# Assignment #2

**Reading:** Sections 2.2 – 2.3, 3.1 – 3.3

**Chapter 2:** 11, 12, 13, 14, 16, 17, 18, 19, 21, 22, 24, 25, 26, 27, 28, 30(a,b,c), 31, 32, 33, 38, 40

**Chapter 3:** 1, 2, 3, 4, 5, 6, 7, 8, 11(a), 12, 13, 15, 17, 19, 20 21, 22, 23, 24(a), 25, 26, 27

**C Exercises:** C−1 through C7 (see below)

## Exercises C

Here are a few more exercises which use lecture material related to Chapters 2 and 3.

**Problem −1.** Two gamblers ($A$ and $B$) play the following game. To start off, each of them puts one dollar in "the pot". One of these dollars is marked with an X. Then the players alternate taking turns starting with $A$ (that is, $A$, $B$, $A$, $B$, $\dots$). Each turn consists of the following: a player reaches into the pot and pulls out a dollar at random. If it is the marked dollar, the player wins all the money in the pot and the game is over. If it is not the marked dollar, the player puts it back into the pot, and then adds one more dollar to the pot and the game continues. Let $Y$ denote the total length of the game, that is, the total number of draws from the pot. Let $Z$ be the winnings of player $A$. (Note that $Z$ is negative if $B$ wins.) Find $EY$ and $EZ$.

**Problem 0.** Use indicator random variables to prove the principle of inclusion-exclusion for the case of 3 events. (The proof works for any number of events, but the notation becomes more complicated.) Hint: Start by using DeMorgan's Law $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$.

**Problem 1.** A simple game consists of the following: A fair coin is tossed repeatedly. Before each toss the players guess the outcome. If a player guesses correctly three times in a row, they win a dollar. To be more precise, a player receives a dollar after the $i$-th toss, if they have correctly guessed the outcomes of tosses $i-2$, $i-1$, and $i$. Note that under these rules, a player guessing correctly 4 times in a row receives a total of 2 dollars; a player guessing correctly 5 times in a row receives a total of 3 dollars, etc. Suppose a player guesses on 12 tosses. Find the mean and variance of this player's total winnings. [This can be done using indicator (0-1 valued) random variables. In lecture we used indicator rv's to calculate the mean and variance of the hypergeometric distribution. Something similar works here.]

**Problem 2.** Now and then someone will take a machine gun, go to the nearest McDonald's and start shooting large numbers of total strangers for no apparent reason. We shall call such behavior "running amok". Consider a nation of $2 \times 10^8$ inhabitants. Let $p_i$ be the probability that person $i$ will run amok at some time during the next year. Suppose $p_i = 10^{-9}$ for $1.5 \times 10^8$ people, $p_i = 10^{-8}$ for $0.4 \times 10^8$ people, and $p_i = 10^{-7}$ for $0.1 \times 10^8$ people. What is the probability that exactly 2 people will run amok during the next year? (Assume that people act independently of each other.)

**Problem 3.** Let $X_1, X_2, \ldots, X_{10}$ be i.i.d. random variables with density $f(x) = 2x$ for $0 < x < 1$. Define $Y = \max X_i$ and $Z = \min X_i$.

**(a)** Find the cdf of $Y$ and calculate $EY$.

**(b)** Find the cdf of $Z$ and calculate $EZ$.

**Problem 4.** A clerk, Bob Cratchit, is working under the light of three lousy light bulbs. These light bulbs have independent lifetimes which have an exponential distribution with a mean of 4 hours. Ebeneezer Scrooge refuses to replace *any* of the light bulbs until they have *all* burned out. When working under the light of all 3 bulbs, Bob makes errors at an average rate of 1 per hour. When working under 2 bulbs, Bob makes an average of 3 errors per hour. When working under 1 bulb, he averages 6 errors per hour. When all the bulbs burn out, Bob quits working. What is the expected value of the total number of errors that Bob makes? (Your solution may be fairly informal. Also, you should think about the more general situation of $n$ light bulbs.)

• The next few problems require using normal approximations. For integer-valued random variables, more accurate answers can often be obtained by using the "continuity correction". This consists of approximating $P(a \leq Y \leq b)$ by $P(a - \frac{1}{2} \leq X \leq b + \frac{1}{2})$ where $X$ is a normally distributed random variable with the same mean and variance as $Y$. (Here $a$ and $b$ are integers and $Y$ is an integer-valued rv.) Use the continuity correction when applicable.

**Problem 5.** Joe is trying to decide whether or not to bother voting in tomorrow's election for Governor. He wishes to calculate the probability that his vote will make a difference, that is, change the outcome of the election. He thinks this probability will be largest when the election is close. To represent a close election he decides to use the following probability model: Assume that (excluding Joe) there are 4,000,000 registered voters with exactly 2,000,000 supporting candidate $A$ and 2,000,000 supporting candidate $B$. Not all of these people will actually vote. Assume (optimistically) that any given registered voter will cast a vote with probability 0.8 and that people decide independently of each other whether or not to vote. Joe's vote will make a difference if there is a tie among the other voters (so that Joe's vote breaks the tie) or if Joe's vote creates a tie (thus forcing a runoff election). Under this model, what is the probability that Joe's vote makes a difference?

**Problem 6.**

**(a)** Suppose $X$ has a Beta(30, 30) distribution. Calculate a normal approximation for $P(.4 < X < .6)$. (For comparison, the exact probability can be computed using the `pbeta` function in S-plus.)

**(b)** Suppose $X$ has a Poisson(80) distribution. Calculate a normal approximation for $P(75 \leq X \leq 90)$. (For comparison, the exact probability can be computed using the `ppois` function in S-plus.)

**Problem 7.** (Should be Problem **0.5** .) Let $X \sim$ Geometric($p$). Compute $EX^3$ by conditioning and using the discrete memoryless property.