Homework Set 2, due Thursday Sept 14.

Last revised, Aug 26, 2006.

Exercise 1

a *Consider the ordinary differential equation*

$$dX_t = A\,X_t\,dt$$

where $X_t \in \mathbf{R}^2$ *and the matrix* A *has two real eigenvalues* $\lambda_1 = 1$ *and* $\lambda_2 = -10^5$. *Then the backward Euler method*

$$X(t_{n+1}) - X(t_n) = AX(t_{n+1})(t_{n+1} - t_n)$$

is an efficient method to solve the problem (why?).

b *Formulate and motivate a backward Euler method for approximation of the Ito SDE*

$$dX_t = aX_t dt + bX_t dW_t;$$

where $a < 0$ *and* $b > 0$ *are constants.*

Exercise 2 *Solve the exercise 3.17 from the lecture notes [GMS$^+$06]*

References

[GMS$^+$06] J. Goodman, K.S. Moon, A. Szepessy, R. Tempone, and Z. Zouraris. Stochastic and Partial Differential Equations with Adapted Numerics. *Lecture Notes*, 2006.