$$R(\mu,\tau) = \hat{\mu}y_{..} + \sum_{i=1}^{4} \hat{\tau}_i y_{i.} = 59.83$$

with 4 degrees of freedom.

$$R(\beta|\mu,\tau) = R(\mu,\tau,\beta) - R(\mu,\tau) = 138.78 - 59.83 = 78.95 = SS_{Blocks}$$

with 7-4=3 degrees of freedom.

Source	*DF*	*SS*(exact)	*SS*(approximate)
Tips	3	39.53	39.98
Blocks	3	78.95	79.53
Error	8	6.22	6.22
Total	14	125.74	125.73

Note that for the exact analysis, $SS_T \neq SS_{Tips} + SS_{Blocks} + SS_E$.

4-31 An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

	Car				
Additive	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	14		13	14	9
4	13	11	11	12	
5	11	12	10		8

There are several computer software packages that can analyze the incomplete block designs discussed in this chapter. The Minitab General Linear Model procedure is a widely available package with this capability. The output from this routine for Problem 4-27 follows. The adjusted sums of squares are the appropriate sums of squares to use for testing the difference between the means of the gasoline additives.

Minitab Output

General Linear Model

```
Factor     Type Levels Values
Additive  fixed      5 1 2 3 4 5
Car      random      5 1 2 3 4 5

Analysis of Variance for Mileage, using Adjusted SS for Tests

Source     DF     Seq SS     Adj SS     Adj MS       F      P
Additive    4    31.7000    35.7333     8.9333    9.81  0.001
Car         4    35.2333    35.2333     8.8083    9.67  0.001
Error      11    10.0167    10.0167     0.9106
Total      19    76.9500
```

4-32 Construct a set of orthogonal contrasts for the data in Problem 4-31. Compute the sum of squares for each contrast.

One possible set of orthogonal contrasts is:

$$H_0 : \mu_4 + \mu_5 = \mu_1 + \mu_2 \quad (1)$$
$$H_0 : \mu_1 = \mu_2 \quad (2)$$
$$H_0 : \mu_4 = \mu_5 \quad (3)$$
$$H_0 : 4\mu_3 = \mu_4 + \mu_5 + \mu_1 + \mu_2 \quad (4)$$

The sums of squares and *F*-tests are:

Brand ->	1	2	3	4	5			
Q_i	33/4	11/4	-3/4	-14/4	-27/4	$\sum c_i Q_i$	*SS*	F_0
(1)	-1	-1	0	1	1	-85/4	30.10	39.09
(2)	1	-1	0	0	0	-22/4	4.03	5.23
(3)	0	0	0	-1	1	-13/4	1.41	1.83
(4)	-1	-1	4	-1	-1	-15/4	0.19	0.25

Contrasts (1) and (2) are significant at the 1% and 5% levels, respectively.

4-33 Seven different hardwood concentrations are being studied to determine their effect on the strength of the paper produced. However the pilot plant can only produce three runs each day. As days may differ, the analyst uses the balanced incomplete block design that follows. Analyze this experiment (use $\alpha = 0.05$) and draw conclusions.

Hardwood	Days						
Concentration (%)	1	2	3	4	5	6	7
2	114				120		117
4	126	120				119	
6		137	114				134
8	141		129	149			
10		145		150	143		
12			120		118	123	
14				136		130	127

There are several computer software packages that can analyze the incomplete block designs discussed in this chapter. The Minitab General Linear Model procedure is a widely available package with this capability. The output from this routine for Problem 4-33 follows. The adjusted sums of squares are the appropriate sums of squares to use for testing the difference between the means of the hardwood concentrations.

Minitab Output

General Linear Model

```
Factor     Type Levels Values
Concentr  fixed      7  2  4  6  8 10 12 14
Days     random      7 1 2 3 4 5 6 7

Analysis of Variance for Strength, using Adjusted SS for Tests

Source     DF     Seq SS     Adj SS     Adj MS       F      P
Concentr    6    2037.62    1317.43     219.57   10.42  0.002
Days        6     394.10     394.10      65.68    3.12  0.070
Error       8     168.57     168.57      21.07
Total      20    2600.29
```