

3a. Assume that S_t is the price of a single stock. Derive a Monte Carlo and a PDE method to determine the price of a contingent claim with the contract $\int_0^T h(t, S_t)dt$, for a given function h , instead of the usual contract $\max(S_T - K, 0)$ for European call options.

3b. Derive the Black and Scholes equation for a general system of stocks $S(t) \in R^d$ solving

$$dS_i = a_i(t, S(t))dt + \sum_{j=1}^d b_{ij}(t, S(t))dW_j(t).$$

and a rainbow option with the contract $f(T, S(T)) = g(S(T))$, for a given function $g : R^d \rightarrow R$, e.g. $g(s) = \max(d^{-1} \sum_{i=1}^d s_i - K, 0)$ where K is a constant. Hint: generalize the derivation we did in class to a portfolio with the option and all stocks.