

[Extra Problem] Find the Fisher information matrix for a k -parameter exponential family with the natural parameter:

$$f(\underline{x} | \underbrace{\theta_1, \dots, \theta_k}_{\boldsymbol{\theta}}) = c(\boldsymbol{\theta}) h(\underline{x}) \exp(\sum \theta_i t_i(\underline{x}))$$

The easiest approach is to use

$$I(\boldsymbol{\theta}) = E\left(-\frac{\partial^2}{\partial \boldsymbol{\theta}^2} \log f(\underline{x} | \boldsymbol{\theta})\right)$$

which means

$$I_{ij} = E\left(-\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log f(\underline{x} | \boldsymbol{\theta})\right).$$

$$\text{Clearly } -\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log f = -\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log c(\boldsymbol{\theta}).$$

Thus, I is a $k \times k$ matrix with entries

$$I_{ij} = -\frac{\partial^2}{\partial \theta_i \partial \theta_j} \log c(\boldsymbol{\theta}).$$