SCIPM

Computes simultaneous confidence intervals on all pairwise differences of means.

Required Arguments

NI — Vector of length `NGROUP` containing the number of observations in each mean. (Input)

YMEANS — Vector of length `NGROUP` containing the means. (Input)

DFS2 — Degrees of freedom for s^2. (Input)

S2 — s^2, the estimated variance of an observation. (Input)
The variance of `YMEANS(I)` is estimated by `S2`/`NI(I)`.

STAT — `NGROUP * (NGROUP –`
1)/2 by 5 matrix containing the statistics relating to the difference of means. (Output)

Col.	Description
1	Group number for the *i*-th mean
2	Group number for the *j*-th mean
3	Difference of means (*i*-th mean) (*j*-th mean)
4	Lower confidence limit for the difference
5	Upper confidence limit for the difference

Optional Arguments

NGROUP — Number of means. (Input)
Default: `NGROUP` = size (`NI`,1).

IMETH — Method used for constructing confidence intervals on all pairwise differences of means. (Input)
Default: `IMETH` = 0.

`IMETH`	Method
0	Tukey (if equal group sizes), Tukey-Kramer method (otherwise)
1	Dunn-Sidak method
2	Bonferroni method
3	Scheffe method
4	One-at-a-time *t* method−*LSD* test

CONPER — Confidence percentage for the simultaneous interval estimation. (Input)
Default: `CONPER` = 95.0.

`IMETH`	`CONPER`
0	Percentage must be greater than or equal to 90.0 and less than or equal to 99.0.
≥ 1	Percentage must be greater than or equal to 0.0 and less than 100.0.

IPRINT — Printing option. (Input)
Default: `IPRINT` = 0.

`IPRINT`	Action
0	No printing is performed.
1	Printing is performed.

LDSTAT — Leading dimension of `STAT`
exactly as specified in the dimension statement in the calling program. (Input)

Default: `LDSTAT` = size (`STAT`,1).

FORTRAN 90 Interface

Generic: `CALL SCIPM (NI, YMEANS, DFS2, S2, STAT [,…])`

Specific: The specific interface names are `S_SCIPM` and `D_SCIPM`.

FORTRAN 77 Interface

Single: `CALL SCIPM (NGROUP, NI, YMEANS, DFS2, S2, IMETH, CONPER, IPRINT, STAT, LDSTAT)`

Double: The double precision name is `DSCIPM`.

Description

Routine `SCIPM` computes simultaneous confidence intervals on all $k^* = k(k - 1)/2$ pairwise comparisons of k means $\mu_1, \mu_2, \ldots, \mu_k$ the one-way analysis of variance model. Any of several methods can be chosen. A good review of these methods is given by Stolir (1981). Also the methods are discussed in many elementary statistics texts, e.g., Kirk (1982, pages 114−127).

Let s^2 (input in `S2`) be the estimated variance of a single observation. Let v be the degrees of freedom (input in `DFS2`) associated w s^2: Let $\alpha = 1 -$ `CONPER`/100.0. The methods are summarized as follows:

Tukey method: The Tukey method gives the narrowest simultaneous confidence intervals for all pairwise differences of means μ_i μ_j in balanced ($n_1 = n_2 = \ldots = n_k = n$) one-way designs. The method is exact and uses the Studentized range distribution. The form for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm q_{1-\alpha;k,v}\sqrt{\frac{s^2}{n}}$$

where $q_{1-\alpha;k,v}$ is the $(1 - \alpha)100$ percentage point of the Studentized range distribution with parameters k and v.

Tukey-Kramer method:
The Tukey-Kramer method is an approximate extension of the Tukey method for the unbalanced case. (The method simplifies to tl Tukey method for the balanced case.) The method always produces confidence intervals narrower than the Dunn-Sidak and Bonferroni methods. Hayter (1984) proved that the method is conservative, *i*.e., the method guarantees a confidence coverage of at least $(1 - \alpha)100\%$. Hayter's proof gave further support to earlier recommendations for its use (Stoline 1981). (Methods that are currently better are restricted to special cases and only offer improvement in severely unbalanced cases, see, e.g., Spurrier and Isha 1985). The formula for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm q_{1-\alpha;k,v}\sqrt{\frac{s^2}{2n_i} + \frac{s^2}{2n_j}}$$

Dunn-Šidák method
The Dunn-Šidák method is a conservative method. The method gives wider intervals than the Tukey-Kramer method. (For large v small α and k, the difference is only slight.) The method is slightly better than the Bonferroni method and is based on an improved Bonferroni (multiplicative) inequality (Miller, pages 101, 254−255). The method uses the t distribution (see IMSL routine `TIN`, in [Chapter 17, Probability and Distribution Functions and Inverses](#)). The formula for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm t_{\frac{1}{2}+\frac{1}{2}(1-\alpha)^{1/k^*};v}\sqrt{\frac{s^2}{n_i} + \frac{s^2}{n_j}}$$

where $t_{f;v}$ is the $100f$ percentage point of the t distribution with v degrees of freedom.

Bonferroni method:

The Bonferroni method is a conservative method based on the Bonferroni (additive) inequality (Miller, page 8). The method uses t distribution. The formula for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm t_{1-\alpha/(2k^*);\nu} \sqrt{\frac{s^2}{n_i} + \frac{s^2}{n_j}}$$

Scheffé method:
The Scheffé method is an overly conservative method for simultaneous confidence intervals on pairwise difference of means. The method is applicable for simultaneous confidence intervals on all contrasts, i.e., all linear combinations

$$\sum_{i=1}^{k} c_i \mu_i \text{ where } \sum_{i=1}^{k} c_i = 0$$

The method can be recommended here only if a large number of confidence intervals on contrasts in addition to the pairwise differences of means are to be constructed. The method uses the *F* distribution (see IMSL routine `FIN`, in Chapter 17, Probability a Distribution Functions and Inverses). The formula for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm \sqrt{(k-1) F_{1-\alpha;k-1,\nu} \left(\frac{s^2}{n_i} + \frac{s^2}{n_j}\right)}$$

where $F_{1-\alpha;k-1,\nu}$ is the $(1-\alpha)100$ percentage point of the *F* distribution with $k-1$ and ν degrees of freedom.

One-at-a-time *t* method (Fisher's LSD): The one-at-a-time *t*
method is the method appropriate for constructing a single confidence interval. The confidence percentage input is appropriate for interval at a time. The method has been used widely in conjunction with the overall test of the null hypothesis $\mu_1 = \mu_2 = \ldots = \mu_k$ by the use of the *F*
statistic. Fisher's LSD (least significant difference) test is a two-stage test that proceeds to make pairwise comparisons of means or if the overall *F* test is significant.

Milliken and Johnson (1984, page 31) recommend LSD comparisons after a significant *F* only if the number of comparisons is sma and the comparisons were planned prior to the analysis. If many unplanned comparisons are made, they recommend Scheffe's method. If the *F*
test is insignificant, a few planned comparisons for differences in means can still be performed by using either Tukey, Tukey-Kran Dunn-Šidák or Bonferroni methods. Because the *F*
test is insignificant, Scheffe's method will not yield any significant differences. The formula for the difference $\mu_i - \mu_j$ is given by

$$\bar{y}_i - \bar{y}_j \pm t_{1-\frac{\alpha}{2};\nu} \sqrt{\frac{s^2}{n_i} + \frac{s^2}{n_j}}$$

Comments

Workspace may be explicitly provided, if desired, by use of `S2IPM`/`DS2IPM`. The reference is:

`CALL S2IPM (NGROUP, NI, YMEANS, DFS2, S2, IMETH, CONPER, IPRINT, STAT, LDSTAT, WK, IWK)`

The additional arguments are as follows:

WK — Real work vector of length `NGROUP`.

IWK — Integer work vector of length `NGROUP`.

Example

Simultaneous 99% confidence intervals are computed for all pairwise comparisons of 5 means from a one-way analysis of variance design. In order to compare the results of each method, all the options for `IMETH` are used for input. The data are given by Kirk (19 Table 3.5-1, page 117). In the output, pairs of means declared not equal are indicated by the letter `N`. The other pairs of means (for which there is insufficient evidence from the data to declare the means are unequal) are indicated by an equal sign (=).

```
      USE SCIPM_INT

      IMPLICIT   NONE
      INTEGER    LDSTAT, NGROUP
      PARAMETER  (NGROUP=5, LDSTAT=NGROUP*(NGROUP-1)/2)
!
      INTEGER    IMETH, IPRINT, NI(NGROUP)
      REAL       CONPER, DFS2, S2, STAT(LDSTAT,5), YMEANS(NGROUP)
!
      DATA YMEANS/36.7, 48.7, 43.4, 47.2, 40.3/
      DATA NI/10, 10, 10, 10, 10/
!
      DFS2   = 45.0
      S2     = 28.8
      CONPER = 99.0
      IPRINT = 1
      DO 10  IMETH=0, 4


      CALL SCIPM (NI, YMEANS, DFS2, S2, STAT, IMETH=IMETH, &
      CONPER=CONPER, IPRINT=IPRINT)
   10 CONTINUE
      END
```

Output

```
                   Simultaneous Confidence Intervals
                 for All Pairwise Differences of Means
                            (Tukey Method)

                                         99.0% Confidence Interval
                                        --------------------------

   Group I  Group J  Mean I - Mean J   Lower Limit   Upper Limit
N        1        2            -12.0       -20.261        -3.739
=        1        3             -6.7       -14.961         1.561
N        1        4            -10.5       -18.761        -2.239
=        1        5             -3.6       -11.861         4.661
=        2        3              5.3        -2.961        13.561
=        2        4              1.5        -6.761         9.761
N        2        5              8.4         0.139        16.661
=        3        4             -3.8       -12.061         4.461
=        3        5              3.1        -5.161        11.361
=        4        5              6.9        -1.361        15.161

               Simultaneous Confidence Intervals
             for All Pairwise Differences of Means
                      (Dunn-Sidak Method)

                                        99.0% Confidence Interval
                                       --------------------------
   Group I  Group J  Mean I - Mean J   Lower Limit   Upper Limit
N        1        2            -12.0       -20.445        -3.555
=        1        3             -6.7       -15.145         1.745
N        1        4            -10.5       -18.945        -2.055
=        1        5             -3.6       -12.045         4.845
=        2        3              5.3        -3.145        13.745
=        2        4              1.5        -6.945         9.945
=        2        5              8.4        -0.045        16.845
=        3        4             -3.8       -12.245         4.645
=        3        5              3.1        -5.345        11.545
=        4        5              6.9        -1.545        15.345

               Simultaneous Confidence Intervals
             for All Pairwise Differences of Means
                      (Bonferroni Method)

                                        99.0% Confidence Interval
                                       --------------------------
   Group I  Group J  Mean I - Mean J   Lower Limit   Upper Limit
N        1        2            -12.0       -20.449        -3.551
```

=	1	3	-6.7	-15.149	1.749
N	1	4	-10.5	-18.949	-2.051
=	1	5	-3.6	-12.049	4.849
=	2	3	5.3	-3.149	13.749
=	2	4	1.5	-6.949	9.949
=	2	5	8.4	-0.049	16.849
=	3	4	-3.8	-12.249	4.649
=	3	5	3.1	-5.349	11.549
=	4	5	6.9	-1.549	15.349

Simultaneous Confidence Intervals
for All Pairwise Differences of Means
(Scheffe Method)

	Group I	Group J	Mean I - Mean J	99.0% Confidence Interval Lower Limit	Upper Limit
N	1	2	-12.0	-21.317	-2.683
=	1	3	-6.7	-16.017	2.617
N	1	4	-10.5	-19.817	-1.183
=	1	5	-3.6	-12.917	5.717
=	2	3	5.3	-4.017	14.617
=	2	4	1.5	-7.817	10.817
=	2	5	8.4	-0.917	17.717
=	3	4	-3.8	-13.117	5.517
=	3	5	3.1	-6.217	12.417
=	4	5	6.9	-2.417	16.217

Simultaneous Confidence Intervals
for All Pairwise Differences of Means
(One-at-a-Time t Method--LSD Test)

	Group I	Group J	Mean I - Mean J	99.0% Confidence Interval Lower Limit	Upper Limit
N	1	2	-12.0	-18.455	-5.545
N	1	3	-6.7	-13.155	-0.245
N	1	4	-10.5	-16.955	-4.045
=	1	5	-3.6	-10.055	2.855
=	2	3	5.3	-1.155	11.755
=	2	4	1.5	-4.955	7.955
N	2	5	8.4	1.945	14.855
=	3	4	-3.8	-10.255	2.655
=	3	5	3.1	-3.355	9.555
N	4	5	6.9	0.445	13.355

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