Homework Set 4, due Thursday Sept 28.

Last revised, Aug 30, 2006.

Exercise 1 *Assume that $S(t)$ is the price of a single stock. Derive a Monte Carlo and a PDE method to determine the price of a contingent claim with with the (path dependent) payoff*

$$\int_0^T h(t, S(t))dt,$$

for a given function h.

Exercise 2 *Derive the Black-Scholes equation for a general system of stocks $S(t) \in \mathbf{R}^d$ of the form*

$$dS_i(t) = a_i(t, S(t))dt + \sum_{j=1}^{d} b_{ij}(t, S(t))dW_j(t), \ i = 1, \dots, d$$

and the European option with final payoff $f(T, S(T)) = g(S(T))$. Here $g : \mathbf{R}^d \to \mathbf{R}$ is a given function e.g. $g(s) = \max\left(\frac{\sum_{i=1}^d s_i}{d} - K, 0\right)$.

Hint: *Generalize the classroom derivation considering a self financing portfolio with all stocks and the option.*

Exercise 3 **a** *Solve Exercise 4.10 from [GMS+06].*

b Optional for extra credit. *Consider the SDE*

$$dX(t) = -aX(t)dt + b(X(t))dW(t),$$

where $a > 0$. Is it possible to determine the function b such that X has a prescribed limit distribution with density $p(x)$ as $t \to \infty$? If the answer is yes, carry out explicit computations for the uniform case

$$p(x) = \begin{cases} 1, \ \textit{if} \ -0.5 \le x \le 0.5, \\ 0, \ \textit{otherwise.} \end{cases}$$

References

[GMS+06] J. Goodman, K.S. Moon, A. Szepessy, R. Tempone, and Z. Zouraris. Stochastic and Partial Differential Equations with Adapted Numerics. *Lecture Notes*, 2006.