Recall: Definition 1.3: European floating strike lookback put option

$$Payoff = \max(M - S_T, 0) \tag{1}$$

where S_T is the final stock price and $M = \max_{0\leq\tau\leq T} S_\tau$.
Let $P = P(S, M, t)$ stand for the option price, which satisfies

$$\begin{cases} \dfrac{\partial P}{\partial t} + \dfrac{\sigma^2}{2} S^2 \dfrac{\partial^2 P}{\partial S^2} + (r_d - r_f) S \dfrac{\partial P}{\partial S} - r_d P = 0, \quad 0 \leq S \leq M \quad \text{and } 0 \leq t < T, \\ P(S, M, T) = M - S \\ \dfrac{\partial P}{\partial M}(M, M, t) = 0 \end{cases} \tag{2}$$

where $r_d > 0$ and $r_f > 0$ represent the risk-free domestic rate and foreign rate, respectively.
With the transformation:

$$x = \ln \frac{M}{S} \qquad V(x,t) = \frac{P(S,M,t)}{S}, \tag{3}$$

Then (2), can be reduced to

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{\sigma^2}{2} \dfrac{\partial^2 V}{\partial x^2} + \left(r_d - r_f - \dfrac{\sigma^2}{2} \right) \dfrac{\partial V}{\partial x} - r_f V = 0, \quad x > 0 \quad \text{and } 0 \leq t < T, \\ V(x, T) = e^x - 1 \\ \dfrac{\partial V}{\partial x}(0, t) = 0 \end{cases} \tag{4}$$

$$u = e^{\sigma\sqrt{\Delta t}} \quad d = \frac{1}{u} \quad K_0 = \frac{M}{S} = 1 \tag{5}$$

$$u^j = \frac{M_n}{S_n} \quad V_j^N = u^j - 1 \tag{6}$$

$$p = \frac{\rho e^{-r_f \Delta t} - d}{u - d} \tag{7}$$

$$K_2 = 1 = u^0 \quad K_2 = u^1 \quad K_2 = \frac{S}{d^2 S} = u^2 \quad u^3 \quad u^4 \tag{8}$$

p $1-p$

$$p = \frac{\rho e^{-r_f \Delta t} - d}{u - d}$$

$\rho = e^{r_d \Delta t}$ $V_0^n = \frac{1}{\rho}\left(puV_0^{n+1} + (1-p)dV_1^{n+1}\right)$ $for\ j = 0$ $V_j^n = \frac{1}{\rho}\left(puV_{j-1}^{n+1} + (1-p)dV_{j+1}^{n+1}\right)$ for $j > 0$

$$\rho = e^{r_d \Delta t} \tag{9}$$

$$V_{-1}^{i+1} = V_1^{i+1} \tag{10}$$

$$V_{j-1}^{i+1} = V_j^{i+1} = V_{j+1}^{i+1} \tag{11}$$

extrapolation T t x

$$V = e^x - 1 \tag{12}$$

Payoff:

$$V_j^N = u^j - 1 \tag{13}$$