The Monte Carlo method and options models

3a. In the risk neutral formulation a stock solves the SDE

$$dS/S(t) = rdt + \sigma dW(t),$$

with constant interest rate r and volatility σ . Show that

$$S(T) = S(0)exp(rT - \sigma^2/2T + \sigma W(T)),$$

and use that to simulate the price

$$f(0, S(0)) = e^{-rT} E[max(S(T) - K, 0)],$$

of an European call option by a Monte Carlo method, where $K = 35, r = 0.04, \sigma = 0.2, T = 1$. Compute also the corresponding delta

$$\Delta = \frac{\partial f(s,0)}{\partial s},$$

by approximating with a difference quotient and determine a good choise of your " ∂s ". Estimate the accuracy of your results. Suggest a better method to solve this problem.

3b. Assume that a system of stocks solve

$$dS_i/S_i(t) = rdt + \sum_{j=1}^d \sigma_{ij} dW_j(t), \quad i = 1, \dots, d,$$

where W_j are independent Brownain motions. Show that

$$S_i(T) = S(0)exp(rT + \sum_{j=1}^d (\sigma_{ij}W_j(T) - \sigma_{ij}^2T/2)).$$

Let

$$S_{av} \equiv \sum_{i=1}^{d} S_i/d,$$

and simulate the price of the option above with S(T) replaced by $S_{av}(T)$, where $d = 10, S_i(0) = 40, i = 1, ... 10$ and some ad hoc non diagonal choice of the volatility matrix σ . Estimate the accuracy of your results. Can you find a better method to solve this problem?