

2a. Formulate and motivate a forward Euler method for approximation of the Stratonovich SDE

$$dX_t = a(t, X_t)dt + b(t, X_t) \cdot dW.$$

2b. Consider the differential equation

$$dX = aX_t dt,$$

where $X_t \in \mathbb{R}^2$ and the matrix a has two real eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -10^5$. Then the backward Euler method

$$X(t_{n+1}) - X(t_n) = aX(t_{n+1})(t_{n+1} - t_n)$$

is an efficient method to solve the problem (why?). Formulate and motivate a backward Euler method for approximation of the Ito SDE

$$dX_t = aX_t dt + bX_t dW,$$

where a and b are constant.