- When $\prod$ is irreducible, this implies that $\exists$ an $n > 0$ s.t. for each $\prod^n$ has an element $\prod_{i,j} > 0$. For any i and any j.

- $\prod_{m\times m}$ is idempotent $\Rightarrow \prod^2 = \prod$

$$\begin{aligned}
\prod \cdot \prod{}^2 &= \prod \cdot \prod \\
\prod{}^3 &= \prod{}^2 \\
&\vdots \\
\prod{}^n &= \prod{}^{n-1} \\
&= \prod
\end{aligned} \tag{1}$$

Therefore $\prod_{i,j} > 0$ for $n = 1$, $\prod$ is <u>aperiodic</u>.

- Since $\prod$ is idempotent, consider the j^{th} column of two rows, m and n.

$$\begin{aligned}
\prod_{m,j} &= \sum_{i=1}^{N} \prod_{m,i} \cdot \prod_{i,j} \\
1 &= \sum_{i=1}^{N} \frac{\prod_{m,i} \cdot \prod_{i,j}}{\prod_{m,j}} \\
1 &= \frac{\prod_{m,1} \cdot \prod_{1,j} + \prod_{m,2} \cdot \prod_{2,j} + \cdots + \prod_{m,j} \cdot \prod_{j,j} + \cdots + \prod_{m,N} \cdot \prod_{N,j}}{\prod_{m,j}}
\end{aligned}$$

$$\begin{aligned}
\prod_{n,j} &= \sum_{i=1}^{N} \prod_{n,i} \cdot \prod_{i,j} \\
1 &= \sum_{i=1}^{N} \frac{\prod_{n,i} \cdot \prod_{i,j}}{\prod_{n,j}} \\
1 &= \frac{\prod_{n,1} \cdot \prod_{1,j} + \prod_{n,2} \cdot \prod_{2,j} + \cdots + \prod_{n,j} \cdot \prod_{j,j} + \cdots + \prod_{n,N} \cdot \prod_{N,j}}{\prod_{n,j}}
\end{aligned}$$

From above

$$\sum_{i=1}^{N} \frac{\prod_{m,i} \cdot \prod_{i,j}}{\prod_{m,j}} = 1 - \prod_{j,j}$$
$$\sum_{i=1}^{N} \frac{\prod_{n,i} \cdot \prod_{i,j}}{\prod_{n,j}} = 1 - \prod_{j,j}$$

Therefore $\prod_{m,j} = \prod_{n,j}$, thus all rows of the $\prod$ are identical