{cases} 6\% \times \frac{x}{100} \times \frac{n}{30} & \text{if } n \leq 30 \\ 6\% \times \frac{x}{100} & \text{if } n > 30 \end{cases} \\ &= 6\% \times \frac{x}{100} \times \min\left(1, \frac{n}{30}\right) \text{ for any } n \end{aligned} \tag{A.14}$$



Graph: 100% PSA

From Table 2[2], can produce a commonly displayed graph to illustrate 100% PSA.

[2] Appendix B

A.3 Mortgage Cash Flow with Prepayments

To create table 2, each of the following formulas were used to illustrate prepayment effect on a pool of mortgages.

$$
\begin{aligned}
\widehat{M}_n &= \text{actual monthly scheduled payment} \\
\widehat{P}_n &= \text{actual scheduled principal} \\
\widehat{I}_n &= \text{actual scheduled interest} \\
\widehat{B}_n &= \text{remaining (end-of month) balance for the } n \text{ month} \\
SMM_n &= \text{Prepayment rate in month } n\text{, such that} \\
& \quad Q_n = (1 - SMM_n)(1 - SMM_{n-1}) \cdots (1 - SMM_1)
\end{aligned}
$$

Total Monthly Payment in month n

$$
\begin{aligned}
\widehat{M}_n &= \frac{\widehat{B}_n i(1+i)^{N-n+1}}{[(1+i)^{N-n+1} - 1]} \\
&= M_n Q_{n-1} \qquad \text{(A.15)}
\end{aligned}
$$

Scheduled Principal Portion of this Monthly Payment

$$
\begin{aligned}
\widehat{P}_n &= \frac{\widehat{B}_n i}{[(1+i)^{N-n+1} - 1]} \\
&= P_n Q_{n-1} \qquad \text{(A.16)}
\end{aligned}
$$

Interest Portion

$$\widehat{I}_n = \widehat{B}_n i$$
$$= I_n Q_{n-1} \tag{A.17}$$

Unscheduled Principal Payment in month n

$$PR_n = (\widehat{B}_{n-1} - \widehat{P}_n) SMM_n \tag{A.18}$$

Remaining Balance at end of month

$$\widehat{B}_n = B_{n-1} - \widehat{P}_n - PR_n$$
$$= B_n Q_n \tag{A.19}$$

Total Cash Flow of an Investor in MBS

$$\widehat{CF}_n = \widehat{P} + PR_n + \left(\frac{C}{C+S}\right) \widehat{I}_n \tag{A.20}$$

A.4 Arctangent Expansion as Taylor Series

A.4.1 Taylor Expansion

The Taylor Series is a method that allow for a function to be represented or approixmated as sums of terms derived from the values of the derivatives of the function at any point. The Taylor Series, defined on an open interval is the following:

$$f(x) \approx \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n \tag{A.21}$$

Therefore, $f(x) = \arctan(x)$, where $a = 0$, can be approximated to obtain:

$$\arctan(x) \approx \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\, x^{2n+1} \;\text{ for }\; |x| \leq 1 \tag{A.22}$$

If let $a = 0$, will obtain the following, (where β_i are parameters):

$$f(x) \approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \cdots$$

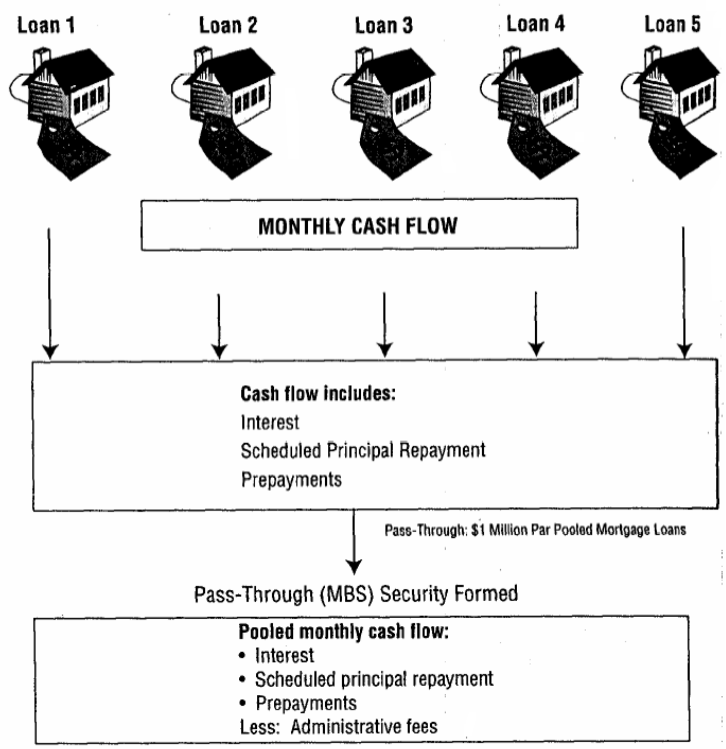
Using the model suggested, $f(x) = \arctan(\beta_1 x + \beta_2) + \beta_0$, will expand as follows

$$\begin{aligned} f(x) &\approx f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \cdots \\ &\approx \arctan(\beta_2) + \beta_0 + \frac{\beta_1}{1+\beta_2^2}\, x \\ &\quad - \frac{\beta_1\beta_2}{(1+\beta_2^2)^2}\, x^2 - \frac{2\beta_1^2(1+9\beta_2^2)}{(1+\beta_2^2)^3}\, x^3 + \cdots \end{aligned} \tag{A.23}$$

Appendix B

Tables

B.1 Formation of a MBS



How a Pass-Through Security Is Created

Exhibit above is drawn from Fabozzi's Handbook of Mortgage-Backed Securities.

B.2 Cash Flow of Single Mortgage with No Servicing Fee

Loan	$1,000,000.00
Years	30
# of Payments	360
Interest Rate (Annl)	9.50%
Monthly Rate	0.007916667
Payment	($8,408.54)
Coupon	9.00%

Payment #	Beg. Balance	Payment	Interest	Principal	End Balance
1	$1,000,000.00	$8,408.54	$7,916.67	$491.88	$999,508.12
2	$999,508.12	$8,408.54	$7,912.77	$495.77	$999,012.36
3	$999,012.36	$8,408.54	$7,908.85	$499.69	$998,512.66
4	$998,512.66	$8,408.54	$7,904.89	$503.65	$998,009.01
5	$998,009.01	$8,408.54	$7,900.90	$507.64	$997,501.37
6	$997,501.37	$8,408.54	$7,896.89	$511.66	$996,989.72
7	$996,989.72	$8,408.54	$7,892.84	$515.71	$996,474.01
8	$996,474.01	$8,408.54	$7,888.75	$519.79	$995,954.22
9	$995,954.22	$8,408.54	$7,884.64	$523.90	$995,430.32
10	$995,430.32	$8,408.54	$7,880.49	$528.05	$994,902.26
⋮	⋮	⋮	⋮	⋮	⋮
350	$88,247.14	$8,408.54	$698.62	$7,709.92	$80,537.22
351	$80,537.22	$8,408.54	$637.59	$7,770.96	$72,766.27
352	$72,766.27	$8,408.54	$576.07	$7,832.48	$64,933.79
353	$64,933.79	$8,408.54	$514.06	$7,894.48	$57,039.31
354	$57,039.31	$8,408.54	$451.56	$7,956.98	$49,082.33
355	$49,082.33	$8,408.54	$388.57	$8,019.97	$41,062.35
356	$41,062.35	$8,408.54	$325.08	$8,083.47	$32,978.89
357	$32,978.89	$8,408.54	$261.08	$8,147.46	$24,831.43
358	$24,831.43	$8,408.54	$196.58	$8,211.96	$16,619.47
359	$16,619.47	$8,408.54	$131.57	$8,276.97	$8,342.50
360	$8,342.50	$8,408.54	$66.04	$8,342.50	($0.00)

Table 1: Cash Flow for a Mortgage with $\$100,000$ Loan, 30 Year Mortgage Rate 9.5%

B.3 Cash Flow of Single Mortgage-Backed Security with Service Fee

						PSA 100%						
Payment #	Beg. Balance	% Mtg's remain	Payment	Interest	Principal	CPR	SMM	Prepayment	Total Principal	End Balance	Coupon	Fee
1	$1,000,000.00	100.00%	$8,408.54	$7,916.67	$491.88	0.002	0.00016682	$166.74	$658.61	$999,341.39	$7,500.00	$416.67
2	$999,341.39	99.98%	$8,407.14	$7,911.45	$495.69	0.004	0.000333946	$333.56	$829.25	$998,512.14	$7,495.06	$416.39
3	$998,512.14	99.95%	$8,404.33	$7,904.89	$499.44	0.006	0.00050138	$500.38	$999.83	$997,512.31	$7,488.84	$416.05
4	$997,512.31	99.90%	$8,400.12	$7,896.97	$503.15	0.008	0.000669124	$667.12	$1,170.27	$996,342.04	$7,481.34	$415.63
5	$996,342.04	99.83%	$8,394.50	$7,887.71	$506.79	0.01	0.000837177	$833.69	$1,340.48	$995,001.56	$7,472.57	$415.14
6	$995,001.56	99.75%	$8,387.47	$7,877.10	$510.37	0.012	0.001005543	$1,000.00	$1,510.38	$993,491.19	$7,462.51	$414.58
7	$993,491.19	99.65%	$8,379.04	$7,865.14	$513.90	0.014	0.00117422	$1,165.97	$1,679.87	$991,811.32	$7,451.18	$413.95
8	$991,811.32	99.53%	$8,369.20	$7,851.84	$517.36	0.016	0.001343212	$1,331.52	$1,848.88	$989,962.44	$7,438.58	$413.25
9	$989,962.44	99.40%	$8,357.96	$7,837.20	$520.75	0.018	0.001512519	$1,496.55	$2,017.30	$987,945.14	$7,424.72	$412.48
10	$987,945.14	99.25%	$8,345.31	$7,821.23	$524.08	0.02	0.001682143	$1,660.98	$2,185.06	$985,760.07	$7,409.59	$411.64
11	$985,760.07	99.08%	$8,331.28	$7,803.93	$527.34	0.022	0.001852084	$1,824.73	$2,352.08	$983,408.00	$7,393.20	$410.73
12	$983,408.00	98.90%	$8,315.85	$7,785.31	$530.53	0.024	0.002022343	$1,987.72	$2,518.25	$980,889.75	$7,375.56	$409.75
13	$980,889.75	98.70%	$8,299.03	$7,765.38	$533.65	0.026	0.002192923	$2,149.85	$2,683.50	$978,206.25	$7,356.67	$408.70
14	$978,206.25	98.48%	$8,280.83	$7,744.13	$536.70	0.028	0.002363825	$2,311.04	$2,847.74	$975,358.52	$7,336.55	$407.59
15	$975,358.52	98.25%	$8,261.25	$7,721.59	$539.67	0.03	0.002535049	$2,471.21	$3,010.88	$972,347.64	$7,315.19	$406.40
16	$972,347.64	98.00%	$8,240.31	$7,697.75	$542.56	0.032	0.002706597	$2,630.28	$3,172.84	$969,174.80	$7,292.61	$405.14
17	$969,174.80	97.73%	$8,218.01	$7,672.63	$545.37	0.034	0.00287847	$2,788.17	$3,333.55	$965,841.25	$7,268.81	$403.82
18	$965,841.25	97.45%	$8,194.35	$7,646.24	$548.11	0.036	0.003050669	$2,944.79	$3,492.90	$962,348.35	$7,243.81	$402.43
19	$962,348.35	97.16%	$8,169.35	$7,618.59	$550.76	0.038	0.003223197	$3,100.06	$3,650.83	$958,697.52	$7,217.61	$400.98
20	$958,697.52	96.84%	$8,143.02	$7,589.69	$553.33	0.04	0.003396053	$3,253.91	$3,807.24	$954,890.28	$7,190.23	$399.46
21	$954,890.28	96.51%	$8,115.37	$7,559.55	$555.82	0.042	0.00356924	$3,406.25	$3,962.07	$950,928.21	$7,161.68	$397.87
22	$950,928.21	96.17%	$8,086.40	$7,528.18	$558.22	0.044	0.003742759	$3,557.01	$4,115.23	$946,812.98	$7,131.96	$396.22
23	$946,812.98	95.81%	$8,056.14	$7,495.60	$560.54	0.046	0.003916611	$3,706.10	$4,266.64	$942,546.34	$7,101.10	$394.51
24	$942,546.34	95.43%	$8,024.59	$7,461.83	$562.76	0.048	0.004090797	$3,853.46	$4,416.22	$938,130.12	$7,069.10	$392.73
25	$938,130.12	95.04%	$7,991.76	$7,426.86	$564.90	0.05	0.004265319	$3,999.01	$4,563.91	$933,566.21	$7,035.98	$390.89
26	$933,566.21	94.64%	$7,957.67	$7,390.73	$566.94	0.052	0.004440178	$4,142.68	$4,709.62	$928,856.59	$7,001.75	$388.99
27	$928,856.59	94.22%	$7,922.34	$7,353.45	$568.89	0.054	0.004615375	$4,284.40	$4,853.29	$924,003.30	$6,966.42	$387.02
28	$924,003.30	93.78%	$7,885.77	$7,315.03	$570.75	0.056	0.004790913	$4,424.08	$4,994.83	$919,008.47	$6,930.02	$385.00
29	$919,008.47	93.33%	$7,847.99	$7,275.48	$572.51	0.058	0.004966792	$4,561.68	$5,134.19	$913,874.28	$6,892.56	$382.92
30	$913,874.28	92.87%	$7,809.01	$7,234.84	$574.18	0.06	0.005143013	$4,697.11	$5,271.29	$908,602.99	$6,854.06	$380.78
31	$908,602.99	92.39%	$7,768.85	$7,193.11	$575.74	0.06	0.005143013	$4,670.00	$5,245.74	$903,357.25	$6,814.52	$378.58
32	$903,357.25	91.92%	$7,728.90	$7,151.58	$577.32	0.06	0.005143013	$4,643.01	$5,220.33	$898,136.92	$6,775.18	$376.40
.....	.....	.....	.....		.....	.....		.....	.....	.....	.....	.....

Table 2: Projected Cash Flow for a MBS with $\$100$ million Balance, 30 Year Mortgage Rate 9.5%, and Fee 0.5%

B.4 Prepayment Table: 100% PSA

Payment #	Principal	Interest	Fee
0	0	0	0
12	19110.2499	89393.16	4966.286
24	42759.62995	86685.31	4815.851
36	60623.058	81981.58	4554.532
48	59547.21639	76493.61	4249.645
60	56203.99646	71273.86	3959.659
72	53069.01416	66346.33	3685.907
84	50130.02976	61692.76	3427.376
96	47375.54634	57296.02	3183.112
108	44794.76559	53139.93	2952.218
120	42377.54613	49209.29	2733.85
132	40114.3644	45489.78	2527.21
144	37996.27792	41967.91	2331.551
156	36014.89076	38630.97	2146.165
168	34162.32103	35466.98	1970.388
180	32431.17041	32464.65	1803.592
192	30814.49553	29613.36	1645.187
204	29305.78094	26903.08	1494.615
216	27898.91393	24324.33	1351.351
228	26588.16065	21868.19	1214.899
240	25368.14388	19526.23	1084.791
252	24233.82198	17290.51	960.584
264	23180.4692	15153.51	841.8614
276	22203.6572	13108.11	728.2284
288	21299.23765	11147.61	619.3119
300	20463.3259	9265.66	514.7589
312	19692.28565	7456.236	414.2354
324	18982.71459	5713.646	317.4248
336	18331.43082	4032.49	224.0272
348	17735.46026	2407.646	133.7581
360	17192.02464	834.2531	46.3474

Table 3: Illustration of Yearly Prepayments of Table 2: Cash Flow Example.[1]

[1]Appendix C for Graph.

B.5 Correlation Matrix

Correlation matrix:

Variables	30 fixed	GT30 Govt	GT10 Govt	GT7 Govt	TBM3	TBM6	CPI	cpi1%
30 fixed	**1.000**	0.914	0.956	0.952	0.764	0.758	-0.713	-0.005
GT30 Govt	0.914	**1.000**	0.912	0.863	0.525	0.521	-0.705	-0.018
GT10 Govt	0.956	0.912	**1.000**	0.991	0.782	0.787	-0.561	0.054
GT7 Govt	0.952	0.863	0.991	**1.000**	0.845	0.851	-0.551	0.063
TBM3	0.764	0.525	0.782	0.845	**1.000**	0.998	-0.359	0.081
TBM6	0.758	0.521	0.787	0.851	0.998	**1.000**	-0.336	0.087
CPI	-0.713	-0.705	-0.561	-0.551	-0.359	-0.336	**1.000**	0.152
cpi1%	-0.005	-0.018	0.054	0.063	0.081	0.087	0.152	**1.000**

Correlation Matrix for each Predictor

B.6 Comparison of Regression Models

Linear Reg (Model 1)

Intercept	0.977602
GT10 Govt	1.103603
	0.903904

Multiple Reg

(Intercept)	0.413432
6month	1.233164
1yr	-1.331633
10yr	1.353058
	0.928879

Multivariate Reg (Model 2)

(Intercept)	0.682673
6month	56.33936
1year	-54.37487
GT10 Govt	1.725656
x6:x13	-12.55167
x8:x13	11.34562
x6:x14	-7.983409
x8:x14	8.267624
x13:x14	-0.08954
x6:x8:x13	0.084385
x6:x8:x14	-0.069003
x6:x13:x14	1.846358
x8:x13:x14	-1.762539
	0.963548

Multivariate Reg (Model 6)

(Intercept)	0.48708
x13	1.271024
x13:x8	-0.126881
x13:x5	0.120253
	0.92747

Multivariate Reg (Model 5(shortened))

(Intercept)	3.721681
x13	0.08191
x13:x6	-0.09747
x13:x8	0.174336
	0.940605

TABLE

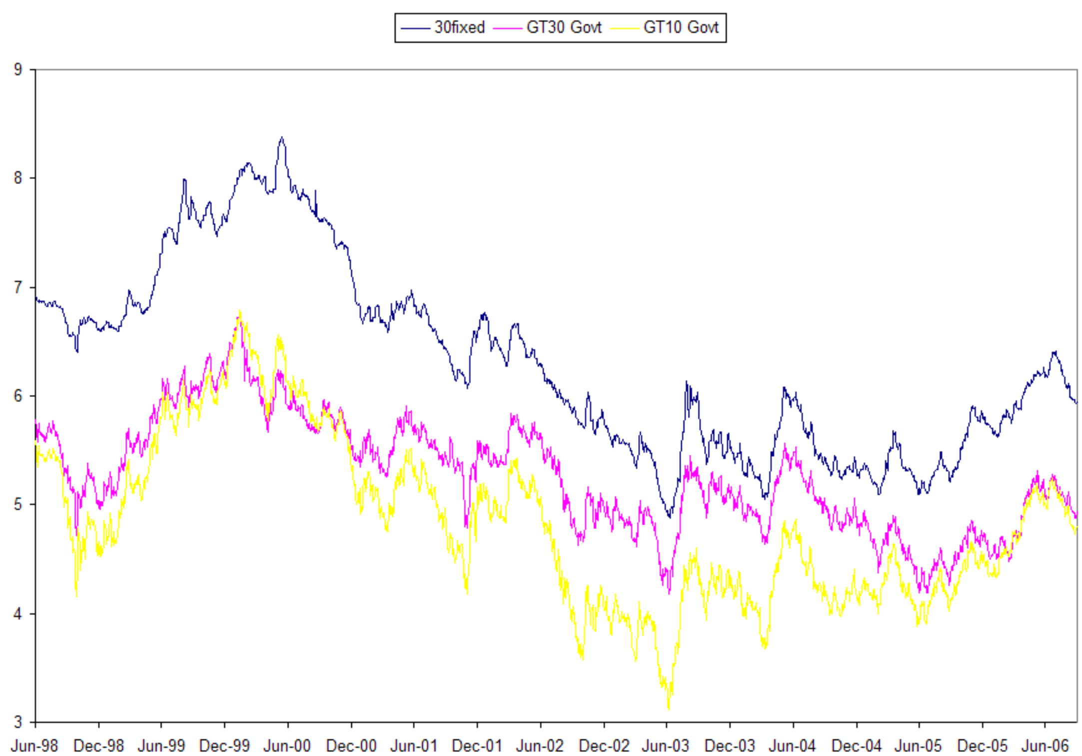
y	30fixed
x4	4wk
x5	3month
x6	6month
x7	TBY1
x8	1year
x9	7 year
x10	GT2 Govt
x11	GT3 Govt
x12	GT5 Govt
x13	GT10 Govt
x14	GT30 Govt

Table 3: Regression Models with estimates and Adjusted R^2

Appendix C

Graphs

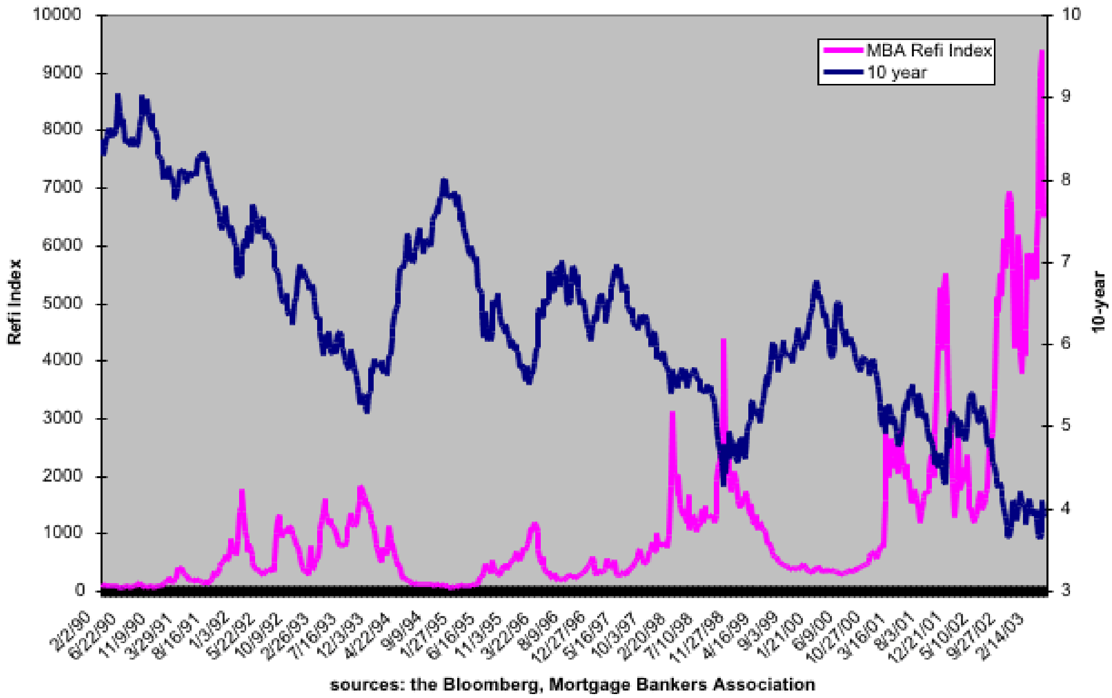
C.1 Time Series plot: 10 and 30 T-Bonds and 30 FRM



Graph: Time-Series Plot: 10 and 30 T-Bonds and 30 FRM (1998-2006)[1]

[1]Data provided by [6]

C.2 Refinance Rates vs. Rates



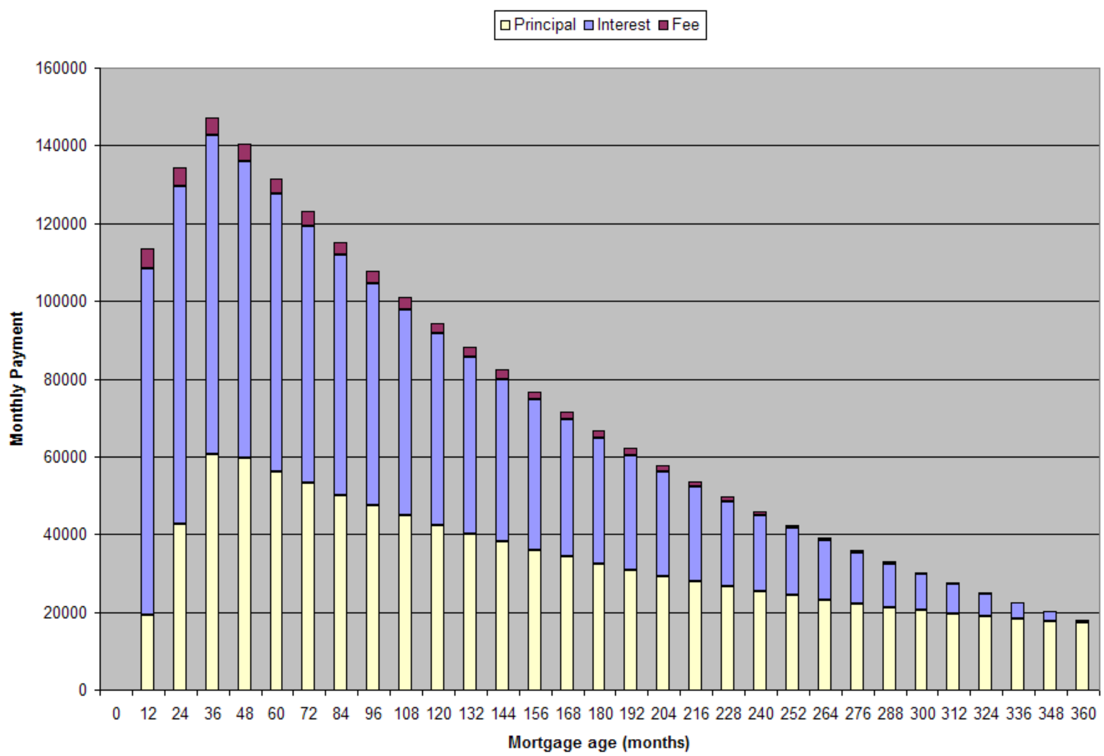
Graph: 10 and MBA Index

The graph above[2] The MBA, a national association representing the real estate finance industry, is responsible for deriving the refinancing index.

As Chart 1 below illustrates, refinancing activity, as measured by the Mortgage Bankers Association Refinancing Index, tends to rise as rates fall and slow down as rates rise.

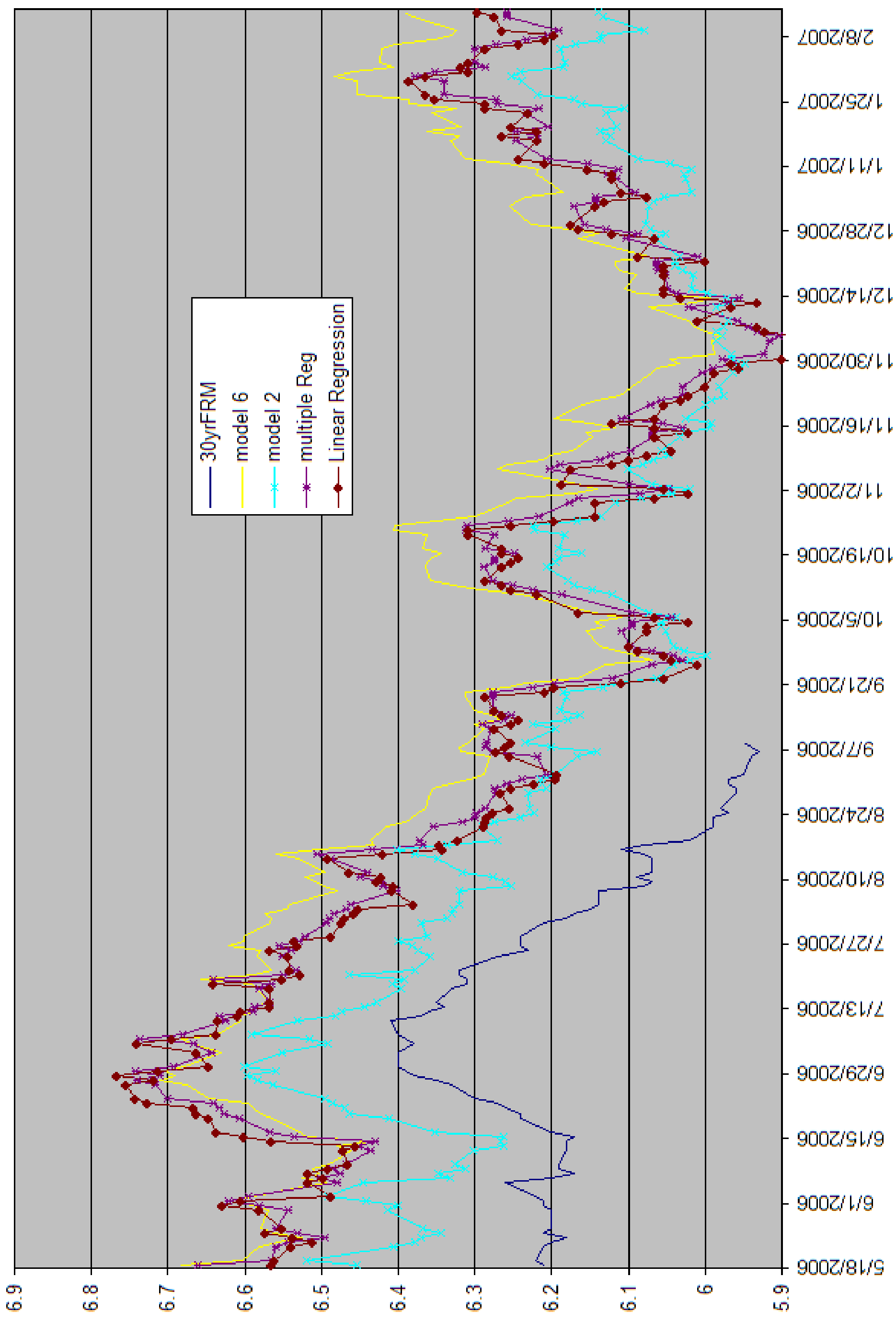
[2] [7] provided by website, displays refinancing activity.

C.3 Prepayment Graph: 100% PSA



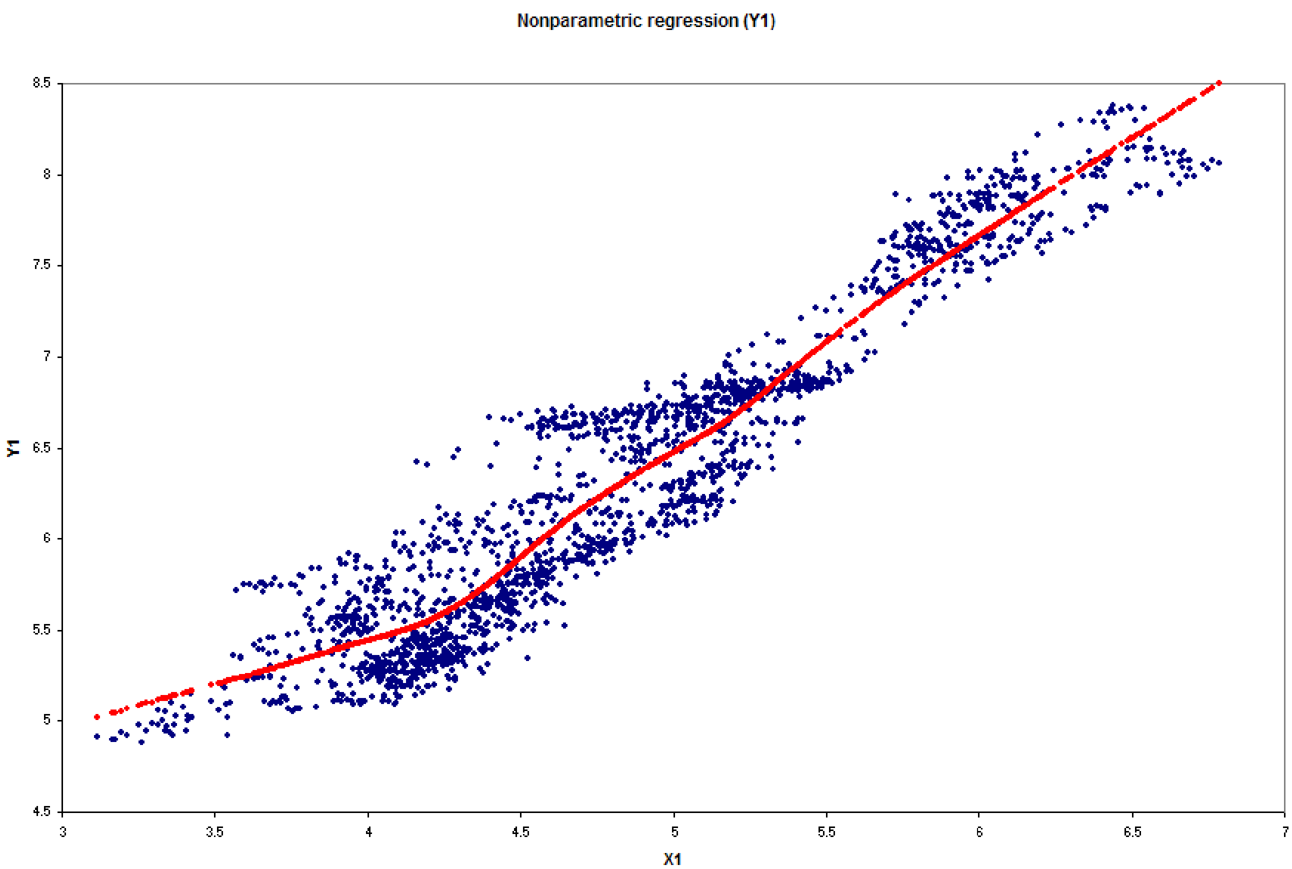
Graph: Illustration of Table 2: Cash Flow Example.

C.4 Regression Models: Prediction



Graph: Analysis of Regression Models

C.5 Nonparametric Regression Model



Graph: Fitted Nonparametric Regression Model

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[6] (www.federalreserve.com)

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