Homework 3 for STA 5166 (Assigned, Oct. 8)

Statistics in Applications I

Due: Oct. 17, 2007 (Wednesday)

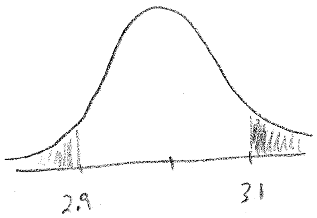
1: BHH Ch.2; Problems 10, 12, 13(a,b,c,d); Pages 62-65. (40)

2: BHH Ch.3; Problem 2 (Pages 124-125). Submit both your summary results and R/Splus program for the problem. (20)

3: BHH Ch.3; Problems 4, 7, and 13 (Pages 125-128). For each of the three problems, perform a t-test on the difference of the two means and perform a test based on a randomization distribution (use R/Splus to generate 10000 samples and plot the histogram of the differences). Submit both your summary results and R/Splus program for each of the problems. (40)

10

a) $X \sim N(3.06, 0.03)$



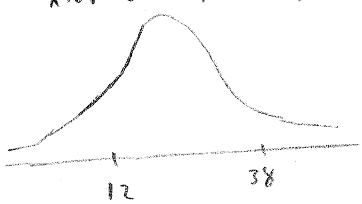
$P(x \leq 2.9) + P(x \geq 3.1)$

$0 + 0.0912 = 9.12\%$

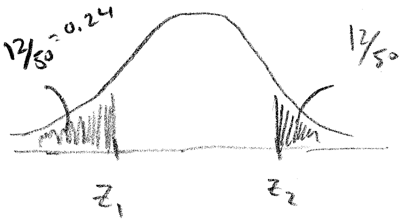
if $n = 397$ then $n_1 = 0$

$n_2 \approx 36.6 = 37 = (397)(9.12\%)$

b) $X \sim N(3.06, 0.03)$



$Z \sim N(0,1)$



$P(x \leq 2.9) = P\left(Z \leq \frac{2.9-\mu}{\sigma}\right) = \frac{12}{50} \rightarrow -0.7063 = \frac{2.9-\mu}{\sigma}$

$P(x \geq 3.1) = P\left(Z \leq \frac{3.1-\mu}{\sigma}\right) = \frac{12}{50} \rightarrow 0.7063 = \frac{3.1-\mu}{\sigma}$

$3.1 - 0.7063\sigma = \mu = 2.9 + 0.7063\sigma$

$\sigma = 0.1415$

$\mu = 2.9 + 0.7063(0.1415)$

$\mu = 2.9 \approx 3$

Assumed that randomly selected bolts which lengths are iid, normally distribution.

(12) $X \sim N(1000, 10)$

a) $P(x < 985) = P\left(z < \frac{985-1000}{10}\right) = 0.066807 = 6.68\%$

b) $P(x > 1020) = P\left(z > \frac{1020-1000}{10}\right) = 0.0227 = 2.27\%$

c) $P(985 < x < 1020) = P\left(\frac{985-1000}{10} < z < \frac{1020-1000}{10}\right)$

$= 0.9104 = 91.07$

(13) $P(1 \text{ or } 2) = p$

x	f	P(x)
0	0	0
1	4	4/64
2	19	19/64
3	15	15/64
4	17	17/64
5	5	5/64
6	2	2/64

theoretical probability of success 1 die

a) $P(1 \text{ or } 2) = P(1) + P(2) = 1/6 + 1/6 = 1/3$

Dice	1	0
P	$\frac{1}{3}$	$\frac{2}{3}$

$E(x) = p = 1/3$

$Var(x) = pq = (1/3)(2/3) = 2/9$

13 d) $X \sim Bin(1/3, 6)$

$$P(X = x_i) = \binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}$$

$$P(X=0) = \binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6 = 0.088$$

$$P(X=1) = \binom{6}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 = 0.263$$

$$P(X=2) = \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 = 0.329$$

$P(X=3) = 0.219$

$P(X=4) = 0.082$

$P(X=5) = 0.016$

$P(X=6) = 0.001$

X	0	1	2	3	4	5	6
expected freq	5.632	16.832	21.056	14.016	5.248	1.024	0.064

↑ $64(P(X=0))$ ↖ $64(P(X=1))$

(13)

for one die

b) $E(x) = 1\left(\frac{4}{64}\right) + 2\left(\frac{19}{64}\right) + 3\left(\frac{15}{64}\right) + 4\left(\frac{17}{64}\right) + 5\left(\frac{7}{64}\right) + 6\left(\frac{2}{64}\right)$

$= 3.15625$

Since 6 die

$E(x) = \frac{3.15625}{6} = 0.526$

$s^2 = Var(x) = E(x^2) - [E(x)]^2 =$

$= 1^2\left(\frac{4}{64}\right) + 2^2\left(\frac{19}{64}\right) + \ldots + (6)^2\left(\frac{2}{64}\right) - [0.526]^2$

$= 0.24937$

c) Testing hypothesis of mean

$H_0: p = p_0 = 1/3$

$H_a: p > p_0 = 1/3$

$z = \frac{0.526 - 1/3}{\sqrt{\frac{2/9}{64 \cdot 6}}} = 8.009 > z_{0.05}$

$P(z > 8.009) =$ p-value < 0.01

Reject Null H_0

accept $p > p_0$

mean and variance

don't have theoritical values

for Bernoulli

Testing variance

$H_0: \sigma^2 = \sigma_0^2$ $\quad p_0 q_0 = \frac{2}{9}$

$H_a: \sigma^2 \neq \sigma_0^2$ $\quad s^2 = 0.2493$

$\chi^2 = \frac{(n-1)s^2}{p_0 q_0}$

$= \frac{(63)(0.2493)}{2/9}$

$= 70.6 \sim \chi^2_{64}$

$P(|\chi^2_{64}| > 70.6)$

Reject H_0

Jaime Frade
STA5166

Chapter 3.2 (三。二)

Summary:
Will try to test the hypothesis to see if there exists a significant difference between the mean values of levels of asbestos fiber in the air of the industrial plant with and without S-142 chemical. From the comparative trail in the plant, the four consecutive readings had a mean difference of -3.5. The null hypothesis is that with or without S-142, the asbestos levels will not change, the alternative is that with S-142, the level will decrease since the mean difference is negative. To test this, used as a reference the past observations of asbestos levels without S-142. From the dataset, obtained a probability that 1/109 (=0.0091743119) that there exists a mean difference less that the comparative trail. Since this probability is less that 5%, we reject the null hypothesis and accept the salesman claim that S-142 is beneficial to reduce the level of asbestos levels in the air of the industrial plant.

Jaime Frade
STA5166

CODE

```
data=scan("C:/Documents and Settings/Jaime/Desktop/FALL07/STA5166/BHH2-Data/datahw3.dat")
data
n1=0
Mean1wout = mean(c(8,6))
Mean2with = mean(c(3,4))
diff_means = Mean2with-Mean1wout
y = c(rep(NA, (109)))
x = c(rep(NA, (109)))
for(i in 1:111){ y[i] = (data[i]+data[i+1])/2}
for(j in 1:109){ x[j] = y[j+2] - y[j];
if(x[j]<= diff_means) n1=n1+1}
x
sort(x)
n1
diff_means
n1/109
```

OUTPUT

```
> data=scan("C:/Documents and Settings/Jaime/Desktop/FALL07/STA5166/BHH2-Data/datahw3.dat")
Read 112 items
> data
  [1]  9 10  8  9  8  8  8  8  7  6  9 10 11  9 10 11 11 11 11 10 11 12 13 12 13 12
 [26] 14 15 14 12 13 13 12 13 13 13 13 13 10  8  9  8  6  7  7  6  5  6  5  6  4
 [51]  5  4  4  2  4  5  4  5  6  5  5  6  5  6  7  8  8  8  7  9 10  9 10  9  8
 [76]  9  8  7  7  8  7  7  7  8  8  8  8  7  6  5  6  5  6  7  6  6  5  6  6  5
[101]  4  3  4  4  5  5  6  5  6  7  6  5
> n1=0
> Mean1wout = mean(c(8,6))
> Mean2with = mean(c(3,4))
> diff_means = Mean2with-Mean1wout
> y = c(rep(NA, (109)))
> x = c(rep(NA, (109)))
> for(i in 1:111){ y[i] = (data[i]+data[i+1])/2}
> for(j in 1:109){ x[j] = y[j+2] - y[j];
+ if(x[j]<= diff_means) n1=n1+1}
> x
  [1] -1.0 -0.5 -0.5 -0.5 -0.5 -1.5  0.0  3.0  3.0  0.5 -1.0  0.5  1.5  0.5  0.0
 [16] -0.5 -0.5  1.0  2.0  1.0  0.0  0.0  0.5  2.0  1.5 -1.5 -2.0  0.0  0.0 -0.5
 [31]  0.5  0.5  0.0  0.0 -1.5 -4.0 -3.0 -0.5 -1.5 -2.0  0.0  0.0 -1.5 -1.0  0.0
 [46]  0.0 -0.5 -1.0 -0.5 -0.5 -1.5 -1.0  1.5  1.5  0.0  1.0  1.0 -0.5  0.0  0.5
 [61]  0.0  1.0  2.0  1.5  0.5 -0.5  0.0  2.0  1.5  0.0  0.0 -1.0 -1.0  0.0 -1.0
 [76] -1.5  0.0  0.5 -0.5 -0.5  0.5  1.0  0.5  0.0 -0.5 -1.5 -2.0 -1.0  0.0  0.0
```

```
 [91]  1.0  1.0 -0.5 -1.0 -0.5  0.5  0.0 -1.5 -2.0 -1.0  0.5  1.0  1.0  1.0  0.5
[106]  0.0  1.0  1.0 -1.0
> x
  [1] -1.0 -0.5 -0.5 -0.5 -0.5 -1.5  0.0  3.0  3.0  0.5 -1.0  0.5  1.5  0.5  0.0
 [16] -0.5 -0.5  1.0  2.0  1.0  0.0  0.0  0.5  2.0  1.5 -1.5 -2.0  0.0  0.0 -0.5
 [31]  0.5  0.5  0.0  0.0 -1.5 -4.0 -3.0 -0.5 -1.5 -2.0  0.0  0.0 -1.5 -1.0  0.0
 [46]  0.0 -0.5 -1.0 -0.5 -0.5 -1.5 -1.0  1.5  1.5  0.0  1.0  1.0 -0.5  0.0  0.5
 [61]  0.0  1.0  2.0  1.5  0.5 -0.5  0.0  2.0  1.5  0.0  0.0 -1.0 -1.0  0.0 -1.0
 [76] -1.5  0.0  0.5 -0.5 -0.5  0.5  1.0  0.5  0.0 -0.5 -1.5 -2.0 -1.0  0.0  0.0
 [91]  1.0  1.0 -0.5 -1.0 -0.5  0.5  0.0 -1.5 -2.0 -1.0  0.5  1.0  1.0  1.0  0.5
[106]  0.0  1.0  1.0 -1.0
> sort(x)
  [1] -4.0 -3.0 -2.0 -2.0 -2.0 -2.0 -1.5 -1.5 -1.5 -1.5 -1.5 -1.5 -1.5 -1.5 -1.5
 [16] -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -1.0 -0.5 -0.5 -0.5
 [31] -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5 -0.5
 [46]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 [61]  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.5  0.5  0.5  0.5  0.5
 [76]  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  0.5  1.0  1.0  1.0  1.0  1.0  1.0
 [91]  1.0  1.0  1.0  1.0  1.0  1.0  1.0  1.5  1.5  1.5  1.5  1.5  1.5  2.0  2.0
[106]  2.0  2.0  3.0  3.0
> n1
[1] 1
> diff_means
[1] -3.5
> n1/109
[1] 0.009174312
```

Jaime Frade
STA5166

Chapter 3.4 (三。四)

Assumptions:
Ratings are both approximately normal distributed. Two samples, A and B, are independent. Ratings in each brand are i.i.d.

Summary:
To test the hypothesis that $\eta_A = \eta_B$, against the $\eta_A \neq \eta_B$. I used a t-test to check if the difference in means is not equal to zero. The p-value obtained 0.3316. Therefore there is not sufficient evidence to reject the null hypothesis. The assumptions were needed to conduct test.

Using a randomization distribution, I also tested the above hypothesis. Here, I did not make assumptions about the distributions of the ratings. The mean of brand A: (3.875) and brand B: (5.285714), to obtain a difference of 1.4107.

There exist 6435 possible permutations of 8 ratings of brand A and 7 ratings of brand B. Assuming that the null hypothesis, then there exist no difference in the ratings of brand A and brand B. Can arrange and for each calculate the differences that are less than 1.4107. Count the number of occurrences and this will lead to a calculation of the p-value.

The p-value obtained after a large number of observations should be approximately equal to the p-value obtained from the t-test above. I obtained the p-value: 0.3551. This also leads to the conclusion that one cannot reject the null hypothesis.

Jaime Frade
STA5166

CODE

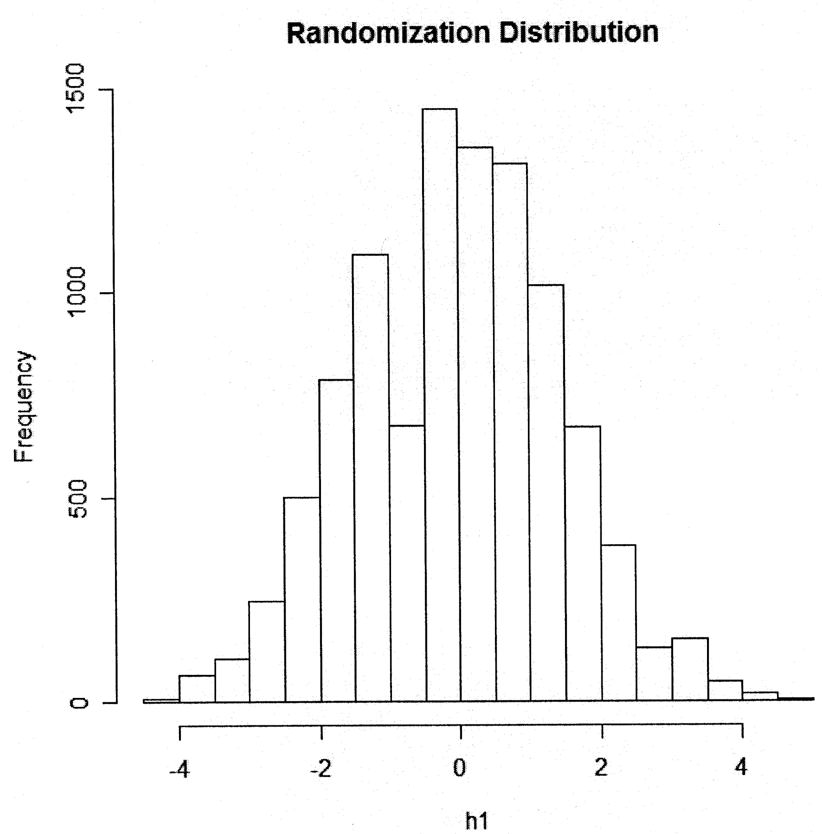
```
brandA = c(2,4,2,1,9,9,2,2)
brandB = c(8,3,5,3,7,7,4)

y = t.test(brandA, brandB)
y
n1=0
h1=0
y1= c(2,4,2,1,9,9,2,2,8,3,5,3,7,7,4)
c1=c(rep("A", 8), rep("B", 7))
d1 = c(rep(0,10000))
diff= 5.285714-3.875
for(i in 1:10000){
c2=sample(c1);
x1=y1[c2=="A"];
x2=y1[c2=="B"];
m1 = mean(x1);
m2 = mean(x2);
d1[i] = m2-m1;
h1=c(h1,d1[i]);
if(abs(d1[i]) >= 1.4107)n1=n1+1
}
n1
hist(h1, main="Randomization Distribution")
pvalue= n1/10000
pvalue
```

OUTPUT

```
> brandA = c(2,4,2,1,9,9,2,2)
> brandB = c(8,3,5,3,7,7,4)
>
> y = t.test(brandA, brandB)
> y

        Welch Two Sample t-test

data:  brandA and brandB
t = -1.0122, df = 11.923, p-value = 0.3316
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -4.449587  1.628159
sample estimates:
mean of x mean of y
 3.875000  5.285714

> n1=0
```

Jaime Frade
STA5166

```
> h1=0
> y1= c(2,4,2,1,9,9,2,2,8,3,5,3,7,7,4)
> c1=c(rep("A", 8), rep("B", 7))
> d1 = c(rep(0,10000))
> diff= 5.285714-3.875
> for(i in 1:10000){
+ c2=sample(c1);
+ x1=y1[c2=="A"];
+ x2=y1[c2=="B"];
+ m1 = mean(x1);
+ m2 = mean(x2);
+ d1[i] = m2-m1;
+ h1=c(h1,d1[i]);
+ if(abs(d1[i]) >= 1.4107)n1=n1+1
+ }
> n1
[1] 3551
> hist(h1, main="Randomization Distribution")
> pvalue= n1/10000
> pvalue
[1] 0.3551
```



Jaime Frade
STA5166

Chapter 3.7 (三。七)

Assumptions:
Results are both approximately normal distributed. Two samples, designs A and B, are independent. Results in each design are i.i.d.

Summary:
Will try to test the hypothesis to see if there exists a significant difference between the mean values for the power attainable for the two designs. The null hypothesis assumes there is no difference in the mean values. I used a t-test to check if the difference in means is not equal to zero. The p-value obtained 0.4343. Therefore there is not sufficient evidence to reject the null hypothesis. The assumptions were needed to conduct test.

Using a randomization distribution, I also tested the above hypothesis. Here, I did not make assumptions about the distributions of the ratings. The mean of design A: (1.55) and brand B: (1.75), to obtain a difference of 0.2

The p-value obtained after a large number of observations should be approximately equal to the p-value obtained from the t-test above. I obtained the p-value: 0.4454. This also leads to the conclusion that one cannot reject the null hypothesis.

Jaime Frade
STA5166

CODE

```
designA = c(1.8, 1.9, 1.1, 1.4)
designB = c(1.9, 2.1, 1.5, 1.5)
y = t.test(designA , designB)
y
n1=0
h1=NULL
y1= c(1.8, 1.9, 1.1, 1.4, 1.9, 2.1, 1.5, 1.5)
c1=c(rep("A", 4), rep("B", 4))
diff=1.75-1.55
d1 = rep(0, 10000)
for(i in 1:10000){
c2=sample(c1);
x1=y1[c2=="A"]; x2=y1[c2=="B"]
m1 = mean(x1); m2 = mean(x2);
d1[i] = m2-m1;
h1=c(h1,d1[i]);
if(abs(d1[i]) >= diff)n1=n1+1
}
n1
hist(h1, main="Randomization Distribution")
pvalue= n1/10000
pvalue
```

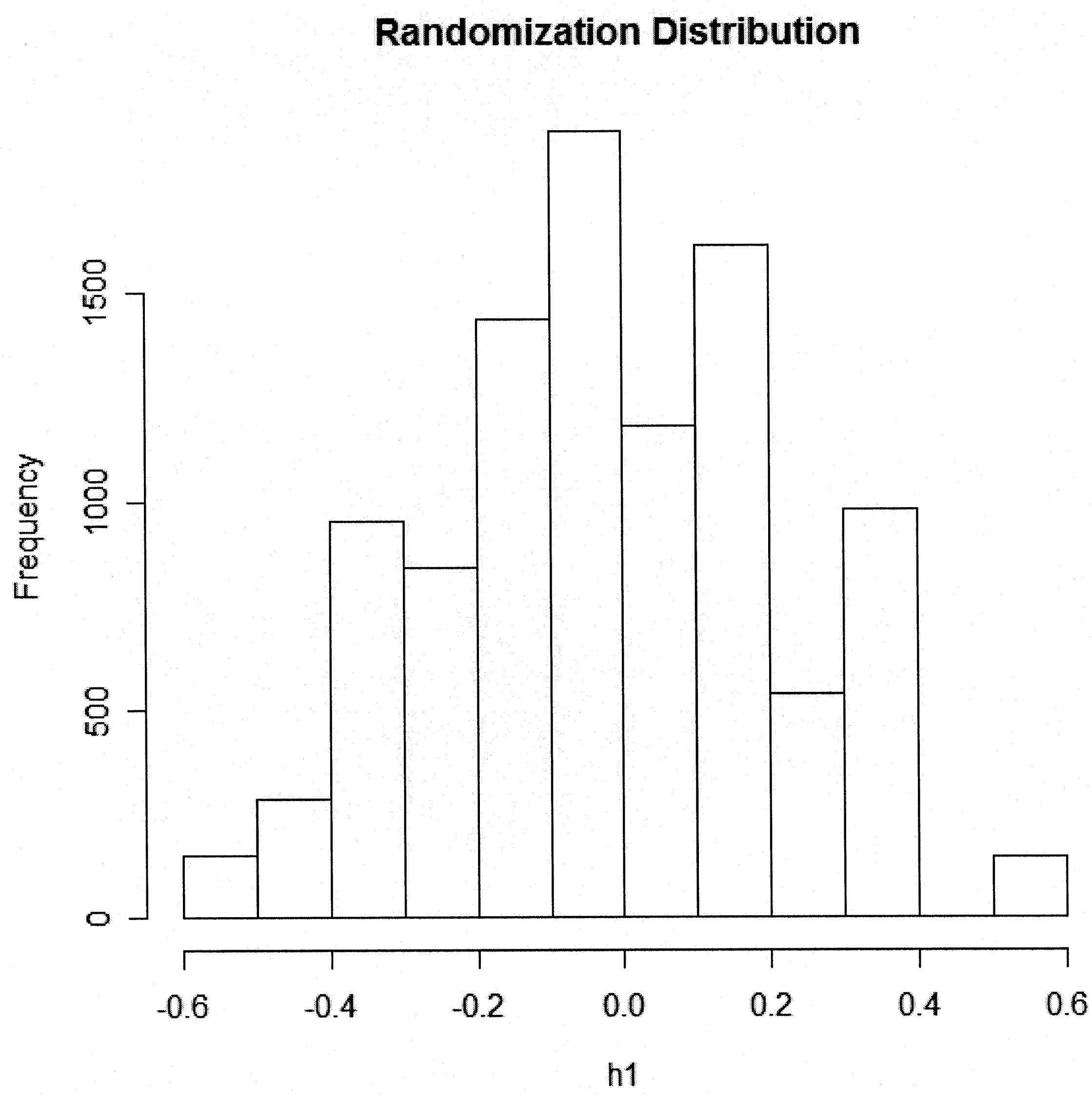
OUTPUT

```
> designA = c(1.8, 1.9, 1.1, 1.4)
> designB = c(1.9, 2.1, 1.5, 1.5)
>
> y = t.test(designA , designB)
> y

        Welch Two Sample t-test

data:  designA and designB
t = -0.8402, df = 5.756, p-value = 0.4343
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.7885191  0.3885191
sample estimates:
mean of x mean of y
     1.55      1.75

> n1=0
> h1=NULL
> y1= c(1.8, 1.9, 1.1, 1.4, 1.9, 2.1, 1.5, 1.5)
> c1=c(rep("A", 4), rep("B", 4))
> diff=1.75-1.55
> d1 = rep(0, 10000)
> for(i in 1:10000){
```

Jaime Frade
STA5166

```
+ c2=sample(c1);
+ x1=y1[c2=="A"]; x2=y1[c2=="B"]
+ m1 = mean(x1); m2 = mean(x2);
+ d1[i] = m2-m1;
+ h1=c(h1,d1[i]);
+ if(abs(d1[i]) >= diff)n1=n1+1
+ }
> n1
[1] 4454
> hist(h1, main="Randomization Distribution")
> pvalue= n1/10000
> pvalue
[1] 0.4454
```

==============================



Jaime Frade
STA5166

hapter 3.13 (三。十三)

Assumptions:
Results of production from each diet are both approximately normal distributed. Two samples, designs A and B, are independent. Results in each diet are i.i.d.

Summary:
Will try to test the hypothesis to see if there exists a significant difference between the mean values for the power attainable for the two designs. The null hypothesis assumes there is no difference in the mean values. I used a t-test to check if the difference in means is not equal to zero. The p-value obtained 0.07842. Therefore there is not sufficient evidence to reject the null hypothesis. The assumptions were needed to conduct test.

Using a randomization distribution, I also tested the above hypothesis. Here, I did not make assumptions about the distributions of the ratings. The mean of diet A: (166.5) and brand B: (156.6667), to obtain a absolute value of the difference of 9.83

The p-value obtained after a large number of observations should be approximately equal to the p-value obtained from the t-test above. I obtained the p-value: 0.0913. This also leads to the conclusion that one cannot reject the null hypothesis.

A 95% confidence interval for the mean difference: [-9.399669, 29.05967]

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t_c s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Here, the 95% confidence interval for the difference in mean hen production between diet A and diet B numbers above. Thus, not only do we estimate the difference to be 9.83 mg/dl, but we are 95% confident it is no less than lower bound or greater than upper bound.

Jaime Frade
STA5166

CODE

```
dietA = c(166,174,150,166,165,178)
dietB = c(158,159,142,163,161,157)

y = t.test(dietA , dietB)
y
n1=0
h1=NULL
y1= c(166,174,150,166,165,178, 158,159,142,163,161,157)
c1=c(rep("A", 6), rep("B", 6))
diff=156.6667-166.5
for(i in 1:10000){
c2=sample(c1)
x1=y1[c2=="A"]; x2=y1[c2=="B"]
m1 = mean(x1); m2 = mean(x2)
d1 = m2-m1
h1=c(h1,d1)
if(d1 <= diff)n1=n1+1
}
n1
hist(h1, main="Randomization Distribution")
pvalue= n1/10000
pvalue

9.83-qt(0.975,10)* sqrt((5*var(dietA)+5*var(dietB))/10)
9.83+qt(0.975,10)* sqrt((5*var(dietA)+5*var(dietB))/10)
```

OUTPUT

```
> dietA = c(166,174,150,166,165,178)
> dietB = c(158,159,142,163,161,157)
>
> y = t.test(dietA , dietB)
> y

        Welch Two Sample t-test

data:  dietA and dietB
t = 1.9735, df = 9.436, p-value = 0.07842
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -1.359600 21.026267
sample estimates:
mean of x mean of y
 166.5000  156.6667
```

Jaime Frade
STA5166

```
> n1=0
> h1=NULL
> y1= c(166,174,150,166,165,178, 158,159,142,163,161,157)
> c1=c(rep("A", 6), rep("B", 6))
> diff=166.5 - 156.6667
> d1 = rep(0,10000)
> for(i in 1:10000){
+ c2=sample(c1);
+ x1=y1[c2=="A"];
+ x2=y1[c2=="B"];
+ m1 = mean(x1);
+ m2 = mean(x2);
+ d1[i] = m2-m1;
+ h1=c(h1,d1[i]);
+ if(abs(d1[i])>=9.83)n1=n1+1
+ }
> n1
[1] 913
> hist(h1, main="Randomization Distribution")
> pvalue= n1/10000
> pvalue
[1] 0.0913
> 9.83-qt(0.975,10)* sqrt((5*var(dietA)+5*var(dietB))/10)
[1] -9.399669
> 9.83+qt(0.975,10)* sqrt((5*var(dietA)+5*var(dietB))/10)
[1] 29.05967
```

Jaime Frade
STA5166