

Homework Set 1, due Thursday Sept 7.

Last revised, Aug 26, 2006.

Exercise 1 *Prove the following identities by taking limits of the Forward Euler method:*

$$\int_0^T t dW(t) = T W(T) - \int_0^T W(t) dt$$

(integration by parts)

$$\int_0^T W(t) dW(t) = \frac{W(T)^2}{2} - \frac{T}{2}$$

Exercise 2 *The Ornstein-Uhlenbeck process can be defined by*

$$X(t) = x_\infty + e^{-at}(x_0 - x_\infty) + b \int_0^t e^{-a(t-s)} dW(s),$$

where $a, b > 0$ are given constants. Compute the expected value and the variance of $X(t)$. Then compute their limits as $t \rightarrow \infty$. Interpret the results.