4.30 (b) Remarks

① Fact: If $U \sim N(a,b)$, then $\frac{U}{c} \sim N\left(\frac{a}{c}, \frac{b}{c^2}\right)$.

Apply: If $Y|X=x \sim N(x, x^2)$,

then $\frac{Y}{X}|X=x \sim N\left(\frac{x}{x}, \frac{x^2}{x^2}\right) = N(1,1)$

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② Fact: If $W|X=x \sim H$ for all x,

then W and X are indep. and $W \sim H$.

Apply to $W = \frac{Y}{X}$ above.

$W|X=x \sim N(1,1)$ for all x

so W and X are indep. and $W \sim N(1,1)$.

Proof of ②: Suppose H has pdf h.

$$f_{X,W}(x,w) = f_X(x) f_{W|X}(w|x)$$

$$= f_X(x)\, h(w) \quad \text{for all } x, w$$

implies X and W indep.

and $f_W(w) = h(w)$.