STA2122 Name ANS Key

Problem Sheet Number:______________________

Solve the problem.

1) A manufacturer claims that the mean lifetime of its fluorescent bulbs is 1500 hours. A homeowner selects 25 bulbs and finds the mean lifetime to be 1640 hours with a standard deviation of 80 hours. Test the manufacturer's claim. Use $\alpha = 0.05$.. Do not use p-value method.

$\bar{X} = 1650$, $s = 80$, $n = 25$, $\alpha = 0.05$, $df = 24$

$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{1650 - 1500}{80/\sqrt{25}}$, $t = 9.375$

a) Setup hypothesis: $H_0: \mu = 1500$ $H_a: \mu \neq 1500$

b) Test statistics: $t = 9.375$ Critical t Test statistics: $t^*_{\frac{0.05}{2}, 24} = 1.711$

c) Conclusion: reject Null hypoth

conclude that the mean lifetime is different

2) The Metropolitan Bus Company claims that the mean waiting time for a bus during rush hour is less than 8 minutes. A random sample of 20 waiting times has a mean of 6.5 minutes with a standard deviation of 2.1 minutes. At $\alpha = 0.01$, test the bus company's claim. If $\alpha = 0.05$, test the Company's claim using P-values.

$\bar{X} = 6.5$, $s = 2.1$, $n = 20$, $df = 19$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{6.5 - 8}{2.1/\sqrt{20}} = -3.19$

a) Setup hypothesis: $H_0: \mu = 8$ $H_a: \mu < 8$

b) p-value method: $0.001 < p < 0.0025$

c) Conclusion: reject Null hypoth

conclude that mean waiting time has decreased

3) Nine students took the SAT. Their scores are listed below. Later on, they took a test preparation course and retook the SAT. Their new scores are listed below. Test the claim that the test preparation had no effect on their scores. Use $\alpha = 0.05$. Assume that the distribution is normally distributed.

Student	1	2	3	4	5	6	7	8	9
Scores before course	720	860	850	880	860	710	850	1200	950
Scores after course	740	860	840	920	890	720	840	1240	970
← differences	-20	0	10	-40	-30	-10	10	-40	-20

a) Find the average difference: $(\bar{d})$: -15.55 b) Find the standard deviation: $(S_{\bar{d}})$: 19.43

c) Setup hypothesis: $H_0: \mu_{Before} = \mu_{After}$ → $\mu_B - \mu_A = 0$; $H_a: \mu_B \neq \mu_A$ → $\mu_B - \mu_A \neq 0$ d) p-value method: $0.04 < p < 0.05$

e) Conclusion: Do Not Reject

cannot conclude there is a difference

no effect on SAT scores

$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{-15.55}{19.43/\sqrt{9}}$

$t = -2.40$ $df = 8$

10. a.

Source	df	Mean Square* Year 1 Spring	Year 1 Winter	Year 2 Spring	Year 2 Winter
Treatment	3	32.55	41.05	193.15	48.65
Error	4	12.99	18.11	12.19	45.31

*Mean square value x 10^3

b. $F\ max = 45.31/12.19 = 3.72$, $F_{.05}\ max = 20.6$, $t = 4, \nu = 4$
Do not reject hypothesis of equal variances.

c.

Source	df	MS*	E(MS)
Year (Y)	1	13.61	$\sigma^2 + 16\sigma_y^2$
Season (S)	1	1,911.01	$\sigma^2 + 8\sigma_{ys}^2 + 16\theta_s^2$
Treatment (T)	3	257.70	$\sigma^2 + 4\sigma_{ty}^2 + 8\theta_t^2$
Y x S	1	94.61	$\sigma^2 + 8\sigma_{ys}^2$
T x S	3	15.94	$\sigma^2 + 2\sigma_{tys}^2 + 4\theta_{ts}^2$
T x Y	3	28.47	$\sigma^2 + 4\sigma_{ty}^2$
T x Y x S	3	13.27	$\sigma^2 + 2\sigma_{tys}^2$
Error	16	22.15	σ^2

*Mean square value x 10^3

d. Season: $F_0 = 1911.01/94.61 = 20.2$, $F(.05, 1, 1) = 161.4$, Do not reject.
Treatments: $F_0 = 257.7/28.47 = 9.05$, $F(.05, 3, 3) = 9.28$, Do not reject.
Y x S: $F_0 = 94.61/22.15 = 4.27$, $F(.05, 1, 16) = 4.49$, Do not reject.
T x S: $F_0 = 15.94/13.27 = 1.20$, $F(.05, 3, 3) = 9.28$, Do not reject.
T x Y: $F_0 = 28.47/22.15 = 1.29$, $F(.05, 3, 16) = 3.24$, Do not reject.
T x Y x S: $F_0 = 13.27/22.15 = 0.60$, $F(.05, 3, 16) = 3.24$, Do not reject.

Chapter 9

1. a. $\lambda = 1$
b. $E = 2/3$
c. $y_{ij} = \mu + \tau_i + \rho_j + e_{ij}$ $\quad i = 1,2,3,4 \quad j = 1,2,3,4,5,6$
τ_i = fixed temperature effect, ρ_j = fixed block effect
e_{ij} = experimental error, mean 0, variance σ^2

Source	df		SS		MS	
Blocks(unadj)	5		613.66		122.73	
Temperature(adj)	3		718.29		239.43	
linear		1		677.08		677.08
quadratic		1		7.33		7.33
cubic		1		33.87		33.87
Error	3		41.00		13.67	

Test for temperature: $F_0 = 17.51$. $F(.05, 3, 3) = 9.28$. Reject.

d. means: 26.46, 23.82, 10.07, 3.61; standard error = 2.50

e. 3.70

f. See analysis of variance table for SS partitions.
linear: $F_0 = 49.53$, $F(.05, 1, 3) = 10.13$, Reject.
quadratic: $F_0 = 0.54$, $F(.05, 1, 3) = 10.13$, Do not reject.

g. No. F_0 for cubic = 2.48, $F(.05, 1, 3) = 10.13$, Do not reject.

3. **a.** $y_{ijm} = \mu + \tau_i + \rho_j + \gamma_m + e_{ijm} \quad i, m = 1, 2, \ldots, 7 \quad j = 1, 2, 3, 4$
τ_i = fixed intersection type effect, ρ_j = fixed observer effect,
γ_m = fixed city effect,
e_{ijm} = experimental error, mean 0, variance σ^2

Source	df	SS	MS
Observer	3	146.51	48.84
City(unadj)	6	1,342.07	223.68
Intersection(adj)	6	9,972.46	1,662.08
Error	12	368.85	30.74

b. 4.19

c. Plan 9B.5, $E = 0.88$

d. $d(.05, 6, 12) = 2.58$, $M = 2.58(4.19) = 10.8$
Select intersection types 2, 3, and 7 with P(CS) = 0.95.

5. **a.** $t = 5, r = 4, k = 4, b = 5$

b. $\lambda = 3$

Chapter 10

1. **a.** $y_{ijk} = \mu + \tau_i + \gamma_j + \rho_{m(j)} + e_{ijm} \qquad i = 1, 2, \ldots, 8 \quad j = 1, 2, \ldots, 7 \quad m = 1, 2$

τ_i = variety effect, γ_j = replication effect, $\rho_{m(j)}$ = block within replication effect,

e_{ijm} = experimental error, mean 0, variance σ^2