Time–varying Beta Risk of Pan–European Industry Portfolios: A Comparison of Alternative Modeling Techniques

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Abstract

This paper investigates the time–varying behavior of systematic risk for eighteen pan–European sectors. Using weekly data over the period 1987−2005, four different modeling techniques in addition to the standard constant coefficient model are employed: a bivariate t–GARCH(1,1) model, two Kalman filter based approaches, a bivariate stochastic volatility model estimated via the efficient Monte Carlo likelihood technique as well as two Markov switching models. A comparison of the different models' ex–ante forecast performances indicates that the random walk process in connection with the Kalman filter is the preferred model to describe and forecast the time–varying behavior of sector betas in a European context.

Keywords: time–varying beta risk; Kalman filter; bivariate t–GARCH; stochastic volatility; efficient Monte Carlo likelihood; Markov switching; European industry portfolios.

JEL Codes: C22; C32; G10; G12; G15.

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1 Introduction

Beta represents one of the most widely used concepts in finance. It is used by financial economists and practitioners to estimate a stock's sensitivity to the overall market, to identify mispricings of a stock, to calculate the cost of capital and to evaluate the performance of asset managers. In the context of the capital asset pricing model (CAPM) beta is assumed to be constant over time and is estimated via ordinary least squares (OLS). However, inspired by theoretical arguments that the systematic risk of an asset depends on microeconomic as well as macroeconomic factors, various studies over the last three decades, e.g. Fabozzi and Francis (1978), Sunder (1980), Bos and Newbold (1984) and Collins et al. (1987), have rejected the assumption of beta stability.

While many papers have concentrated on testing the constancy of beta, only minor efforts have been made to explicitly model the stochastic behavior of beta. In this study, different techniques will be approached to model and to analyze the time–varying behavior of systematic risk. As from a practical perspective betas prove to be especially useful in the context of sectors, the focus will be on betas at the industry rather than at the stock level.[1] The increasing importance of the sector perspective in Europe, induced by the advancement of European integration and the introduction of a single currency, is reflected in the widespread sectoral organization of most institutional investors as well as in the creation of sector specific financial products such as sector exchange tradable funds, sector futures and sector swaps in recent years. In spite of the empirical evidence generated that systematic risk on the industry level in Australia, India, New Zealand, the UK and the US is time–variant, similar work in a pan–European context is still missing. This paper aims to close this gap by empirically analyzing the stochastic behavior of beta for eighteen European sector portfolios.

The first technique for estimating time–varying betas is based upon the multivariate generalized autoregressive conditional heteroskedasticity (M–GARCH) model, first proposed by Bollerslev (1990), which belongs to the class of GARCH models, introduced by Engle (1982) and Bollerslev (1986). The conditional variance estimates as produced by a GARCH(1,1) model are utilized to generate the series of conditional time–varying betas. This approach has been applied in various studies to model time–varying betas. For example, Giannopoulos (1995) uses weekly local stock market data over the period from 1984 until 1993 to estimate time–varying country betas. Brooks et al. (1998) estimate conditional time–dependent betas for Australian industry portfolios using monthly data covering the period from January 1974 to March 1996. Li (2003) studies the time–varying beta risk for New Zealand sector portfolios by analyzing daily data from January 3, 1997 to August 28, 2002. Although the popular GARCH(1,1) model is able to describe the volatility clustering in financial time series as well as other prominent stylized facts of returns, such as excess kurtosis, the standard GARCH model does not capture other important properties of volatility, e.g. asymmetric effects on conditional volatility of positive and negative shocks.[2] Therefore, nonlinear extensions of the basic GARCH model have been proposed and adopted to the modeling of time–varying betas. For example, Braun et al.

[1] See Yao and Gao (2004) for details.

[2] A review of GARCH and related models and their empirical applications in finance can be found in Bollerslev et al. (1992), Pagan (1996) and Franses and van Dijk (2000, Chap. 4).

(1995) employ an exponential GARCH (EGARCH) model to test for predictive asymmetry in beta and Faff et al. (2000) estimate time varying systematic risk of UK industry indices by an EGARCH and a threshold ARCH (TARCH) specification.

Although GARCH can be considered as being practitioners' preferred tool to model and forecast volatility, the class of Stochastic Volatility (SV) models represents an attractive alternative. By adding an additional contemporaneous shock to the return variance, SV models are more flexible in characterizing volatility dynamics than GARCH models. Koopman et al. (2004) and Yu (2002), for example, compare the ability of SV models to that of alternative ARCH–type models to predict stock price volatility and conclude that the SV model outperforms its competitors. However, despite its theoretical appeal and its empirical superiority over GARCH models, the SV model is rarely used in practice for volatility forecasting or to model time–varying betas. This can be mainly attributed to the difficulties related to parameter estimation which is substantially more difficult for SV models. Nevertheless, the results presented by Li (2003) who estimates and compares non–constant betas for New Zealand industry portfolios based on different techniques, including GARCH and SV, encourage further research in the applicability of SV models to estimate time–varying betas.

An alternative way of modeling the time–varying behavior of beta is based on the state space form of the CAPM. In contrast to volatility–based models where time–varying betas are calculated indirectly by utilizing estimated conditional variance series, the state space approach allows to model and estimate time–varying betas directly by using the Kalman filter (KF). Different models for the dynamic process of conditional betas have been proposed. For US data Fabozzi and Francis (1978) and Collins et al. (1987) modeled beta as a random coefficient. The RC model has also been applied by Wells (1994) for Swedish stocks and by Faff et al. (2000) for UK industry indices. Two of the most prominent alternatives to the model time–varying betas are the random walk (RW) model, recently employed by Lie et al. (2000) for Australian financial stocks and by Li (2003) for New Zealand industry portfolios, and the mean–reverting (MR) model which has been used by Bos and Newbold (1984) for US data, by Brooks et al. (1998) and by Groenewold and Fraser (1999) for Australian sectors. For their investigation of the systematic risk of Australian industrial stock returns Yao and Gao (2004) also considered an autoregressive moving average model (ARMA) as well as an MR model in which the mean beta is allowed to vary over time as proposed by Wells (1994).

The last approach that will be considered in this study uses a Markov switching framework which belongs to the large class of Markov switching models introduced in the seminal works of Hamilton (1989, 1990). Although Markov switching regression models have been applied in many different settings, the literature dealing with time–varying betas is relatively thin. Fridman (1994) considered monthly data from 1980 to 1991 to analyze the excess returns of three oil corporation securities by fitting a two–state regression model resulting in an improved assessment of systematic risk associated with each security. Besides, he noted two effects: beta increases whenever the process is in the more volatile state and the state associated with higher volatility tends to be less persistent than the state associated with lower volatility. Huang (2000) also considered a Markov switching model with one high–risk and one low–risk state.

Using monthly return data from April 1986 to December 1993, he performed several test to check the consistency of different states with the CAPM and rejected the hypothesis that the data were from the same state.

The main purpose of the present paper is to apply various modeling techniques to describe the time–varying behavior of European sector betas and to compare their respective ability to explain sector returns by movements of the overall market. This paper aims at contributing a comparison of modeling techniques to the literature that also incorporates non–standard procedures such as stochastic volatility and Markov switching.

The remainder of this study is organized as follows. Section 2 outlines the competing modeling techniques. Section 3 describes the data and reports standard summary statistics. The empirical results are discussed in section 4 and section 5 concludes.

2 Methodology

2.1 The Unconditional Beta in the CAPM

As a starting point, market risk is treated as being constant. The benchmark for time–varying betas is the excess-return market model with constant coefficients where an asset's unconditional beta can be estimated via OLS:

$$R_{it} = \alpha_i + \beta_i R_{0t} + \epsilon_{it}, \quad \epsilon_{it} \sim (0, \sigma_i^2), \tag{1}$$

with

$$\hat{\beta}_i = \frac{Cov(R_0, R_i)}{Var(R_0)}, \tag{2}$$

where R_{0t} denotes the excess return of the market portfolio and R_{it} denotes the excess return to sector i for $i = 1, \ldots, I$, each for period $t = 1, \ldots, T$. The error terms ϵ_{it} are assumed to have zero mean, constant variance σ_i^2 and to be independently and identically distributed (IID). Following the Sharpe (1964) and Lintner (1965) version of the CAPM, where investors can borrow and lend at a risk-free rate, all returns are in excess over a risk-free interest rate and α_i is expected to be zero.[3] Table 3 summarizes the OLS estimates of the excess market model. As expected the intercept is not different from zero at the 5% level for any sector. If not mentioned otherwise, α_i is assumed to be zero for the rest of this paper.

2.2 GARCH Conditional Betas

While in the traditional CAPM returns are assumed to be IID, it is well established in the empirical finance literature that this is not the case for returns in many financial markets. Signs of autocorrelation and regularly observed volatility clusters where quiet periods with small absolute returns are followed by volatile periods with large absolute returns contradict the assumption of independence and an identical return distribution over time. In this case the variance–covariance matrix of sector and market returns is time–dependent and

[3] See Campbell et al. (1997, Ch. 5) for a review of the CAPM.

a non–constant beta can be defined as

$$\hat{\beta}_{it}^{GARCH} = \frac{Cov(R_{it}, R_{0t})}{Var(R_{0t})}, \tag{3}$$

where the conditional beta is based on the calculation of the time–varying conditional covariance between a sector and the overall market and the time–varying conditional market variance.

A bivariate version of the M–GARCH model as introduced by Bollerslev (1990) is used to compute time–varying betas. Let $\boldsymbol{R}_t$ be a (2×1) time–series vector (R_{it}, R_{0t}) where R_{it} represents the return series of sector i and R_{0t} denotes the return series of the broad market. Consider a system of $n = 2$ conditional mean equations:

$$\boldsymbol{R}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \tag{4}$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2)'$ is a (2×1) vector of constants and $\boldsymbol{\epsilon}_t = (\epsilon_{1t}, \epsilon_{2t})'$ denotes a (2×1) time series vector of residuals, conditioned by the complete information set $\boldsymbol{\Omega}_{t-1}$. A general bivariate GARCH model for the two–dimensional process $\boldsymbol{\epsilon}_t|\boldsymbol{\Omega}_{t-1}$ is given by

$$\boldsymbol{\epsilon}_t = \boldsymbol{z}_t \boldsymbol{H}_t^{1/2}, \tag{5}$$

where $\boldsymbol{z}_t$ is a two–dimensional IID process with zero mean and the identity matrix $\boldsymbol{I}_2$ as covariance matrix. These properties of $\boldsymbol{z}_t$ together with equation (5) imply that $E[\boldsymbol{\epsilon}_t|\Omega_{t-1}] = \boldsymbol{0}$ and $E[\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t'|\Omega_{t-1}] = \boldsymbol{H}_t$ with

$$\boldsymbol{H}_t = \begin{bmatrix} h_{11t} & h_{12t} \\ h_{21t} & h_{22t} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{H}_t$ should depend on lagged errors $\boldsymbol{\epsilon}_{t-1}$ and on lagged conditional covariance matrices $\boldsymbol{H}_{t-1}$. The most influential parameterizations of $\boldsymbol{H}_t$ can be summarized as follows.

The *vech model* represents a general form of the bivariate GARCH(1,1) model. Engle and Kroner (1995) employed the vech$(\cdot)$ operator[4] to stack all the non–redundant elements of $\boldsymbol{H}_t$ into a column vector:

$$\text{vech}(\boldsymbol{H}_t) = \boldsymbol{\Psi^*} + \boldsymbol{\Gamma^*}\text{vech}(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}') + \boldsymbol{\Delta^*}\text{vech}(\boldsymbol{H}_{t-1}), \tag{7}$$

with $\boldsymbol{\Psi^*}$ being a $(n(n+1)/2 \times 1)$ vector and $\boldsymbol{\Gamma^*}$ and $\boldsymbol{\Delta^*}$ being $(n(n+1)/2 \times n(n+1)/2)$ matrices. In the bivariate case equation (7) becomes

$$\begin{aligned} \begin{bmatrix} h_{11t} \\ h_{21t} \\ h_{22t} \end{bmatrix} &= \begin{bmatrix} \psi_{11}^* \\ \psi_{21}^* \\ \psi_{22}^* \end{bmatrix} + \begin{bmatrix} \gamma_{11}^* & \gamma_{12}^* & \gamma_{13}^* \\ \gamma_{21}^* & \gamma_{22}^* & \gamma_{23}^* \\ \gamma_{31}^* & \gamma_{32}^* & \gamma_{33}^* \end{bmatrix} \times \begin{bmatrix} \epsilon_{1,t-1}^2 \\ \epsilon_{2,t-1}\epsilon_{1,t-1} \\ \epsilon_{2,t-1}^2 \end{bmatrix} \\ &+ \begin{bmatrix} \delta_{11}^* & \delta_{12}^* & \delta_{13}^* \\ \delta_{21}^* & \delta_{22}^* & \delta_{23}^* \\ \delta_{31}^* & \delta_{32}^* & \delta_{33}^* \end{bmatrix} \times \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}. \end{aligned} \tag{8}$$

Though this general representation of a bivariate t–GARCH(1,1) model is very flexible, it has two major drawbacks. The total number of to be estimated

[4]The vech$(\cdot)$ operator vertically stacks the matrix elements on or below the principal diagonal and thus transforms an $(n \times n)$ matrix into an $((n(n+1)/2) \times 1)$ vector, see Hamilton (1994, pp. 300–301).

parameters equals $n(n+1)/2 + n^2(n+1)^2/2$ and grows at a polynomial rate.[5] Besides, it is not easy to find restrictions for $\mathbf{\Gamma^*}$ and $\mathbf{\Delta^*}$ which guarantee positive definiteness of $\boldsymbol{H}_t$.

The *diagonal vech model* is a first way to restrict equation (7) and to reduce the number of parameters. Bollerslev et al. (1988) restrict the matrices $\mathbf{\Gamma^*}$ and $\mathbf{\Delta^*}$ to be diagonal such that the conditional covariance between ϵ_{1t} and ϵ_{2t} depends only on lagged cross–products of the residuals and lagged values of $\boldsymbol{H}_t$. In this specification each element of the conditional covariance matrix follows a univariate GARCH(1,1) model:

$$h_{jkt} = \psi_{jk} + \gamma_{jk}\epsilon_{j,t-1}\epsilon_{k,t-1} + \delta_{jk}h_{jk,t-1}, \tag{9}$$

where ψ_{jk}, γ_{jk} and δ_{jk} each denote the jkth element of the $(n \times n)$ matrices $\mathbf{\Psi}$, $\mathbf{\Gamma}$ and $\mathbf{\Delta}$, respectively. These matrices are implicitly given by $\mathbf{\Psi^*} = \text{vech}(\mathbf{\Psi})$, $\mathbf{\Gamma^*} = \text{diag}(\text{vech}(\mathbf{\Gamma}))$ and $\mathbf{\Delta^*} = \text{diag}(\text{vech}(\mathbf{\Delta}))$. As each element of $\boldsymbol{H}_t$ has three parameters, only nine parameters remain to be estimated. Positive definiteness of $\boldsymbol{H}_t$ is guaranteed if $\mathbf{\Psi}$ is positive definite and $\mathbf{\Gamma}$ and $\mathbf{\Delta}$ are positive semi–definite.[6]

The *BEKK model* of Engle and Kroner (1995) is another way to restrict the number of parameters in equation (7). Instead of restricting $\mathbf{\Gamma}$ and $\mathbf{\Delta}$, quadratic forms of these matrices are used in order to guarantee positive definiteness of $\boldsymbol{H}_t$. The model is given by

$$\boldsymbol{H}_t = \mathbf{\Psi} + \mathbf{\Gamma}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}'\mathbf{\Gamma} + \mathbf{\Delta}'\boldsymbol{H}_{t-1}\mathbf{\Delta}, \tag{10}$$

with $\mathbf{\Psi}$ being symmetric and positive definite. The number of parameters equals $(5n^2+n)/2$ so that for the bivariate case two more unknowns than in the diagonal vech setting have to be estimated.

In this study, the *constant correlation model* by Bollerslev (1990) is employed to reduce the computational complexity of the general multivariate GARCH(1,1) model. By introducing the assumption of constant conditional correlations between ϵ_{it} and ϵ_{jt} the diagonal vech model with constant correlations is defined as

$$h_{jjt} = \psi_{jj} + \gamma_{jj}\epsilon^2_{j,t-1} + \delta_{jj}h_{jj,t-1}, \tag{11}$$

$$h_{jkt} = \rho_{jk}\sqrt{h_{jjt}}\sqrt{h_{kkt}}, \qquad \text{for all } j \neq k, \tag{12}$$

where equation (11) and (12) denote the conditional variance of the returns of index j and the conditional covariance between the returns of indices j and k, respectively, with ρ_{jk} being the constant correlation coefficient between R_{it} and R_{jt}. In the bivariate case we are left with only seven ($= n(n+5)/2$) parameters.[7] The univariate GARCH(1,1) models in equation (11) are covariance–stationary if and only if $\gamma_{jj} + \delta_{jj} < 1$. The conditional covariance matrix is guaranteed to be positive definite for positive values of the constant correlation ρ_{jk} and the parameters ψ_{jj}, γ_{jj} and δ_{jj}.

It can be seen from the descriptive statistics in Table 2 that the sector returns in the selected sample are highly leptokurtic. To account for the widely

[5] See Pagan (1996).

[6] See Franses and van Dijk (2000, Ch. 4).

[7] The computations are carried out using the object–oriented matrix programming language `Ox 3.30` of Doornik (2001) together with the package `G@RCH 2.3` by Laurent and Peters (2002).

recognized fact that the unconditional distributions of high–frequency financial return data have fat tails, a standardized Student's t–distribution as proposed by Bollerslev (1987) is considered for the innovations $\boldsymbol{z}_t$ in equation (5). The results in Table 4 suggest that the chosen t–GARCH setup, where the 't' refers to the Student's t–distribution, with non–zero constants in the mean equation is superior to the alternatives of zero constants or normally distributed innovations.

Recalling equation (2), the conditional time–varying beta of sector i can now be estimated as the ratio of the conditional covariance between sector i and the broad market and the conditional market variance:

$$\hat{\beta}_{it}^{tGARCH} = \frac{Cov(R_{0t}, R_{it})}{Var(R_{0t})} = \frac{h_{0it}}{h_{00t}} = \frac{\rho_{0i}\sqrt{h_{iit}}}{\sqrt{h_{00t}}}. \tag{13}$$

2.3 Stochastic Volatility Conditional Betas

While in the GARCH framework with only one error term, the conditional mean and the conditional volatility of the return series are characterized by the same shocks, an alternative way of modeling time–varying volatility was introduced by Taylor (1986) who included an additional contemporaneous shock to the return variance. These models, referred to as SV models, offer a higher degree of flexibility and imply excess kurtosis which qualifies them to be more appropriate in describing financial time series.[8] Therefore, SV models should be an alternative to GARCH–type approaches in the econometric modeling of time–varying betas.

SV models are usually represented by their first two moments. The mean equation is given by

$$R_{it} = \mu_{it} + \sigma_{it}\epsilon_{it}, \quad \epsilon_{it} \sim NID(0, 1), \quad t = 1, \ldots, T, \tag{14}$$

where R_{it} is the return series of index i and μ_{it} denotes the expectation of R_{it}. Following Hol and Koopman (2002) the mean is usually either modeled before estimating the process of volatility or taken to be zero for SV models, implying $\mu_{it} = 0$.[9] The disturbances are assumed to be identically and independently normally distributed with zero mean and unit variance. The variance equation is given by

$$\sigma_{it}^2 = \sigma_i^{*2} exp(v_{it}), \tag{15}$$

where the actual volatility σ_{it}^2 is the product of a positive scaling factor σ^{*2} and the exponential of the stochastic process v_{it} which is modeled as a first–order autoregressive process:

$$v_{it} = \phi_i v_{i,t-1} + \sigma_{\eta i}\eta_{it}, \quad \eta_{it} \sim NID(0, 1), \quad v_{i1} \sim NID\left(0, \frac{\sigma_{\eta i}^2}{1-\phi_i^2}\right), \tag{16}$$

with the persistence parameter ϕ_i being restricted to be positive and smaller than one to ensure stationarity of v_{it}. The disturbances ϵ_{it} and η_{it} are assumed

[8]For further discussion and a general introduction to SV models, see e.g. Ghysels et al. (1996) or Shephard (1996).

[9]Alternatively, some authors use mean–corrected returns, R_{it}^*, defined as $R_{it}^* = \ln(P_{it}) - \ln(P_{i,t-1}) - (1/T)\sum_{i=0}^{T}(\ln(P_{it}) - \ln(P_{i,t-1}))$, see e.g. Kim et al. (1998).

to be uncorrelated, contemporaneously and at all lags. Franses and van Dijk (2000, Ch. 4) offer a useful interpretation of the two different shocks where ϵ_{it} represents the contents of new information (good or bad news) and η_{it} reflects the shocks to the intensity of the flow of news.

Due to the inclusion of an unobservable shock to the return variance, the variance becomes a latent process which cannot be characterized explicitly with respect to observable past information. As a consequence, the parameters of the SV model cannot be estimated by a direct application of standard maximum likelihood techniques. Several procedures for estimating SV models have been proposed, ranging from various method of moments estimators as proposed by Taylor (1986) or Melino and Turnbull (1990), quasi–maximum likelihood as proposed by Harvey et al. (1994), a Bayesian approach employing a Monte Carlo Markov Chain (MCMC) technique as presented by Jacquier et al. (1994), the Monte Carlo likelihood (MCL) estimator as proposed by Danielsson (1994) to the efficient MCL developed by Sandmann and Koopman (1998). The fact that there is still no consensus on how to estimate SV models explains why this class of volatility models has been rarely used in practice so far.

In this study, SV models are estimated via the efficient MCL technique whose finite sample performance compares well to that of MCMC while being less computationally intense.[10] Once the smoothed conditional variance series of market and sector returns, σ^2_{0t} and σ^2_{it}, have been obtained, equation (12) is recalled to construct the time–varying sector betas as

$$\hat{\beta}_{it}^{SV} = \frac{\rho_{0i}\sigma_{it}}{\sigma_{0t}}. \tag{17}$$

2.4 Kalman Filter Based Approaches

In contrast to the volatility–based techniques where the conditional beta series could only be constructed after the conditional variances of the market and sector i have been obtained first, the state space approach allows to model and to estimate the time–varying structure of beta directly. Based on the assumption of normality, state space models are estimated numerically through a recursive algorithm known as the Kalman filter.[11]

In state space form, the excess-return market model in equation (1) with α_{it} treated as zero is modified to become an *observation equation*:

$$R_{it} = \beta_{it} R_{0t} + \epsilon_{it}, \tag{18}$$

where the dynamic process of the unobserved time–varying *state vector*, β_{it}, is defined by the *state equation*:

$$\beta_{it} = \phi_i \beta_{i,t-1} + \eta_{it}, \tag{19}$$

with ϕ_i denoting the constant *transition parameter*. The *observation error* ϵ_{it}

[10] The SV models are estimated using `Ox 3.30` by Doornik (2001) together with the package `SsfPack` by Koopman et al. (1999). The relevant `Ox` code for estimating the SV models has been downloaded from `www.feweb.vu.nl/koopman/sv/`.

[11] For introductory surveys on the KF and its application, see e.g. Meinhold and Singpurwella (1983), Harvey (1989) or Hamilton (1994, Ch. 13).

and the *state equation error* η_{it} are assumed to be Gaussian:

$$E(\epsilon_{it}\epsilon'_{i\tau}) = \begin{cases} \sigma_i^2 & \text{for } t = \tau \\ 0 & \text{otherwise,} \end{cases} \tag{20}$$

$$E(\eta_{it}\eta'_{i\tau}) = \begin{cases} \sigma_{\eta i}^2 & \text{for } t = \tau \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

and to be uncorrelated at all lags:

$$E(\epsilon_{it}\eta'_{i\tau}) = 0 \quad \text{for all } t \text{ and } \tau. \tag{22}$$

The constant variances σ_i^2 and $\sigma_{\eta i}^2$ and the transition parameter ϕ_i are the hyperparameters of the system. A number of alternative specifications for the stochastic process of β_{it} may be derived by formulating different assumptions on ϕ_i.

The *random walk* (RW) model represents the first state space specification of the evolution of the time–varying beta in this paper. By setting the transition parameter ϕ_i to unity, the beta coefficient develops as a random walk:

$$\hat{\beta}_{it}^{KFRW} = \beta_{i,t-1} + \eta_{it}, \tag{23}$$

where the two hyperparameters σ_i^2 and $\sigma_{\eta i}^2$ have to be estimated.[12]

Alternatively, the dynamic process of beta can be modeled as being mean-reverting. In the *mean–reverting* (MR) model an autoregressive process of order one, AR(1), with a constant mean is used for the evolution of beta:

$$\hat{\beta}_{it}^{KFMR} = \bar{\beta}_i^* + \phi_i\beta_{i,t-1} + \eta_{it}, \tag{24}$$

with a constant $\bar{\beta}_i^*$ and the AR(1) parameter $|\phi_i| < 1$. In the empirical literature, equation (24) is often rearranged to allow for a meaningful economic interpretation according to which $\bar{\beta}_i$ can be interpreted as the mean beta over the entire sample and ϕ_i as the "speed parameter" which measures how fast the time–varying beta returns to its mean:[13]

$$\hat{\beta}_{it}^{KFMR} = \bar{\beta}_i + \phi_i\left(\beta_{i,t-1} - \bar{\beta}_i\right) + \eta_{it}, \tag{25}$$

where overall four parameters (σ_i^2, $\sigma_{\eta i}^2$, $\bar{\beta}_i$, ϕ_i) have to be estimated.

2.5 The Markov Switching Approach

The Markov switching approach also belongs to the class of state space models. The implicit assumption of models switching between different regimes is that the data result from a process that undergoes abrupt changes, induced e.g. by political or environmental events.

In the Markov switching framework, the systematic risk of an asset is determined by the different regimes of beta, driven by an unobserved Markov chain. The switching behavior of beta is governed by a *transition probability*

[12] The KF models were computed using `Ox 3.30` by Doornik (2001) together with the package `SsfPack` by Koopman et al. (1999).

[13] See e.g. Bos and Newbold (1984), Faff et al. (2000) or Yao and Gao (2004).

matrix (TPM). Under the assumption of a model with two states, the TPM is of the form

$$\Gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \tag{26}$$

where the entries of each line describe the interaction of the two regimes beta is drawn from: γ_{11} is the probability of staying in the first state from period t to period $t+1$, γ_{12} is the probability of switching from the first to the second state. The second row of the Γ can be interpreted analogously.

In this paper two Markov switching models are employed. The first one is a simple *Markov switching regression* (MS) model. Let $\{s_1, \ldots, s_T\}$ denote the state sequence representing the different regimes; driven by the TPM of a stationary Markov chain, the states take values in $\{1, \ldots, m\}$. Following Huang (2000) the regime–switching CAPM is specified by

$$R_{it} = \alpha_{is_t} + \beta_{is_t} R_{0t} + \eta_{it},\ \eta_{it} \sim N(0, \sigma^2_{is_t}), \tag{27}$$

which means that the regression coefficients $(\alpha_{is_t}, \beta_{is_t})$ are selected according to the value of state s_t. Note that the model is designed to accommodate both the correlations across return series and the serial correlation of the individual series.

The second approach entails additional assumptions on the market returns to synchronize the switching times of beta with different market conditions and will be denoted as *Markov switching market* (MSM) model. Ryden et al. (1998) showed that the temporal and distributional properties of daily return series can be well governed by a hidden Markov model with normal or double–exponential variables. Following their approach, the dynamics of the assets' returns follow the same regime–switching regression of equation (27) with the distribution of the market returns being given by:

$$R_{0t} = \mu_{s_t} + \epsilon_{s_t}, \quad \epsilon_{s_t} \sim N(0, \sigma^2_{0s_t}). \tag{28}$$

This means that in the MSM model the regime of the market changes together with the regime of the regression setup as they depend on the same state sequence. This synchronous behavior offers the advantage of allowing for direct conclusions from the market conditions on the asset's risk represented by beta.

The estimation procedures for our Markov switching models are based on the maximum likelihood method for hidden Markov models.[14] The likelihood L_T of both models is available in an explicit form and hence the parameters of the models can be estimated directly by numerical maximization of the log-likelihood function (MacDonald and Zucchini, 1997, cf.). The estimates for the model parameters include inter alia the state–dependent betas for each asset i and state j denoted by $\hat{\beta}^{MS}_{ij}$ or $\hat{\beta}^{MSM}_{ij}$.

As mentioned above, the state sequence cannot be observed. Therefore, information about the state–distribution at time t has to be derived in order to obtain in–sample estimates as well as out–of–sample forecasts of conditional betas. The desired probabilities of a sojourn in state j at time t can be computed by so–called smoothing, filtering and state prediction algorithms (see

[14] All estimations procedures were carried out using the statistical software package R 2.1.1 (R Development Core Team, 2005) which can be downloaded from `www.r-project.org`. The code for the estimation, decoding and forecasting algorithms are provided upon request.

e.g. Ephraim and Merhav, 2002). Given the state–distribution at time t, estimates for the time–varying betas can be calculated by weighting the state–dependent $\hat{\beta}_{ij}^{MS/MSM}$ with the probability of a sojourn in the corresponding state:

$$\hat{\beta}_{it}^{MS/MSM} = \sum_{j=1}^{m} \Big[\beta_{ij} \cdot P(s_t = j | R_{01}, \ldots, R_{0T}, R_{i1}, \ldots, R_{iT}) \Big], \qquad (29)$$

with

$$P(S_t = j | R_{01}, \ldots, R_{0T}, R_{11}, \ldots, R_{1T}) = \begin{cases} \frac{\alpha_t(j)\beta_t(j)}{L_T} & \text{for } 1 \leq t \leq T \\ \frac{\alpha_t(j)(\Gamma^{t-T})_{\bullet j}}{L_T} & \text{for } T < t, \end{cases} \qquad (30)$$

where $\alpha_t(j)$, $\beta_t(j)$ are the forward-/backward probabilities from the forward–backward algorithm (Rabiner, 1989) and $(\Gamma^{t-T})_{\bullet j}$ denotes the jth column of the matrix Γ^{t-T}.

3 Data and Preliminary Analysis

3.1 Data Series

The data used in this paper are weekly excess returns calculated from the total return indices for eighteen pan–European industry portfolios, covering the period from 2 December 1987 to 2 February 2005. All sector indices are from STOXX Ltd. (2004), a joint venture of Deutsche Boerse AG, Dow Jones & Company and the SWX Group that develops a global free-float weighted index family, the Dow Jones (DJ) STOXX℠ indices. Table 1 presents the first two tiers of the ICB sector structure.

Table 1: The DJ STOXX℠ sector classification

Industries	Supersectors
Basic Materials	Basic Resources, Chemicals
Consumer Goods	Automobiles & Parts, Food & Beverage, Personal & Household Goods
Consumer Services	Media, Retail, Travel & Leisure
Financials	Banks, Financial Services, Insurance
Health Care	Health Care
Industrials	Construction & Materials, Industrial Goods & Services
Oil & Gas	Oil & Gas
Technology	Technology
Telecommunications	Telecommunications
Utilities	Utilities

The DJ STOXX℠ 600 return index, which includes the 600 largest stocks in Europe, serves as a proxy for the overall market. All indices are expressed in euros as common currency.[15] Weekly excess returns between period t and $t-1$ for index i are computed continuously as

$$R_{it} = \ln(P_{it}) - \ln(P_{i,t-1}) - r_t^f, \tag{31}$$

where P_{it} is Wednesday's index closing price in week t, ln is the natural logarithm and r_t^f is the risk–free rate of return, calculated from the 3–month Frankfurt Interbank Offered Rate (FIBOR).[16] All data were obtained from Thomson Financial Datastream.

3.2 Univariate Statistics

Descriptive statistics for the data are provided in Table 2. Over the entire sample, the Healthcare sector offered the highest mean excess return per week (0.17%), while the lowest was seen in Automobiles & Parts (0.02%). The risk as measured by the standard deviation ranges from 0.0203 for the defensive Utilities to 0.0422 for the high risk sector Technology. The market and all its segments are leptokurtic. Except for Healthcare and Travel & Leisure all sectors and the market are negatively skewed. The Jarque–Bera statistic confirmes the departure from normality for all return series at the 1% significance level.

4 Empirical Results

4.1 Unconditional Beta Estimates

The estimated parameters of the OLS model are reported in Table 3. According to the efficient market hypothesis and the implications of the Sharpe–Lintner version of the CAPM, all alphas should be zero. It can be seen from the first column that none of the estimated alphas is different from zero at an acceptable significance level. In comparison, the estimated betas are all significant at a higher than 1% level. Over the entire sample the lowest beta was estimated for Food & Beverages (0.65), while the beta for Technology (1.49) was the highest, confirming the sector's high-risk profile as discussed in subsection 3.2. From the reported coefficients of determination (R^2) it can be seen that depending on the respective sector between 43% (Oil & Gas) and 83% (Industrial Goods & Services) of the total return variation can be explained by movements of the overall market.

The last two columns provide the results of the classical Lagrange multiplier (LM) ARCH test for heteroskedasticity, as proposed by Engle (1982). With the exception of Retail, the null hypothesis of homoskedastic disturbances can be rejected at the 3% level for all sectors for both lag orders tested.

4.2 Modeling Conditional Betas

In the chosen *bivariate GARCH* framework, fitting univariate t–GARCH(1,1) models to the excess returns of each sector and the overall market is the first step

[15] As foreign exchange fluctuations have an impact on the results and currency risk cannot be segregated from market risk when returns are translated into a common currency, caution is needed when interpreting any results.

[16] The FIBOR yields fib_t are percentage per annum. They were converted to weekly rates r_t^f, where $r_t^f = (1 + fib_t/100)^{1/52} - 1$.

in computing time–varying betas. The results are summarized in Table 5. The coefficients for the ARCH and GARCH terms, γ_i and δ_i, are always significantly different from zero. Besides, they are all positive and sum up to less than unity so that positive definiteness and stationarity is guaranteed. For Basic Resources, Technology and Telecommunications the models exhibit the highest level of persistence while the models for Travel & Leisure, Utilities and Retail are the least persistent. With the exception of five sectors, the constant ψ_i is different from zero at the 5% level, while all but two constant terms μ_i in the mean equation are statistically significant at the 1% level. As outlined in subsection 2.2, the correlation coefficient ρ_{oi} between a sector and the overall market is the other factor that is needed to calculate GARCH conditional betas. The correlations, estimated over the entire sample, are reported in the last column of Table 5. All correlations are higher than 0.65, indicating a strong linear association between the market index and each sector.

Stochastic volatility models represent the second technique from the class of volatility models used in this study to model time–varying betas. Table 6 summarizes the estimation results of the considered SV models for European sectors over the full sample period. The asymmetric 95% confidence intervals for the persistence parameter ϕ_i are generally narrow indicating a high level of significance. The degree of volatility persistence ranges from a low for Travel & Leisure to the highest level for Technology and Telecommunications which compares well to the GARCH results with the difference that the degree of persistence is generally closer to unity for the SV models. For the two other parameters, both the asymmetric confidence intervals as well as the range of parameter estimates across sectors, are wider. For the sectors Retail, Travel & Leisure and Utilities the combination of a low persistence parameter and a high value for $\sigma^2_{\eta i}$, which measures the variation of the volatility process, implies that the process of volatility is less predictable for these three sectors. The highest levels of volatility as indicated by a high scaling parameter σ_i^{*2} are found for Automobiles & Parts and the three sectors Telecommunications, Media and Technology (TMT) which broadly corresponds to the calculated standard deviations of weekly returns in subsection 3.2.

The *Kalman filter* has been applied to the two proposed state equations (23) and (25) according to which the state vector β_{it} is either modeled as a random walk or as a mean–reverting process. Even though the mean-reverting model requires the estimation of two additional parameters, the AIC is generally smaller than for the simpler random walk specification (Table 7). While the estimated variance for the observation equation, σ^2_i, is generally higher in case of the RW parameterization, the opposite is true with respect to $\sigma^2_{\eta i}$, the variance of the dynamic process of the time–varying beta. For the MR model two additional parameters have been estimated with $\bar{\beta}_i$ comparing well to the estimated OLS betas. Across all sectors the estimates for the speed parameter, the second extra parameter of the MR model, can be clustered into three groups. In the first group, ϕ_i is close to unity, so that the resulting series of conditional betas become similar to the RW series. In the second group with values for ϕ_i around 0.5 the conditional betas return faster to their individual means which implies more noisy series of conditional betas. In the third group where ϕ_i is close to zero, the resulting beta series follow a random coefficient model.

Depending on the chosen starting values, the Kalman filter is likely to produce large outliers in the first stages of estimation. In order to avoid an unfair bias against the Kalman filter, the first fifty conditional beta estimates for any of the chosen modeling techniques will not be included in the subsequent analyses.

The fit of the *Markov switching* MS and MSM models to the data has been tested with a different number of regimes. According to the AIC, two states turned out to be sufficient and therefore the results summarized in Table 8 always concern two–state models. As expected, all alphas are very close to zero and for almost all sectors the high– and the low–risk states can be well identified. However, the two state–dependent betas are lying quite close together in case of the MS model for the sectors Industrials and Retail and in case of the MSM model for Industrials. Generally, the MSM model is characterized by a less well separation of the two regimes; the state–dependent betas are lying closer together than the betas of the corresponding MS model. This phenomenon can be explained by the lack of flexibility of the former model due to the enforced synchronous switching with the market regimes. Besides, it should be mentioned that the estimates for the expected market returns μ_1 and μ_2 of the MSM model are very close to zero which supports Ryden et al. (1998) who proposed means equal to zero for daily return series. The estimates for γ_{11} and γ_{22}, mostly taking values between 95% and 99%, show a high persistence for both the high– and the low–risk state. We cannot confirm the observation of Fridman (1994) who reported lower persistence of the high–risk state.

4.3 Comparison of Conditional Beta Estimates

According to the discussed estimation results for the various modeling techniques, time–varying betas have been calculated for eighteen sectors. All conditional beta series are summarized by their respective mean and range in Table 9.

Even though the mean conditional betas are usually close to their OLS peers, a wide range of mean betas can be observed for every individual sector. Outstanding in this context are the means of the SV based conditional beta series which are smaller than unity for every sector. Theoretically, this is not meaningful as the aggregate of all sectors constitutes the overall market. The widest beta range across sectors is observed for the MR model, followed by the GARCH and the RW approach. On the other hand, the minimum and maximum of conditional betas estimated by the two Markow switching approaches do not deviate far from their respective mean.

Figure 1 illustrates general similarities and differences between the alternative conditional beta series for the Insurance sector. As already indicated by the range of conditional betas, the KF and GARCH based techniques display the greatest variation. The time series of systematic risk exhibit the greatest amplitude when modeled by the MR model which seems to be the technique that is most flexible in capturing changes in a sector's sensitivity to the overall market over time. With the exception of the Markov switching framework the evolution of the Insurance beta during the TMT bubble and its aftermath is described in a similar way by all techniques. Between observations 600 and 650, which corresponds to the twelve months period before the peak of the TMT bubble, a sharp fall of the Insurance beta below unity is indicated. In the subsequent two years the sector's beta more than doubles where the highest values are reached within the MR and the GARCH framework. The Markov switching models are not able to reflect the developments and dramatic shifts in terms of market risk in the course of the TMT bubble. Especially the MSM model switches back and force between the different states without giving a clear direction of the sector's sensitivity to the overall market.

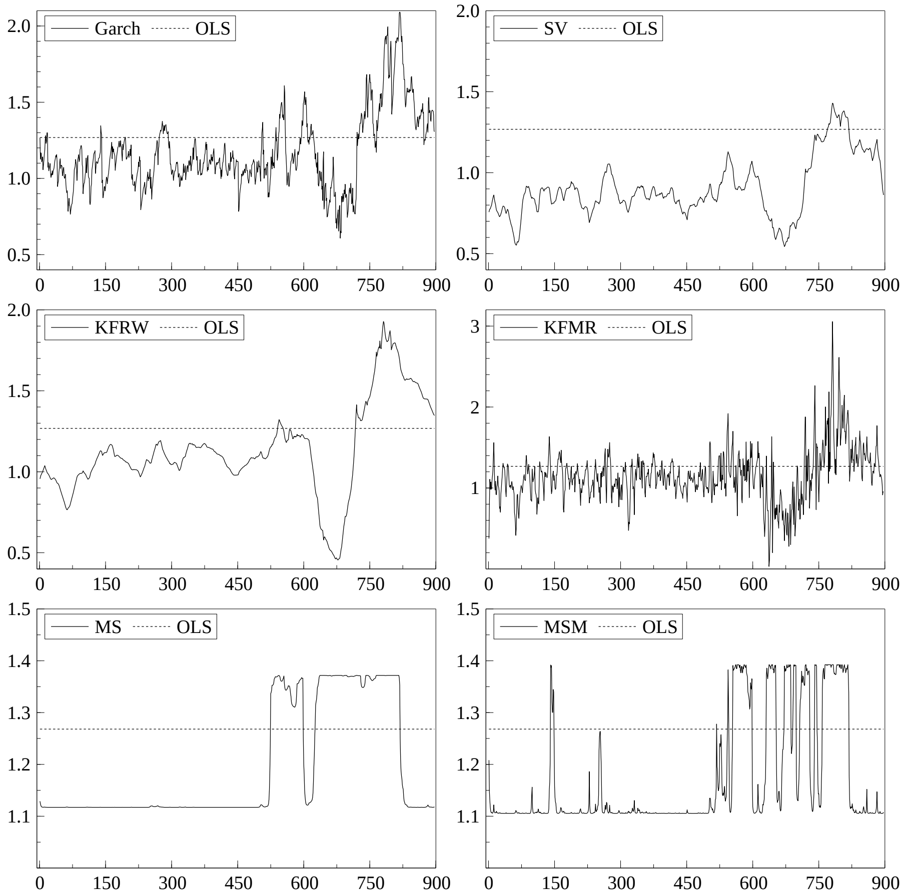


Figure 1: Various conditional betas for the Insurance sector

As mentioned in subsection 4.2, within the KF family the characteristics of the stochastic process of systematic risk depend on the speed parameter. Figures 6 & 7 illustrate that a beta though originally modeled as a mean–reverting process will resemble its RW counterpart the more close to unity the speed parameter gets. In case of Food & Beverages, Healthcare and Personal & Household Goods the MR betas literally follow a random walk. On the other hand Automobiles & Parts, Banks, Construction & Materials, Financial Services, Industrial Goods & Services, Insurance, Media, Oil & Gas, Retail and Travel & Leisure are highly mean-reverting.

Irrespective of the chosen modeling technique, the return of betas to their pre–bubble levels can be observed for most sectors at the end of the sample period. Visualizations of the conditional beta time series that are based on one of the volatility or Markov switching models are shown in Figures 4 & 5 and Figures 8 & 9, respectively.

4.4 In–Sample Forecasting Accuracy

The results above strongly indicate that systematic risk is not stationary and that the nature of the time–varying behavior of beta depends on the chosen modeling technique. To determine which approach generates the relatively best measure of time–varying systematic risk, the different techniques are formally ranked based on their in–sample forecast performance. Following previous studies, the first two criteria used to evaluate and compare the respective in–sample forecasts are the mean absolute error (MAE) and the mean squared error (MSE):

$$MAE_i = \frac{1}{T}\sum_{t=1}^{T}\frac{|\hat{R}_{it} - R_{it}|}{T}, \tag{32}$$

$$MSE_i = \frac{1}{T}\sum_{t=1}^{T}\frac{(\hat{R}_{it} - R_{it})^2}{T}, \tag{33}$$

where T is the number of forecast observations and $\hat{R}_{it} = \hat{\beta}_{it}R_{0t}$ denotes the series of return forecasts for sector i, calculated as the product of the conditional beta series estimated over the entire sample and the series of market returns which is assumed to be known in advance. The forecast quality is inversely related to the size of these two error measures. The resulting MAE and MSE for the different modeling techniques are reported in Tables 10 & 11.

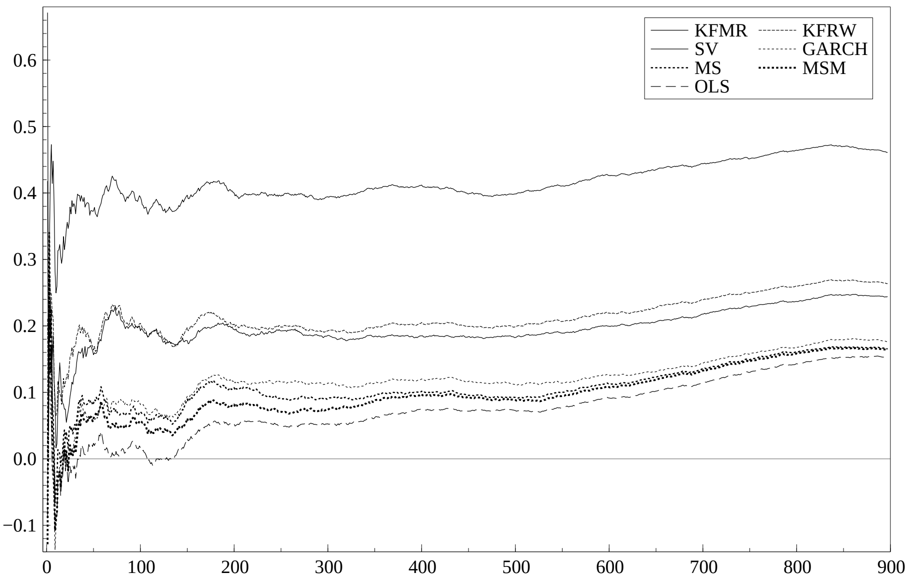
A comparison of the different modeling techniques confirms the expectation that the forecast performance of standard OLS is indeed worse than for any time–varying technique. However, compared to the volatility based techniques and the Markov switching approaches the degree of OLS' inferiority is remarkably low.

For the investigated sample, the two KF techniques clearly outperform their competitors. With respect to both error measures, the MR model ranks first on each occasion. In case where the conditional betas are modeled as a random walk the second lowest MAE is generated fifteen (MSE: seventeen) times. Whenever the RW model does not rank second, it is outperformed by the SV model. On average the MAE (MSE) for the RW model is 15.5% (29.2%) higher than the error measures for the overall best model. Within the class of volatility models, the SV approach seems to be better qualified to model the time–varying behavior of systematic risk than the well established GARCH model. Although the average errors are higher for the SV model, which is mainly due to its bad performance in connection with Technology, the MAE (MSE) of the SV model is lower in 11 (13) out of 18 occasions. Within the Markov switching framework, the conditional MSM betas produce approximately the same forecast errors as the volatility based beta series while the MS betas lead to lower average errors than the MSM and the volatility techniques.

While the mean error criteria can be used to evaluate the average forecast performance over a specified period of time for each model and each sector individually, they do not allow for an analysis of forecast performances across sectors. As from a practical perspective it is interesting how close the rank order of forecasted sector returns corresponds to the order of realized sector returns at any time, Spearman's rank correlation coefficient (ρ_t^S), a non–parametric measure of correlation that can be used for ordinal variables in a cross–sectional context, is introduced as the third evaluation criteria. After ranking the forecasted and observed sector returns separately for each point of time, where the sector with the highest return ranks first, ρ_t^S can be computed as

$$\rho_t^S = 1 - \frac{6 \sum_{i=1}^{I_t} D_{it}^2}{I_t(I_t^2 - 1)}, \tag{34}$$

with D_{it} being the difference between the corresponding ranks for each sector and I_t being the number of pairs of sector ranks, each at time t. Figure 2 illustrates how the average in–sample rank correlations develop over time.



This figure shows the recursively estimated in–sample means of Spearman's rank correlations for the various modeling techniques.

Figure 2: In–sample rank correlation coefficients

The highest in–sample rank correlations are observed for the MR ($\rho^S = 0.46$) and the RW model (0.26). In contrast to the used error criteria above, the third best result is observed for the SV (0.24) and not for the MS (0.17) model which only ranks fifth behind the GARCH approach (0.18). The MSM model (0.16) does only slightly better than OLS (0.15).

To sum up, the in–sample comparison suggests that time–varying European sector betas as estimated by a KF approach are superior to the analyzed alternatives. This is in line with previous findings presented by Brooks et al. (1998) and Faff et al. (2000) for industry portfolios in Australia and the UK.

4.5 Out–Of–Sample Forecasting Accuracy

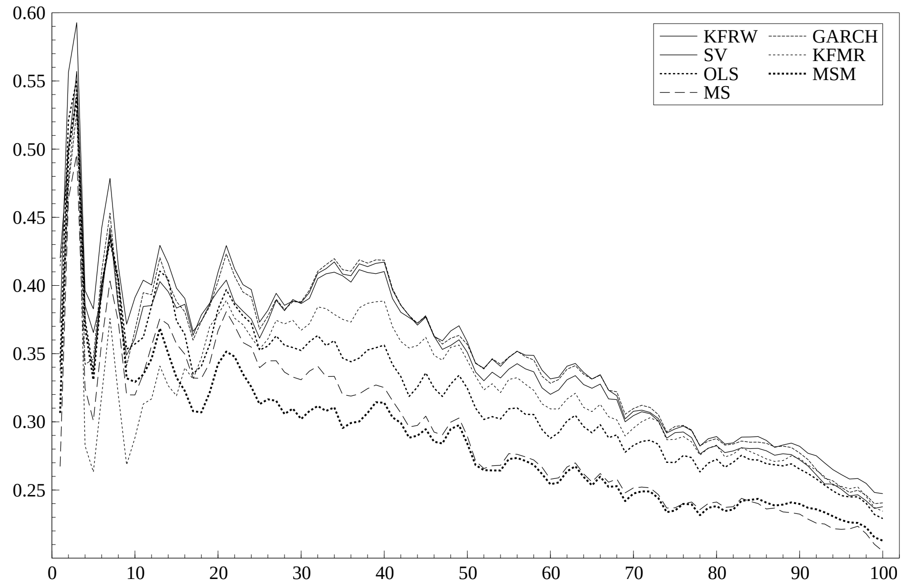
While the in–sample analysis is useful to assess the various techniques' ability to fit the data, the indispensable extension is to evaluate the forecast performances out–of–sample. For that purpose 100 beta and return forecasts based on 100 samples of 520 weekly observations are estimated for each technique. Within this rolling window forecast procedure, the sample is rolled forward by one week while the sample size is kept constant at 520. The first sample, starting 24 March 1993 and ending 5 March 2003, is used to calculate the out–of–sample conditional beta forecasts on 12 March 2003 based on the chosen modeling technique. The 100th beta forecast is then generated based on the last sample starting 15 February 1995 and ending 26 January 2005.

Tables 12 & 13 present the resulting out–of-sample mean error measures. It is again a KF approach that offers the best forecast performance. However, out–of–sample it is the RW not the MR model that yields the lowest average errors. Within the class of volatility models, no clear winner can be proclaimed as GARCH and SV approximately produce the same forecast errors. The worst forecast performances are observed for the two Markov switching models which do even worse than standard OLS. While the average errors related to OLS are higher than for the volatility based techniques, the average relative ranks are even lower, being only inferior to the KF models. Altogether, the superiority of the KF is not as dominant as in–sample. For both error measures the average rank of the overall best model drops to around 3, compared to an average rank of 1 for the best technique in–sample. Only in four (four) occasions the RW (MR) model yields the lowest MSE. The SV model and OLS each yield the lowest MSE three times. The remaining four first ranks are distributed between GARCH (2) and the two Markov switching models with one top rank each.

These findings are broadly confirmed in a cross–sectional setting as shown in Figure 3. The RW model ($\rho^S = 0.25$) produces the best out–of–sample forecasts for beta, followed closely by the GARCH and the SV model (each 0.24). According to the rank correlation criteria, the out–of–sample forecast performance of the MR model (0.23) is only equivalent to that of OLS (each 0.23). The Markov switching techniques (each 0.21) produce the worst forecasts.

Generally, it can be observed that the estimated in– and out–of–sample forecast errors depend positively on the standard deviation of a sector and negatively on the reported R^2 of the excess market model. On the one hand this suggests that the ability to generate precise forecasts diminishes with an increasing level of return volatility. On the other hand there might be third factors that influence the time–varying behavior of systematic risk which haven't been taken into account. On average the highest forecast errors are observed for Automobiles & Parts and the high–risk TMT sectors which have in common that they first strongly outperformed the overall market during the New Economy bubble at the end of the 1990s and then jointly collapsed in the course of the subsequent bear market. Non–high risk sectors with a high level of forecast errors are Basic Resources, Healthcare, Oil & Gas and Retail. While the first three sectors show a low R^2 in the excess market model, Retail is neither considered being a high–risk sector nor is the estimated R^2 particularly low. A possible explanation for a forecast error above average might be the sector's dependency on the British Pound: as more than 55% of the Retail sector is composed of UK stocks,[17] the sector's volatility is positively related to the volatility of the Pound. Thus, an increase in Retail's beta is not necessarily caused by a shift in the sector's sensi-

[17] On April 26th 2005 58.2% of the Retail sector's market cap was listed in British Pound (source: www.stoxx.com).



This figure shows the recursively estimated out–of–sample means of Spearman's rank correlations for the various modeling techniques.

Figure 3: Out–of–sample rank correlation coefficients

tivity to the overall market but could rather result from an additional currency risk.

5 Conclusion and Outlook

Despite the considerable empirical evidence that systematic risk is not constant over time, only a few studies deal with the explicit modeling of the time–varying behavior of betas. Previous studies focused on Australia, India, New Zealand, the US and the UK and employed primarily Kalman filter and GARCH based techniques. The present paper contributes an investigation of time–varying betas for pan–European industry portfolios and extends the spectrum of modeling techniques by a) incorporating two Markov switching approaches whose capabilities to model time–varying betas have not been compared to the proposed alternatives yet and b) by incorporating the stochastic volatility model which so far has only be used by Li (2003) to model time–varying betas.

The in–sample forecast performances of the various techniques suggest that independent from the utilized modeling approach, the extent to which sector returns can be explained by movements of the overall market is always higher for time–varying betas than in connection with standard OLS. This implies confirmation of previous findings that sector betas are not stable over time. Based on the employed evaluation criteria the in– and out–of–sample forecast performances of the various techniques are compared. The results of this study indicate that time–varying sector betas are best described by a random walk process, estimated by the use of the Kalman filter. While the in–sample results overwhelmingly support the KF approach, its superiority is only partly maintained out–of–sample where the advantage over its competitors is less pronounced. The findings of Li (2003) who reports that the SV approach outperforms the other techniques cannot be confirmed in a European context. Remarkably, the out–of–sample forecast performance of the two proposed Markov switching models is inferior to that of any time–varying alternative and also to OLS.

The methodology used in this study can be extended in a couple of directions. First of all, it would be of interest to see how the forecasting accuracy of the various models depends on the chosen length of the forecasting period. Secondly, the performance of the Kalman filter could be further improved by following the proposition of Moonis and Shah (2002) who apply a modified Kalman filter with heteroskedastic errors to account for the phenomenon of volatility clustering. Another way to further optimize beta forecasts, is to use exogenous factors to explain the time–varying behavior of systematic risk. Some first steps into this direction have been made by Abell and Krueger (1989) and Andersen et al. (2005) who link betas to macroeconomics and by Liodakis et al. (2003) who use company fundamentals, momentum and liquidity data as determinants of time–varying betas.

6 Appendix: Tables and Figures

Table 2: Descriptive statistics of excess weekly returns

This table summarizes the weekly excess returns data of the eighteen DJ STOXX[SM] sector indices and the DJ STOXX[SM] Broad as European market portfolio, covering the period from 2 December 1987 to 2 February 2005.

Sector	N^a	Mean	Std. Dev.	Skew.	Kurt.	JB^b
Broad	*897*	*0.0010*	*0.0231*	*−0.30*	*6.83*	*560.81*
Automobiles	897	0.0002	0.0330	−0.56	6.30	452.55
Banks	897	0.0014	0.0270	−0.28	7.49	765.94
Basics	897	0.0012	0.0284	−0.24	5.13	177.41
Chemicals	897	0.0009	0.0257	−0.19	7.87	890.35
Construction	897	0.0008	0.0245	−0.32	4.97	159.58
Financials	897	0.0007	0.0259	−0.63	8.73	1286.90
Food	897	0.0010	0.0212	−0.27	5.86	317.60
Healthcare	897	0.0017	0.0253	0.18	5.52	242.96
Industrials	897	0.0007	0.0248	−0.47	5.69	303.08
Insurance	897	0.0004	0.0334	−0.85	13.97	4606.70
Media	897	0.0007	0.0342	−0.62	9.89	1832.40
Oil & Gas	897	0.0015	0.0267	−0.02	5.56	245.73
Personal	683	0.0009	0.0257	−0.22	4.95	113.83
Retail	683	0.0006	0.0298	−0.78	10.32	1594.50
Technology	897	0.0007	0.0422	−0.55	6.68	553.00
Telecom	897	0.0013	0.0344	−0.18	5.36	212.89
Travel	683	0.0007	0.0234	0.10	6.36	321.69
Utilities	897	0.0015	0.0203	−0.45	5.15	203.02

[a]In September 2004 STOXX Ltd. switched its sector definitions from the DJ Global Classification Standard to the Industry Classification Benchmark and replaced the sectors Cyclical Goods & Services, Non–Cyclical Goods & Services and Retail (old) by the new sectors Travel & Leisure, Personal & Household Goods and Retail (new), respectively. As the history for the newly formed sectors only goes back to 31 December 1991, for these three sectors only 683 instead of 897 weekly observations are available.

[b]JB is the Jarque–Bera statistic for testing normality. In the selected sample the null hypothesis of normality can be rejected at the 1% significance level for every sector as well as for the overall market.

Table 3: OLS estimates of excess market model

This table presents summary statistics for OLS estimation of the excess market model. Figures in parentheses denote p–values.

Sector	α	β	R^2	$ARCH(1)^a$	$ARCH(6)$
Automobiles	−0.001	1.148	0.64	26.14	53.38
	(0.150)	(0.000)		(0.000)	(0.000)
Banks	0.000	1.062	0.82	14.95	44.81
	(0.409)	(0.000)		(0.000)	(0.000)
Basics	0.000	0.902	0.54	61.08	171.64
	(0.610)	(0.000)		(0.000)	(0.000)
Chemicals	0.000	0.907	0.66	39.15	86.82
	(0.989)	(0.000)		(0.000)	(0.000)
Construction	0.000	0.886	0.69	25.91	47.81
	(0.776)	(0.000)		(0.000)	(0.000)
Financials	0.000	0.997	0.79	8.45	79.92
	(0.470)	(0.000)		(0.004)	(0.000)
Food	0.000	0.648	0.50	17.74	184.76
	(0.443)	(0.000)		(0.000)	(0.000)
Healthcare	0.001	0.777	0.50	5.00	58.89
	(0.121)	(0.000)		(0.025)	(0.000)
Industrials	0.000	0.977	0.83	12.44	58.00
	(0.391)	(0.000)		(0.000)	(0.000)
Insurance	−0.001	1.268	0.77	16.03	74.91
	(0.106)	(0.000)		(0.000)	(0.000)
Media	−0.001	1.215	0.67	21.26	74.82
	(0.423)	(0.000)		(0.000)	(0.000)
Oil & Gas	0.001	0.758	0.43	28.53	114.59
	(0.268)	(0.000)		(0.000)	(0.000)
Personal	0.000	0.907	0.74	84.39	95.63
	(0.924)	(0.000)		(0.000)	(0.000)
Retail	0.000	0.949	0.61	1.61	7.26
	(0.519)	(0.000)		(0.204)	(0.297)
Technology	−0.001	1.489	0.66	15.38	98.73
	(0.337)	(0.000)		(0.000)	(0.000)
Telecom	0.000	1.194	0.64	31.76	65.18
	(0.910)	(0.000)		(0.000)	(0.000)
Travel	0.000	0.770	0.65	7.15	38.80
	(0.863)	(0.000)		(0.008)	(0.000)
Utilities	0.001	0.694	0.62	11.44	36.20
	(0.068)	(0.000)		(0.001)	(0.000)

[a]ARCH(p) is the LM statistic of Engle's ARCH test for lag order p. With the exception of Retail the null of no heteroskedasticity can be rejected at the 3% significance level for both lag orders tested for all sectors.

Table 4: Comparison of different GARCH(1,1) specifications

To decide i) whether a non–zero constant should be included in the specification of the conditional mean equation (4) and ii) whether the innovations $\boldsymbol{z}_t$ in equation (5) should be modeled by a normal or a standardized Student's t-distribution, four different setups have been analyzed. This table reports the estimated values of the corresponding log–likelihood functions and the Akaike Information Criteria. In the selected sample, the specification with a non–zero constant in the conditional mean equation and a standardized Student's t-distribution for the innovations $\boldsymbol{z}_t$ offers the best fit.

Sector	$[\boldsymbol{z}_t \sim N, \boldsymbol{\mu} \neq 0]$		$[\boldsymbol{z}_t \sim N, \boldsymbol{\mu} = 0]$		$[\boldsymbol{z}_t \sim t, \boldsymbol{\mu} \neq 0]$		$[\boldsymbol{z}_t \sim t, \boldsymbol{\mu} = 0]$	
	$\ln L^a$	AIC^b	$\ln L$	AIC	$\ln L$	AIC	$\ln L$	AIC
Broad	*2215.9*	*−4.93*	*2209.6*	*−4.92*	*2228.5*	*−4.96*	*2220.1*	*−4.94*
		(3)		*(4)*		*(1)*		*(2)*
Automobiles	1871.9	−4.17	1870.7	−4.16	1892.5	−4.21	1890.6	−4.21
		(3)		(4)		(1)		(2)
Banks	2112.8	−4.70	2108.0	−4.69	2132.4	−4.74	2124.1	−4.73
		(3)		(4)		(1)		(2)
Basics	1988.2	−4.42	1985.5	−4.42	2013.8	−4.48	2009.6	−4.47
		(3)		(4)		(1)		(2)
Chemicals	2110.1	−4.70	2106.3	−4.69	2117.5	−4.71	2112.2	−4.70
		(3)		(4)		(1)		(2)
Construction	2112.9	−4.70	2109.8	−4.70	2129.7	−4.74	2124.3	−4.73
		(3)		(4)		(1)		(2)
Financials	2144.4	−4.77	2137.4	−4.76	2175.7	−4.84	2168.1	−4.83
		(3)		(4)		(1)		(2)
Food	2240.5	−4.99	2238.1	−4.98	2257.3	−5.02	2252.1	−5.01
		(3)		(4)		(1)		(2)
Healthcare	2083.3	−4.64	2076.7	−4.62	2090.4	−4.65	2083.8	−4.64
		(3)		(4)		(1)		(2)
Industrials	2144.2	−4.77	2137.8	−4.76	2158.9	−4.80	2150.9	−4.79
		(3)		(4)		(1)		(2)
Insurance	1973.8	−4.39	1971.3	−4.39	2006.3	−4.46	2003.1	−4.46
		(3)		(4)		(1)		(2)
Media	1927.4	−4.29	1925.1	−4.29	1941.8	−4.32	1937.5	−4.31
		(3)		(4)		(1)		(2)
Oil & Gas	2048.7	−4.56	2044.0	−4.55	2056.8	−4.58	2052.6	−4.57
		(3)		(4)		(1)		(2)
Personal	1570.8	−4.59	1567.7	−4.58	1577.9	−4.61	1574.2	−4.60
		(3)		(4)		(1)		(2)
Retail	1466.1	−4.28	1464.6	−4.28	1521.2	−4.44	1517.6	−4.43
		(3)		(4)		(1)		(2)
Technology	1780.0	−3.96	1776.9	−3.96	1793.5	−3.99	1788.3	−3.98
		(3)		(4)		(1)		(2)
Telecom	1863.2	−4.15	1858.5	−4.14	1866.1	−4.15	1861.0	−4.14
		(2)		(4)		(1)		(3)
Travel	1640.8	−4.79	1637.4	−4.79	1652.3	−4.82	1649.7	−4.82
		(3)		(4)		(1)		(2)
Utilities	2270.9	−5.05	2265.7	−5.05	2284.4	−5.08	2276.1	−5.07
		(3)		(4)		(1)		(2)

[a] $\ln L$ denotes the value of the corresponding log–likelihood function.

[b] AIC is the Akaike Information Criterion, calculated as: $AIC = -2(logL/n) + 2(k/n)$, with k parameters and n observations. Figures in parentheses denote the rank of the respective AIC where the model with the smallest AIC ranks first.

Table 5: Parameter estimates for t–GARCH(1,1) models

This table reports the estimated parameters for the t–GARCH(1,1) models for the eighteen DJ STOXX℠ sectors and the DJ STOXX℠ broad as market index. Figures in parentheses denote p-values.

Sector	μ	$\psi \times 10^4$	γ	δ	DF^a	$\gamma + \delta$	ρ_{0i}
Broad	*0.0025*	*0.1645*	*0.1323*	*0.8370*	*9.2864*	*0.9693*	–
	(0.000)	*(0.033)*	*(0.000)*	*(0.000)*	*(0.000)*		
Automobiles	0.0017	0.2467	0.0969	0.8812	7.0286	0.9781	0.8030
	(0.055)	(0.042)	(0.000)	(0.000)	(0.000)		
Banks	0.0026	0.1229	0.1189	0.8658	7.8772	0.9847	0.9067
	(0.000)	(0.036)	(0.000)	(0.000)	(0.000)		
Basics	0.0022	0.0521	0.0571	0.9383	6.6280	0.9954	0.7323
	(0.004)	(0.257)	(0.004)	(0.000)	(0.000)		
Chemicals	0.0022	0.2654	0.1434	0.8175	9.4763	0.9608	0.8142
	(0.001)	(0.024)	(0.000)	(0.000)	(0.001)		
Construction	0.0023	0.2404	0.1085	0.8515	8.2580	0.9600	0.8328
	(0.001)	(0.027)	(0.000)	(0.000)	(0.000)		
Financials	0.0025	0.1627	0.1345	0.8435	6.2656	0.9780	0.8866
	(0.000)	(0.089)	(0.002)	(0.000)	(0.000)		
Food	0.0020	0.2325	0.1088	0.8383	7.7865	0.9471	0.7040
	(0.001)	(0.016)	(0.000)	(0.000)	(0.000)		
Healthcare	0.0027	0.2343	0.1012	0.8626	10.5350	0.9637	0.7085
	(0.000)	(0.058)	(0.001)	(0.000)	(0.001)		
Industrials	0.0026	0.1624	0.1370	0.8407	8.2634	0.9777	0.9089
	(0.000)	(0.027)	(0.000)	(0.000)	(0.000)		
Insurance	0.0018	0.1358	0.0901	0.8958	6.0133	0.9859	0.8751
	(0.012)	(0.025)	(0.000)	(0.000)	(0.000)		
Media	0.0023	0.1793	0.1019	0.8801	7.7610	0.9820	0.8176
	(0.003)	(0.036)	(0.000)	(0.000)	(0.000)		
Oil & Gas	0.0022	0.1992	0.0780	0.8918	9.4255	0.9699	0.6559
	(0.004)	(0.105)	(0.003)	(0.000)	(0.001)		
Personal	0.0023	0.5125	0.1612	0.7644	12.1030	0.9255	0.8622
	(0.006)	(0.034)	(0.001)	(0.000)	(0.003)		
Retail	0.0024	0.6665	0.1395	0.7801	6.1656	0.9197	0.7789
	(0.007)	(0.025)	(0.002)	(0.000)	(0.000)		
Technology	0.0029	0.1389	0.1048	0.8887	8.6957	0.9935	0.8129
	(0.001)	(0.048)	(0.000)	(0.000)	(0.000)		
Telecom	0.0029	0.1440	0.0855	0.9017	15.4200	0.9872	0.8004
	(0.001)	(0.077)	(0.000)	(0.000)	(0.034)		
Travel	0.0017	0.5387	0.1523	0.7469	8.4281	0.8992	0.8038
	(0.022)	(0.027)	(0.003)	(0.000)	(0.000)		
Utilities	0.0024	0.3829	0.1473	0.7626	7.7585	0.9098	0.7878
	(0.000)	(0.012)	(0.000)	(0.000)	(0.000)		

[a]DF denotes the number of degrees of freedom of the Student's t–distribution which has been estimated along with the other parameters of the t–GARCH(1,1) models.

Table 6: Parameter estimates for SV models

This table reports the estimated parameters for the SV models for the eighteen DJ STOXX℠ sectors and the DJ STOXX℠ broad as market index. Figures in parentheses denote the lower and upper bounds of the asymmetric asymptotic 95% confidence intervals.

Sector	ϕ	σ_η^2	σ^{*2}	$\ln L$	$Q(12)^a$
Broad	*0.966*	*0.039*	*3.677*	*−1913.77*	*16.07*
	(0.924;0.986)	*(0.018;0.084)*	*(2.498;5.414)*		
Automobiles	0.964	0.041	7.651	−2244.29	10.77
	(0.920;0.984)	(0.020;0.087)	(5.255;11.139)		
Banks	0.977	0.035	4.362	−2009.16	14.89
	(0.947;0.990)	(0.018;0.068)	(2.585;7.360)		
Basics	0.973	0.029	6.156	−2127.72	13.93
	(0.925;0.991)	(0.010;0.085)	(4.066;9.320)		
Chemicals	0.958	0.047	4.737	−2021.83	8.42
	(0.906;0.982)	(0.021;0.108)	(3.346;6.706)		
Construction	0.955	0.039	4.707	−2010.39	12.42
	(0.903;0.980)	(0.018;0.086)	(3.482;6.363)		
Financials	0.958	0.067	4.013	−1965.59	31.58
	(0.911;0.981)	(0.033;0.139)	(2.662;6.049)		
Food	0.941	0.053	3.446	−1880.94	11.30
	(0.878;0.972)	(0.025;0.114)	(2.630;4.516)		
Healthcare	0.953	0.038	5.031	−2048.26	13.10
	(0.882;0.982)	(0.014;0.103)	(3.783;6.689)		
Industrials	0.965	0.043	4.314	−1983.11	26.75
	(0.938;0.981)	(0.023;0.083)	(2.831;6.574)		
Insurance	0.979	0.035	5.833	−2135.02	15.40
	(0.952;0.991)	(0.018;0.067)	(3.267;10.414)		
Media	0.978	0.037	6.829	−2196.90	7.71
	(0.947;0.991)	(0.018;0.074)	(3.950;11.808)		
Oil & Gas	0.963	0.031	5.518	−2080.60	10.87
	(0.894;0.988)	(0.010;0.098)	(4.004;7.605)		
Personal	0.947	0.042	5.219	−1572.87	19.61
	(0.864;0.980)	(0.015;0.117)	(3.842;7.088)		
Retail	0.917	0.093	5.975	−1633.80	14.18
	(0.830;0.962)	(0.042;0.205)	(4.442;8.035)		
Technology	0.989	0.021	9.470	−2346.38	14.98
	(0.970;0.996)	(0.010;0.046)	(4.138;21.672)		
Telecom	0.989	0.013	7.590	−2269.84	17.80
	(0.966;0.997)	(0.006;0.030)	(3.895;14.789)		
Travel	0.905	0.089	4.074	−1496.92	8.27
	(0.807;0.956)	(0.039;0.206)	(3.138;5.289)		
Utilities	0.908	0.084	3.215	−1854.46	19.99
	(0.837;0.950)	(0.044;0.159)	(2.561;4.038)		

[a] $Q(l)$ is the test statistic of the Ljung-Box portmanteau test for the null hypothesis of no autocorrelation in the errors up to order l. The Q-statistic is asymptotically χ^2 distributed with $l-p$ degrees of freedom where p denotes the total number of estimated parameters. The relevant critical values at the 95% (99%) level are 16.92 (21.67).

Table 7: Parameter estimates for KF models

This table reports the estimated parameters for the two KF–based models for the eighteen DJ STOXX℠ sectors. All estimates for σ^2 and σ_η^2 are significant at the 1% level. *** means that ϕ is significant at the 1% level (**: 5%, *: 10%).

Sector	Model	$\sigma^2 \times 10^2$	$\sigma_\eta^2 \times 10^2$	$\bar{\beta}$	ϕ	$\ln L$	AIC
Automobiles	RW	0.0320	0.7984			2289.90	−5.10
	MR	0.0261	16.6070	1.149	0.549*	2304.50	−5.13
Banks	RW	0.0106	0.1940			2792.90	−6.22
	MR	0.0075	9.3822	1.030	0.409***	2808.80	−6.25
Basics	RW	0.0322	0.4722			2297.70	−5.12
	MR	0.0307	2.0769	0.961	0.920*	2303.60	−5.13
Chemicals	RW	0.0185	0.2343			2547.90	−5.68
	MR	0.0178	0.8909	0.905	0.940	2551.50	−5.68
Construction	RW	0.0156	0.0578			2638.40	−5.88
	MR	0.0120	11.3370	0.936	0.359**	2629.80	−5.85
Financials	RW	0.0126	0.1605			2721.20	−6.06
	MR	0.0088	9.9813	0.944	0.197***	2756.70	−6.14
Food	RW	0.0168	0.1975			2593.00	−5.78
	MR	0.0167	0.2321	0.675	0.992	2593.70	−5.77
Healthcare	RW	0.0284	0.1828			2364.90	−5.27
	MR	0.0282	0.3078	0.809	0.982	2366.20	−5.27
Industrials	RW	0.0098	0.0631			2843.10	−6.33
	MR	0.0059	11.0350	0.997	0.000	2881.80	−6.42
Insurance	RW	0.0183	0.3019			2548.50	−5.68
	MR	0.0126	13.6660	1.152	0.629***	2570.50	−5.72
Media	RW	0.0300	0.5107			2326.50	−5.18
	MR	0.0192	37.6910	1.179	0.258***	2338.50	−5.21
Oil & Gas	RW	0.0378	0.1355			2242.60	−5.00
	MR	0.0324	12.7640	0.754	0.442***	2249.50	−5.01
Personal	RW	0.0153	0.0807			2012.10	−5.89
	MR	0.0152	0.1333	0.949	0.982	2012.50	−5.88
Retail	RW	0.0298	0.7802			1765.70	−5.16
	MR	0.0240	16.4740	0.903	0.333*	1782.30	−5.21
Technology	RW	0.0443	2.0363			2128.80	−4.74
	MR	0.0414	6.2424	1.481	0.919**	2137.50	−4.76
Telecom	RW	0.0351	0.5212			2258.20	−5.03
	MR	0.0340	1.5511	1.251	0.949*	2262.60	−5.04
Travel	RW	0.0180	0.0505			1960.80	−5.74
	MR	0.0136	10.9060	0.755	0.036***	1971.90	−5.76
Utilities	RW	0.0138	0.1421			2682.90	−5.98
	MR	0.0135	0.3713	0.741	0.966*	2685.50	−5.98

Table 8: Parameter estimates for MS models

This table reports the estimated parameters for the MS/MSM model for the eighteen DJ STOXX℠ sectors.

Sector	Model	$\alpha_1, \alpha_2 \times 10^4$	β_1, β_2	$\sigma^2_{i1}, \sigma^2_{i2} \times 10^2$	γ_{11}	γ_{22}	$\mu_1, \mu_2 \times 10^3$	$\sigma^2_{01}, \sigma^2_{02} \times 10^2$
Automobiles	MS	-13.4; -4.87	1.26; 1.03	1.51; 2.89	0.993	0.980	–	–
	MSM	-3.05; -35.5	1.22; 1.07	1.47; 2.92	0.976	0.926	2.40; -2.97	1.56; 3.70
Banks	MS	1.57; 8.70	1.01; 1.11	0.70; 1.77	0.988	0.970	–	–
	MSM	2.02; 1.07	0.97; 1.10	0.67; 1.75	0.962	0.904	2.30; -2.02	1.58; 3.36
Basics	MS	-3.26; 8.62	1.05; 0.84	1.04; 2.64	0.998	0.998	–	–
	MSM	-3.90; 9.56	1.02; 0.82	1.08; 2.95	0.972	0.917	-2.58; -2.28	1.61; 3.31
Chemicals	MS	1.70; -4.37	1.00; 0.87	0.93; 2.05	0.998	0.996	–	–
	MSM	1.04; -12.6	0.98; 0.85	1.05; 2.38	0.978	0.901	2.65; -4.16	1.63; 3.70
Construction	MS	-7.04; 0.22	1.14; 0.77	0.92; 1.75	0.997	0.993	–	–
	MSM	-4.38; -4.11	1.07; 0.79	0.94; 1.89	0.982	0.959	2.55; -3.26	1.55; 3.42
Financials	MS	-0.61; -3.31	0.91; 1.08	0.84; 1.82	0.997	0.990	–	–
	MSM	-0.05; 3.33	0.91; 1.05	0.81; 1.89	0.969	0.883	2.52; -3.25	1.60; 3.62
Food	MS	-0.95; 1.96	0.91; 0.49	0.76; 2.23	0.993	0.984	–	–
	MSM	-1.48; -1.47	0.89; 0.52	0.78; 2.34	0.985	0.928	2.59; -2.51	1.60; 3.33
Healthcare	MS	14.5; -0.88	0.91; 0.68	1.13; 2.39	0.981	0.971	–	–
	MSM	10.0; 2.72	0.90; 0.73	1.16; 2.40	0.969	0.952	2.62; -1.31	1.51; 3.11
Industrials	MS	-0.59; -7.62	1.02; 0.96	0.62; 1.45	0.998	0.996	–	–
	MSM	1.52; -1.41	1.00; 0.97	0.64; 1.57	0.975	0.951	2.24; -1.45	1.59; 3.38
Insurance	MS	-5.81; -8.25	1.12; 1.37	1.01; 2.48	0.996	0.989	–	–
	MSM	-6.14; 1.58	1.11; 1.39	1.05; 2.64	0.971	0.901	2.44; -2.93	1.64; 3.72
Media	MS	-0.81; -12.4	1.05; 1.41	1.20; 3.40	0.989	0.960	–	–
	MSM	-2.57; -1.51	1.04; 1.32	1.21; 3.34	0.991	0.935	2.37; -4.21	1.65; 3.60
Oil & Gas	MS	6.02; 3.57	0.96; 0.58	1.37; 2.86	0.992	0.983	–	–
	MSM	6.12; 1.92	0.91; 0.71	1.38; 2.94	0.987	0.926	2.20; -1.68	1.62; 3.45
Personal	MS	-2.33; 6.06	0.95; 0.80	1.03; 1.87	0.995	0.980	–	–
	MSM	-1.76; -5.45	0.99; 0.85	1.72; 1.84	0.986	0.962	2.79; -4.19	1.72; 3.88
Retail	MS	2.29; -11.9	0.90; 0.97	1.12; 2.48	0.909	0.891	–	–
	MSM	2.50; -19.6	0.87; 0.99	1.45; 2.83	0.967	0.872	1.68; -5.45	1.68; 4.04
Technology	MS	2.45; -1.61	1.18; 1.71	1.25; 3.85	0.996	0.990	–	–
	MSM	3.67; -7.46	1.17; 1.60	1.25; 3.80	0.993	0.947	2.48; -2.35	1.62; 3.38
Telecom	MS	6.73; -8.51	1.09; 1.31	1.49; 3.24	0.997	0.989	–	–
	MSM	3.29; -3.24	1.12; 1.21	1.49; 3.22	0.992	0.974	2.76; -4.10	1.69; 3.60
Travel	MS	0.05; -0.20	0.82; 0.50	1.09; 2.71	0.967	0.733	–	–
	MSM	-1.36; -8.04	0.84; 0.73	1.05; 2.13	0.976	0.922	3.04; -5.01	1.67; 3.94
Utilities	MS	6.97; -8.05	0.82; 0.56	1.03; 1.63	0.994	0.977	–	–
	MSM	7.07; -3.89	0.81; 0.63	1.03; 1.67	0.989	0.960	2.90; -5.67	1.59; 3.69

Table 9: Comparison of OLS betas and various conditional beta series

This table summarizes the various conditional beta series by reporting the mean betas and their range (in brackets).[a]

Sector	β^{OLS}	β^{tGARCH}	β^{SV}	β^{KFRW}	β^{KFMR}	β^{MSM}	β^{MS}
Automobiles	1.148	1.191	0.810	1.145	1.145	1.182	1.203
		(0.555; 1.707)	(0.455; 1.057)	(0.123; 1.609)	(0.025; 2.370)	(1.065; 1.225)	(1.029; 1.262)
Banks	1.062	1.034	0.943	1.019	1.034	1.014	1.041
		(0.588; 1.489)	(0.656; 1.228)	(0.367; 1.337)	(−0.156; 1.978)	(0.971; 1.103)	(1.011; 1.109)
Basics	0.902	0.961	0.735	0.956	0.945	0.955	0.950
		(0.507; 1.597)	(0.479; 1.364)	(−0.018; 1.489)	(−0.364; 1.616)	(0.815; 1.025)	(0.839; 1.047)
Chemicals	0.907	0.928	0.810	0.913	0.900	0.947	0.941
		(0.496; 1.493)	(0.526; 1.051)	(0.122; 1.299)	(0.031; 1.395)	(0.849; 0.980)	(0.865; 0.996)
Construction	0.886	0.941	0.833	0.964	0.933	0.980	0.992
		(0.506; 1.389)	(0.551; 1.186)	(0.617; 1.358)	(−0.036; 1.581)	(0.794; 1.070)	(0.766; 1.142)
Financials	0.997	0.968	0.888	0.937	0.947	0.948	0.956
		(0.651; 1.436)	(0.560; 1.241)	(0.552; 1.267)	(0.139; 2.081)	(0.911; 1.049)	(0.906; 1.083)
Food	0.648	0.690	0.699	0.710	0.708	0.773	0.763
		(0.346; 1.181)	(0.445; 0.961)	(−0.345; 1.115)	(−0.362; 1.116)	(0.519; 0.894)	(0.486; 0.910)
Healthcare	0.777	0.842	0.725	0.809	0.806	0.830	0.811
		(0.380; 1.369)	(0.410; 1.098)	(0.055; 1.142)	(0.010; 1.173)	(0.731; 0.903)	(0.678; 0.913)
Industrials	0.977	0.992	0.907	0.994	0.996	0.990	0.992
		(0.683; 1.446)	(0.692; 1.192)	(0.816; 1.233)	(−0.198; 1.836)	(0.974; 0.997)	(0.957; 1.017)
Insurance	1.268	1.173	0.914	1.144	1.155	1.177	1.197
		(0.608; 2.091)	(0.544; 1.429)	(0.456; 1.929)	(0.032; 3.055)	(1.105; 1.392)	(1.117; 1.372)
Media	1.215	1.168	0.847	1.184	1.181	1.110	1.132
		(0.681; 2.513)	(0.547; 1.780)	(0.667; 2.586)	(−0.538; 3.820)	(1.039; 1.322)	(1.049; 1.406)
Oil & Gas	0.758	0.807	0.665	0.781	0.753	0.850	0.834
		(0.372; 1.393)	(0.374; 1.006)	(0.318; 1.056)	(−0.217; 1.372)	(0.713; 0.912)	(0.584; 0.958)
Personal	0.907	0.991	0.821	0.956	0.952	0.955	0.913
		(0.580; 1.544)	(0.539; 1.223)	(0.619; 1.186)	(0.576; 1.186)	(0.853; 0.992)	(0.802; 0.949)
Retail	0.949	0.997	0.734	0.907	0.898	0.903	0.934
		(0.563; 1.769)	(0.481; 1.111)	(0.264; 1.599)	(−0.470; 2.110)	(0.876; 0.994)	(0.903; 0.972)
Technology	1.489	1.399	0.884	1.460	1.488	1.313	1.356
		(0.684; 3.299)	(0.479; 1.720)	(0.853; 3.134)	(0.761; 3.438)	(1.174; 1.597)	(1.181; 1.709)
Telecom	1.194	1.234	0.887	1.246	1.266	1.146	1.145
		(0.645; 2.716)	(0.546; 1.683)	(0.738; 2.256)	(0.679; 2.290)	(1.122; 1.213)	(1.088; 1.314)
Travel	0.770	0.835	0.755	0.791	0.752	0.810	0.781
		(0.499; 1.312)	(0.505; 1.131)	(0.500; 0.981)	(−0.342; 1.453)	(0.728; 0.837)	(0.501; 0.814)
Utilities	0.694	0.760	0.799	0.753	0.742	0.762	0.744
		(0.417; 1.096)	(0.354; 1.206)	(0.239; 1.024)	(0.175; 1.018)	(0.626; 0.812)	(0.561; 0.819)

[a] As the Kalman filter is likely to produce large outliers in the first stages of estimation, the first fifty conditional beta estimates for any of the chosen techniques to model conditional betas are not included in the subsequent analyses to avoid an unfair bias against the Kalman filter.

Table 10: In–sample mean absolute errors

This table reports the estimated in–sample MAE ($\times 10^2$) for the eighteen DJ STOXX℠ sectors. For each sector i, figures in parentheses denote the relative rank of a model's MAE where the model with the smallest MAE ranks first.

Sector	β^{OLS}	β^{tGARCH}	β^{SV}	β^{KFRW}	β^{KFMR}	β^{MSM}	β^{MS}
Automobiles	1.4693	1.4838	1.5433	1.3497	1.1195	1.4682	1.4556
	(5)	(6)	(7)	(2)	(1)	(4)	(3)
Banks	0.7964	0.7954	0.7663	0.7357	0.5357	0.7862	0.7906
	(7)	(6)	(3)	(2)	(1)	(4)	(5)
Basics	1.3645	1.3826	1.4208	1.2404	1.1914	1.3553	1.3532
	(5)	(6)	(7)	(2)	(1)	(4)	(3)
Chemicals	1.0948	1.0902	1.0681	1.0066	0.9733	1.0901	1.0890
	(7)	(6)	(3)	(2)	(1)	(5)	(4)
Construction	1.0153	0.9821	0.9704	0.9315	0.7243	0.9877	0.9786
	(7)	(5)	(3)	(2)	(1)	(6)	(4)
Financials	0.8686	0.8783	0.7967	0.8100	0.6000	0.8559	0.8537
	(6)	(7)	(2)	(3)	(1)	(5)	(4)
Food	1.0423	1.0389	0.9926	0.8837	0.8811	0.9784	0.9558
	(7)	(6)	(5)	(2)	(1)	(4)	(3)
Healthcare	1.3274	1.3437	1.3143	1.2499	1.2415	1.3153	1.3041
	(6)	(7)	(4)	(2)	(1)	(5)	(3)
Industrials	0.7348	0.7259	0.6973	0.6915	0.4552	0.7344	0.7322
	(7)	(4)	(3)	(2)	(1)	(6)	(5)
Insurance	1.1457	1.0761	1.0975	0.9883	0.7265	1.1303	1.1318
	(7)	(3)	(4)	(2)	(1)	(5)	(6)
Media	1.3616	1.2762	1.3602	1.1835	0.8120	1.3404	1.3243
	(7)	(3)	(6)	(2)	(1)	(5)	(4)
Oil & Gas	1.4861	1.4981	1.4934	1.4329	1.2310	1.4725	1.4510
	(5)	(7)	(6)	(2)	(1)	(4)	(3)
Personal	0.9712	0.9629	0.9779	0.9175	0.9122	0.9658	0.9667
	(6)	(3)	(7)	(2)	(1)	(4)	(5)
Retail	1.3502	1.3688	1.3477	1.2461	1.0258	1.3470	1.3497
	(6)	(7)	(4)	(2)	(1)	(3)	(5)
Technology	1.6932	1.5501	1.8403	1.4021	1.3275	1.6160	1.5945
	(6)	(3)	(7)	(2)	(1)	(5)	(4)
Telecom	1.5154	1.4690	1.5824	1.3970	1.3565	1.5143	1.4991
	(6)	(3)	(7)	(2)	(1)	(5)	(4)
Travel	0.9930	1.0147	0.9435	0.9591	0.7465	0.9953	0.9698
	(5)	(7)	(2)	(3)	(1)	(6)	(4)
Utilities	0.9795	0.9651	0.8970	0.9032	0.8809	0.9636	0.9491
	(7)	(6)	(2)	(3)	(1)	(5)	(4)
Average MAE	1.1783	1.1612	1.1728	1.0738	0.9300	1.1620	1.1527
Average Rank	6.22	5.28	4.56	2.17	1.00	4.72	4.06

Table 11: In–sample mean squared errors

This table reports the estimated in–sample MSE ($\times 10^3$) for the eighteen DJ STOXX℠ sectors. For each sector i figures in parentheses denote the relative rank of a model's MSE where the model with the smallest MSE ranks first.

Sector	β^{OLS}	β^{tGARCH}	β^{SV}	β^{KFRW}	β^{KFMR}	β^{MSM}	β^{MS}
Automobiles	0.3935	0.3978	0.4244	0.3069	0.2129	0.3925	0.3855
	(5)	(6)	(7)	(2)	(1)	(4)	(3)
Banks	0.1336	0.1328	0.1173	0.1043	0.0571	0.1314	0.1323
	(7)	(6)	(3)	(2)	(1)	(4)	(5)
Basics	0.3821	0.4022	0.3983	0.3158	0.2894	0.3781	0.3778
	(5)	(7)	(6)	(2)	(1)	(4)	(3)
Chemicals	0.2308	0.2304	0.2140	0.1852	0.1727	0.2289	0.2291
	(7)	(6)	(3)	(2)	(1)	(4)	(5)
Construction	0.1889	0.1741	0.1664	0.1568	0.0958	0.1800	0.1724
	(7)	(5)	(3)	(2)	(1)	(6)	(4)
Financials	0.1472	0.1502	0.1220	0.1240	0.0682	0.1447	0.1428
	(6)	(7)	(2)	(3)	(1)	(5)	(4)
Food	0.2333	0.2442	0.2239	0.1680	0.1670	0.2170	0.2099
	(6)	(7)	(5)	(2)	(1)	(4)	(3)
Healthcare	0.3324	0.3509	0.3277	0.2884	0.2842	0.3288	0.3238
	(6)	(7)	(4)	(2)	(1)	(5)	(3)
Industrials	0.1116	0.1105	0.1003	0.0994	0.0421	0.1115	0.1112
	(7)	(4)	(3)	(2)	(1)	(6)	(5)
Insurance	0.2713	0.2325	0.2459	0.1811	0.0968	0.2608	0.2624
	(7)	(3)	(4)	(2)	(1)	(5)	(6)
Media	0.4035	0.3443	0.3911	0.2966	0.1344	0.3909	0.3836
	(7)	(3)	(6)	(2)	(1)	(5)	(4)
Oil & Gas	0.4173	0.4366	0.4240	0.3810	0.2879	0.4106	0.3937
	(5)	(7)	(6)	(2)	(1)	(4)	(3)
Personal	0.1741	0.1816	0.1739	0.1535	0.1515	0.1718	0.1713
	(6)	(7)	(5)	(2)	(1)	(4)	(3)
Retail	0.3559	0.3766	0.3397	0.2855	0.1931	0.3538	0.3551
	(6)	(7)	(3)	(2)	(1)	(4)	(5)
Technology	0.6282	0.5652	0.7143	0.4282	0.3803	0.6002	0.5900
	(6)	(3)	(7)	(2)	(1)	(5)	(4)
Telecom	0.4412	0.4091	0.4654	0.3495	0.3295	0.4406	0.4340
	(6)	(3)	(7)	(2)	(1)	(5)	(4)
Travel	0.1928	0.1997	0.1777	0.1765	0.1089	0.1912	0.1798
	(6)	(7)	(3)	(2)	(1)	(5)	(4)
Utilities	0.1613	0.1571	0.1367	0.1365	0.1308	0.1556	0.1498
	(7)	(6)	(3)	(2)	(1)	(5)	(4)
Average MSE	0.2888	0.2831	0.2868	0.2298	0.1779	0.2827	0.2780
Average Rank	6.22	5.61	4.44	2.06	1.00	4.67	4.00

Table 12: Out–of–sample mean absolute errors

This table reports the estimated out–of–sample MAE ($\times 10^2$) for the eighteen DJ STOXX℠ sectors. For each sector i figures in parentheses denote the relative rank of a model's MAE where the model with the smallest MAE ranks first.

Sector	β^{OLS}	β^{tGARCH}	β^{SV}	β^{KFRW}	β^{KFMR}	β^{MSM}	β^{MS}
Automobiles	1.3089	1.2865	1.2940	1.2889	1.3355	1.2791	1.2990
	(6)	(2)	(4)	(3)	(7)	(1)	(5)
Banks	0.4555	0.4887	0.4824	0.4780	0.4466	0.4586	0.4545
	(3)	(7)	(6)	(5)	(1)	(4)	(2)
Basics	1.5837	1.4955	1.5166	1.4852	1.4954	1.5680	1.5831
	(7)	(3)	(4)	(1)	(2)	(5)	(6)
Chemicals	1.1064	0.9908	1.0259	0.9307	0.9592	1.0829	1.0900
	(7)	(3)	(4)	(1)	(2)	(5)	(6)
Construction	0.9378	0.8721	0.8634	0.8697	0.9260	0.9091	0.9163
	(7)	(3)	(1)	(2)	(6)	(4)	(5)
Financials	0.6787	0.7099	0.6987	0.6941	0.6665	0.6733	0.6727
	(4)	(7)	(6)	(5)	(1)	(3)	(2)
Food	0.9411	0.9588	0.9575	0.9314	FTC[a]	0.9333	0.9510
	(3)	(6)	(5)	(1)	(–)	(2)	(4)
Healthcare	1.0324	1.0964	1.0954	1.0243	1.0218	1.0461	1.0114
	(4)	(7)	(6)	(3)	(2)	(5)	(1)
Industrials	0.7940	0.7383	0.7281	0.7403	0.7884	0.8059	0.7995
	(5)	(2)	(1)	(3)	(4)	(7)	(6)
Insurance	1.0975	0.9913	1.0110	1.0168	1.0795	1.1271	1.0973
	(6)	(1)	(2)	(3)	(4)	(7)	(5)
Media	0.9409	1.0115	1.0276	0.9639	0.9561	0.9747	1.0319
	(1)	(5)	(6)	(3)	(2)	(4)	(7)
Oil & Gas	1.2316	1.2609	1.2513	1.2439	1.2385	1.2594	1.2611
	(1)	(6)	(4)	(3)	(2)	(5)	(7)
Personal	0.6649	0.6656	0.6668	0.6621	0.6549	0.6615	0.6680
	(4)	(5)	(6)	(3)	(1)	(2)	(7)
Retail	1.0722	1.1413	1.1230	1.1223	1.1083	1.1067	1.0991
	(1)	(7)	(6)	(5)	(4)	(3)	(2)
Technology	1.8179	1.7783	1.7524	1.8199	1.8134	1.8996	1.8854
	(4)	(2)	(1)	(5)	(3)	(7)	(6)
Telecom	1.1611	1.0889	1.0779	1.0569	1.0533	1.1470	1.1286
	(7)	(4)	(3)	(2)	(1)	(6)	(5)
Travel	0.7384	0.7568	0.7531	0.7393	0.7412	0.7478	0.7606
	(1)	(6)	(5)	(2)	(3)	(4)	(7)
Utilities	0.8333	0.8379	0.8243	0.8461	0.8426	0.9025	0.8310
	(3)	(4)	(1)	(6)	(5)	(7)	(2)
Average MAE	1.0220	1.0094	1.0083	0.9952	1.0075	1.0324	1.0300
Average Rank	4.11	4.44	3.94	3.11	2.94	4.50	4.72

[a]Failed to converge.

Table 13: Out–of–sample mean squared errors

This table reports the estimated out–of–sample MSE ($\times 10^3$) for the eighteen DJ STOXX℠ sectors. For each sector i figures in parentheses denote the relative rank of a model's MSE where the model with the smallest MSE ranks first.

Sector	β^{OLS}	β^{tGARCH}	β^{SV}	β^{KFRW}	β^{KFMR}	β^{MSM}	β^{MS}
Automobiles	0.3146	0.3101	0.3205	0.3176	0.3223	0.3027	0.3128
	(4)	(2)	(6)	(5)	(7)	(1)	(3)
Banks	0.0349	0.0373	0.0362	0.0364	0.0328	0.0344	0.0344
	(4)	(7)	(5)	(6)	(1)	(3)	(2)
Basics	0.4062	0.3566	0.3744	0.3538	0.3606	0.3962	0.4057
	(7)	(2)	(4)	(1)	(3)	(5)	(6)
Chemicals	0.2177	0.1670	0.1789	0.1424	0.1443	0.2198	0.2184
	(5)	(3)	(4)	(1)	(2)	(7)	(6)
Construction	0.1500	0.1317	0.1297	0.1312	0.1462	0.1443	0.1441
	(7)	(3)	(1)	(2)	(6)	(5)	(4)
Financials	0.0878	0.0959	0.0886	0.0894	0.0817	0.0879	0.0826
	(3)	(7)	(5)	(6)	(1)	(4)	(2)
Food	0.1384	0.1471	0.1457	0.1374	FTC[a]	0.1952	0.1476
	(2)	(4)	(3)	(1)	(–)	(6)	(5)
Healthcare	0.1864	0.2179	0.2151	0.1838	0.1815	0.1952	0.1780
	(4)	(7)	(6)	(3)	(2)	(5)	(1)
Industrials	0.1095	0.1026	0.0992	0.1027	0.1082	0.1124	0.1114
	(5)	(2)	(1)	(3)	(4)	(7)	(6)
Insurance	0.2695	0.1996	0.2134	0.2495	0.2362	0.2737	0.2622
	(6)	(1)	(2)	(4)	(3)	(7)	(5)
Media	0.1499	0.1740	0.1788	0.1564	0.1535	0.1626	0.1845
	(1)	(5)	(6)	(3)	(2)	(4)	(7)
Oil & Gas	0.2510	0.2665	0.2646	0.2528	0.2549	0.2613	0.2641
	(1)	(7)	(6)	(2)	(3)	(4)	(5)
Personal	0.0715	0.0737	0.0745	0.0696	0.0686	0.0702	0.0722
	(4)	(6)	(7)	(2)	(1)	(3)	(5)
Retail	0.2237	0.2623	0.2527	0.2356	0.2360	0.2329	0.2369
	(1)	(7)	(6)	(3)	(4)	(2)	(5)
Technology	0.5790	0.5409	0.5428	0.5822	0.5735	0.6448	0.6322
	(4)	(1)	(2)	(5)	(3)	(7)	(6)
Telecom	0.2316	0.2045	0.1993	0.1905	0.1895	0.2274	0.2213
	(7)	(4)	(3)	(2)	(1)	(6)	(5)
Travel	0.0944	0.0976	0.0986	0.0911	0.0938	0.0977	0.1014
	(3)	(4)	(6)	(1)	(2)	(5)	(7)
Utilities	0.1127	0.1103	0.1087	0.1194	0.1192	0.1269	0.1150
	(3)	(2)	(1)	(6)	(5)	(7)	(4)
Average MSE	0.2016	0.1942	0.1957	0.1912	0.1943	0.2103	0.2069
Average Rank	3.94	4.11	4.11	3.11	2.94	4.89	4.67

[a]Failed to converge.

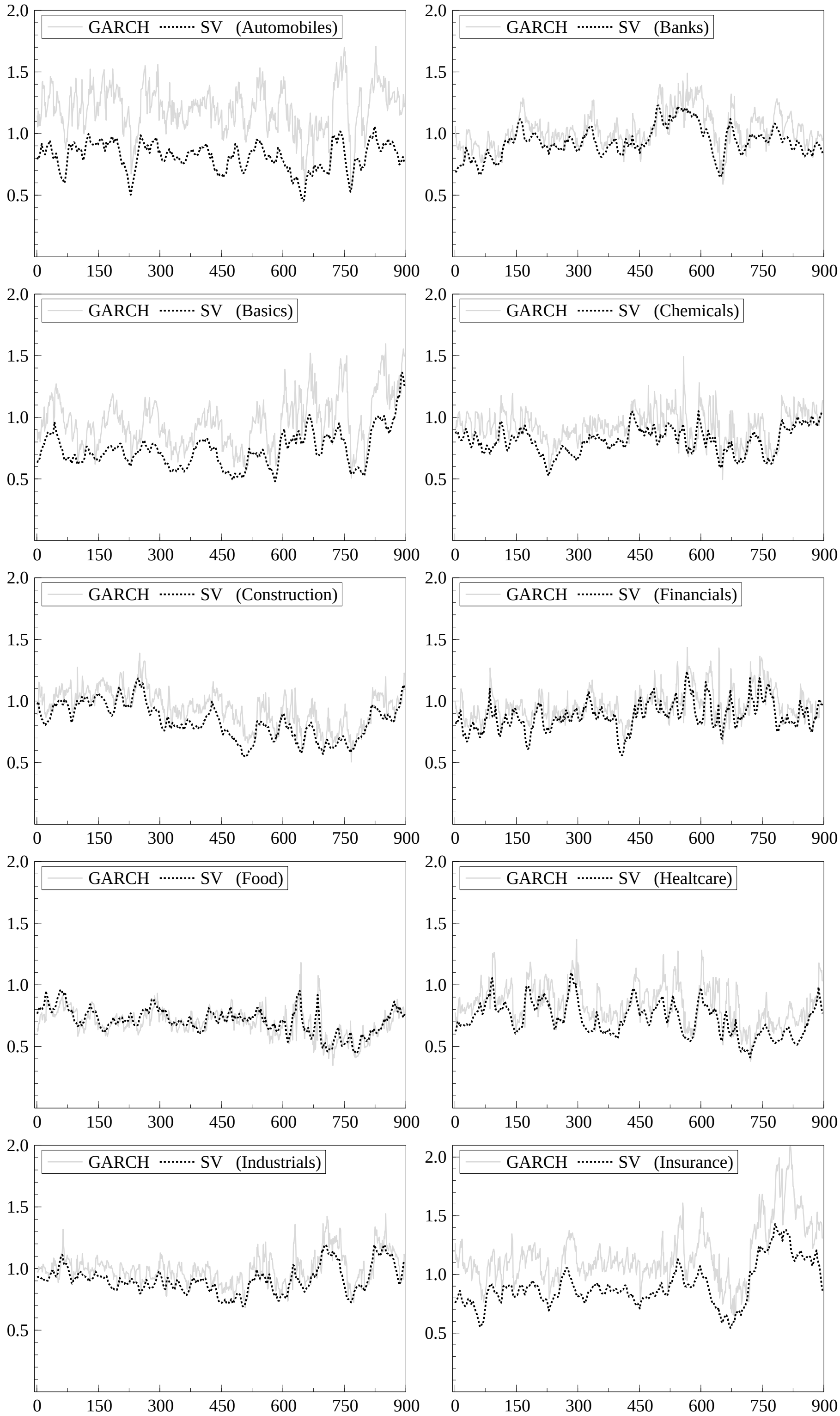


Figure 4: t–GARCH and SV conditional betas (for $i \leq 10$)

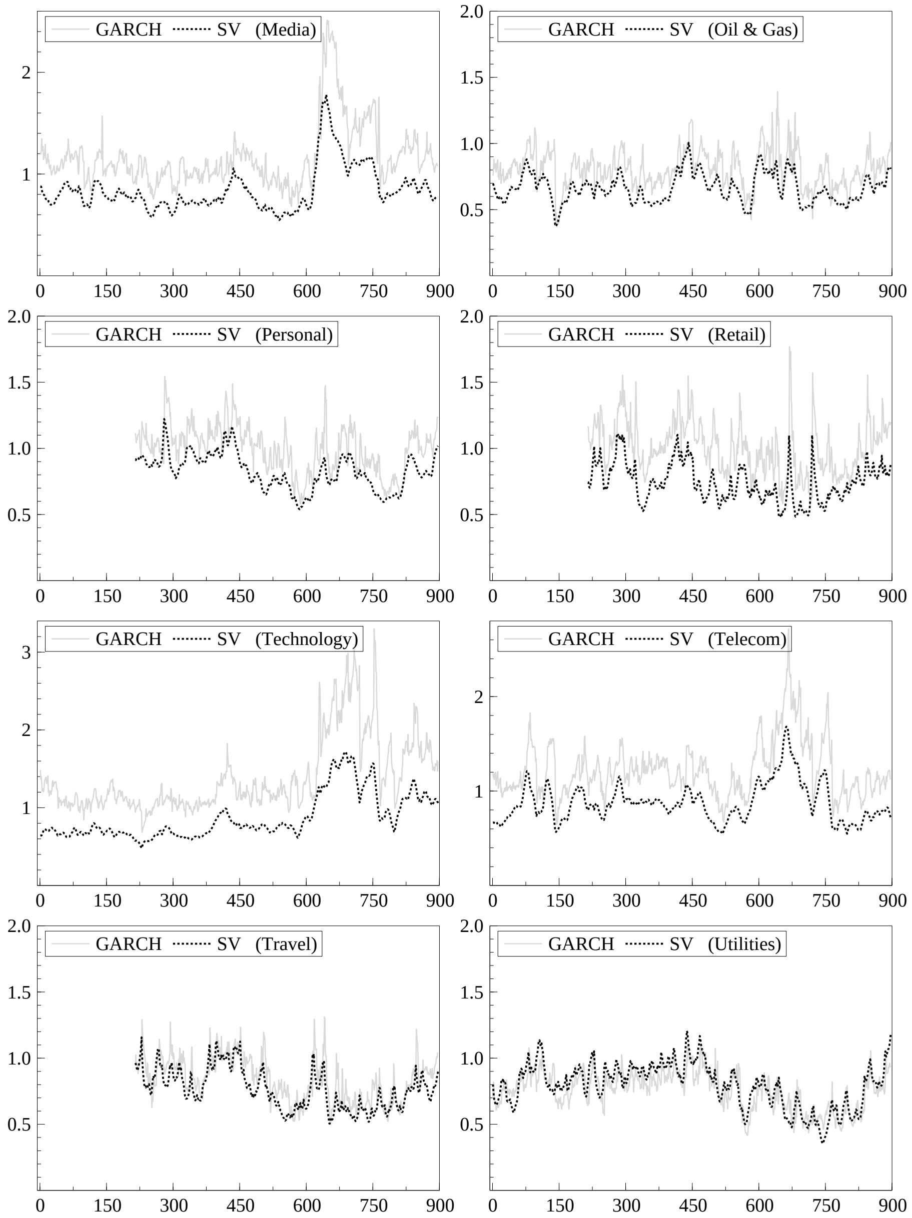


Figure 5: t–GARCH and SV conditional betas (for $i > 10$)

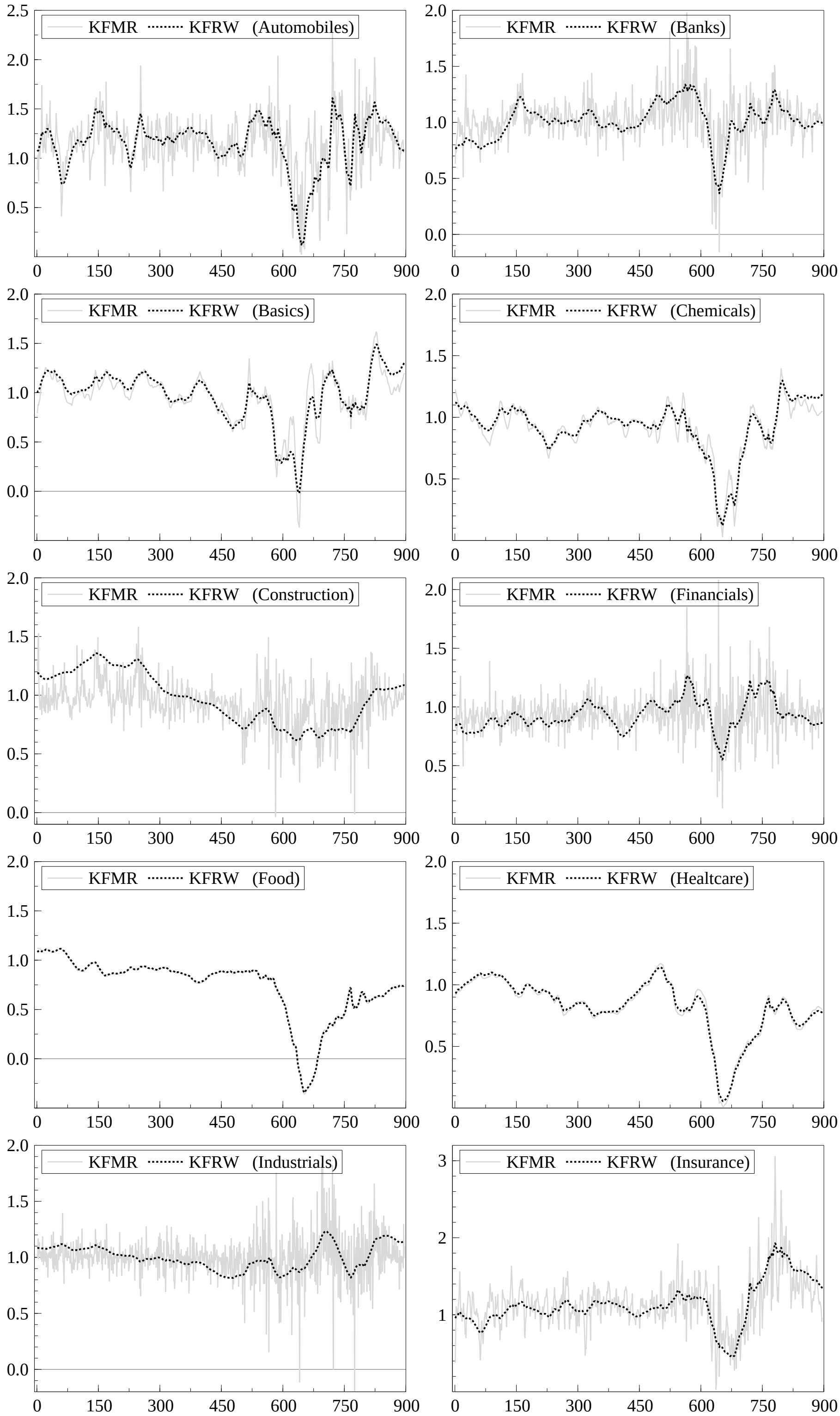


Figure 6: Kalman filter conditional betas (for $i \leq 10$)

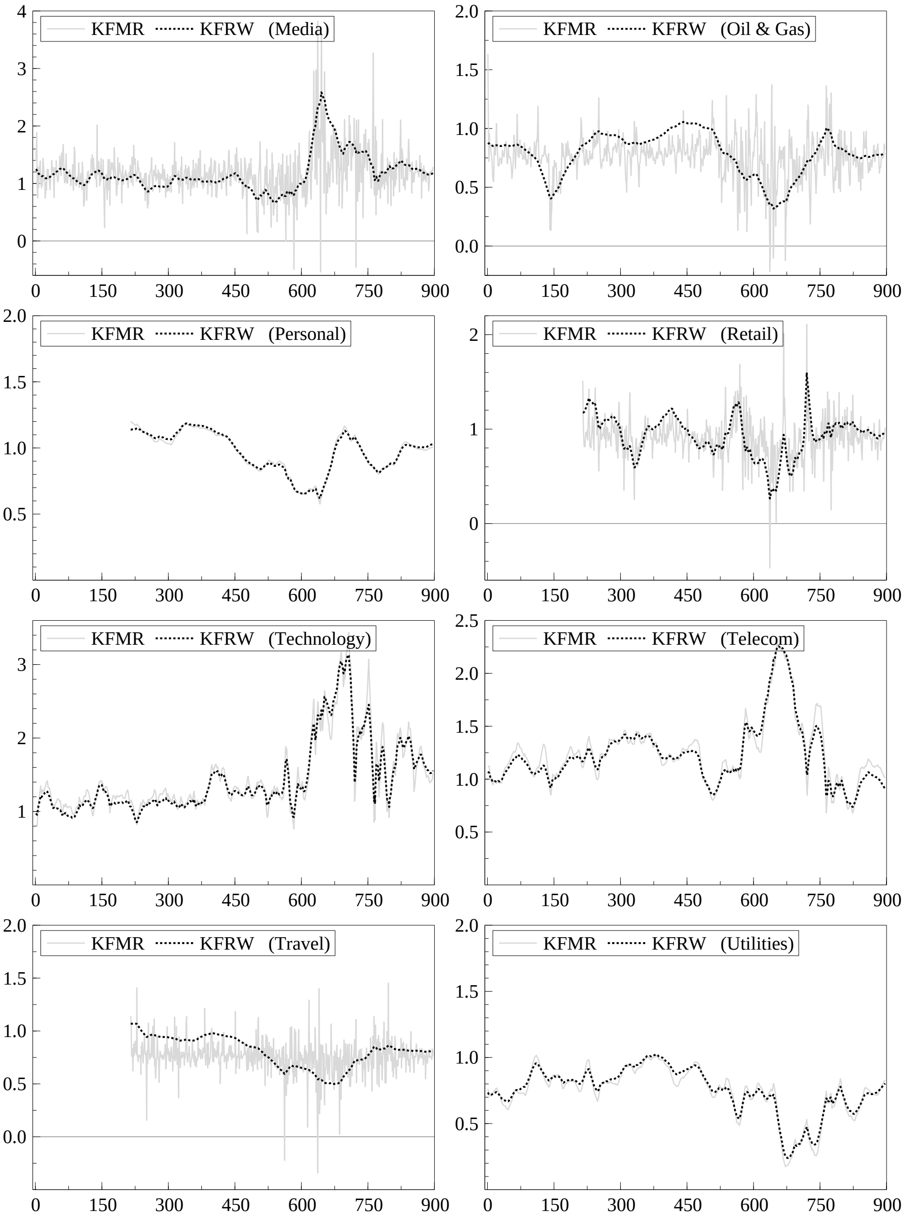


Figure 7: Kalman filter conditional betas (for $i > 10$)

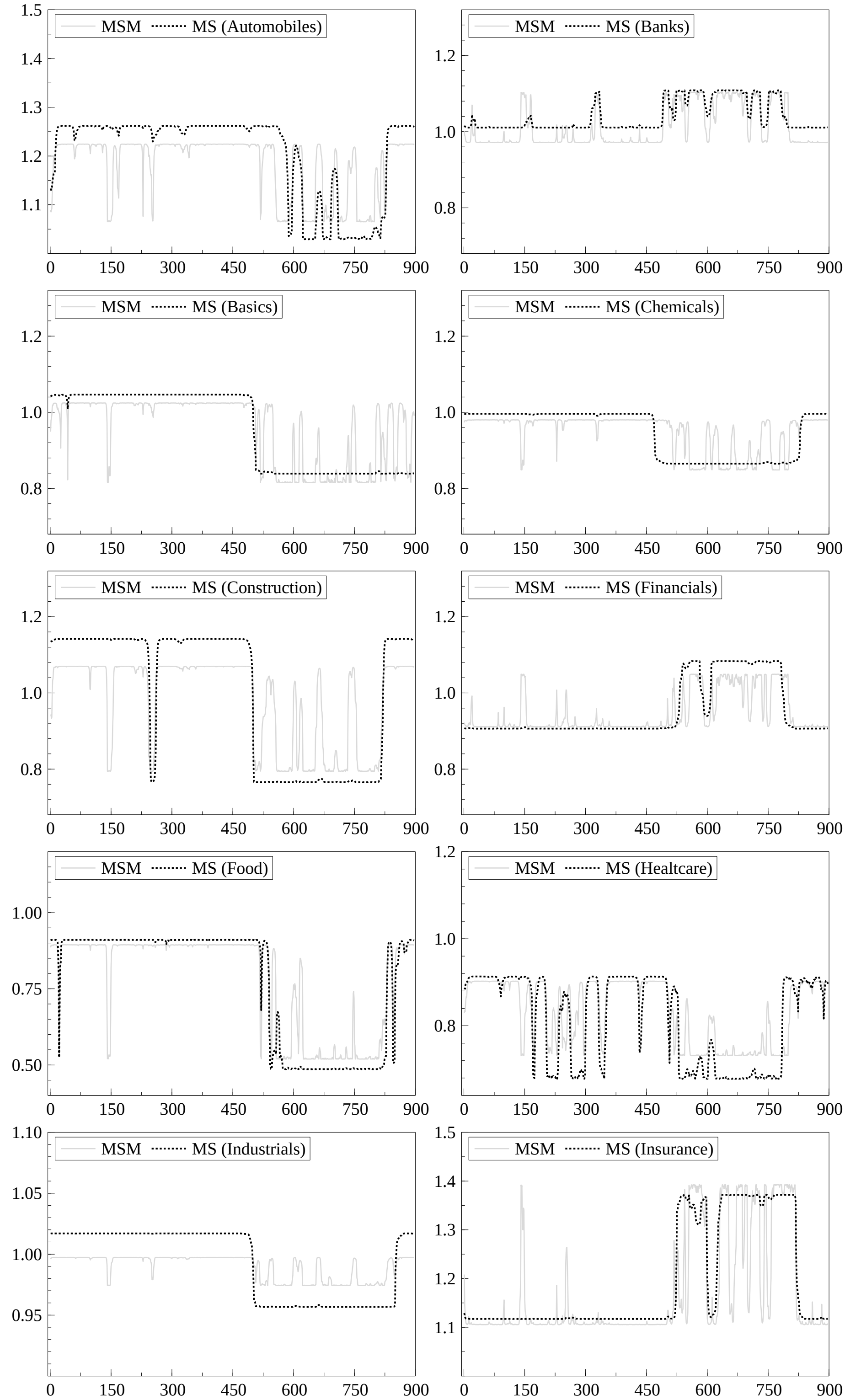


Figure 8: Markov switching conditional betas (for $i \leq 10$)

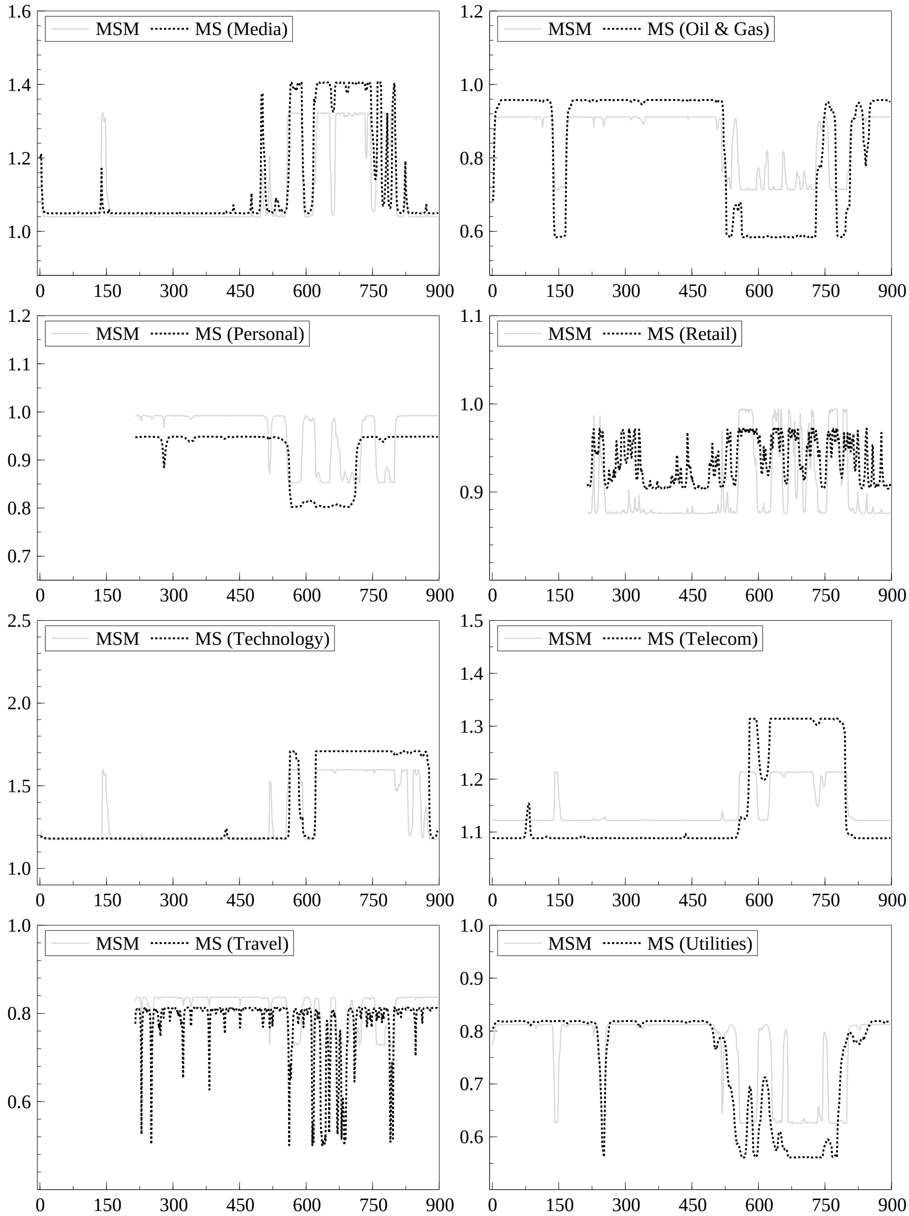


Figure 9: Markov switching conditional betas (for $i > 10$)

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