Further Details on Exercise 7.9

$\hat{\theta} = X_{(n)}$ so that

$$
\begin{aligned}
E\hat{\theta} &= \int x f_{X_{(n)}}(x)\,dx = \int_0^\theta x\,\frac{nx^{n-1}}{\theta^n}\,dx = \frac{n}{\theta^n}\int_0^\theta x^n\,dx = \frac{n}{\theta^n}\,\frac{\theta^{n+1}}{n+1} = \frac{n}{n+1}\,\theta \\
E\hat{\theta}^2 &= \int x^2 f_{X_{(n)}}(x)\,dx = \int_0^\theta x^2\,\frac{nx^{n-1}}{\theta^n}\,dx = \frac{n}{\theta^n}\int_0^\theta x^{n+1}\,dx = \frac{n}{\theta^n}\,\frac{\theta^{n+2}}{n+2} = \frac{n}{n+2}\,\theta^2 \\
\mathrm{Var}(\hat{\theta}) &= \frac{n}{n+2}\,\theta^2 - \left(\frac{n}{n+1}\,\theta\right)^2 = \frac{n\theta^2}{(n+2)\,(n+1)^2}\,. \\
\mathrm{MSE} &= \mathrm{Bias}^2 + \mathrm{Var} = \left(\frac{-\theta}{n+1}\right)^2 + \frac{n\theta^2}{(n+2)\,(n+1)^2} = \frac{2\theta^2}{(n+1)\,(n+2)} \le \frac{\theta^2}{3n}\ \forall\, n
\end{aligned}
$$

with strict inequality for $n \ge 3$. So the MLE beats the MOM estimate (by a very large margin when n is large).