Review Midterm

STA 5446- Fall '05

1. - -----

SEE BELOW

2. Show that $\mathcal{F}_\mathcal{A} = \mathcal{A}$ if and only if $\mathcal{A}$ is a σ field. $\mathcal{F}_\mathcal{A} = \bigcap\{\mathcal{F} : \mathcal{F}$ is a σ field such that $\mathcal{A} \subset \mathcal{F}\}$

3. If P_1 and P_2 are probability measures, then $P(A) = \alpha_1 P_1(A) + \alpha_2 P_2(A)$ is also a probaility measure provided that α_1 and α_2 are non negative and $\alpha_1 + \alpha_2 = 1$.

4. Let $\mathcal{F}$ is a σ field on Ω and let $A \subset \Omega$. Show that $\tilde{\mathcal{F}} = \{A \cap B : B \in \mathcal{F}\}$ is a σ field on A.

5. Show that $|P(A) - P(B)| \leq P(A \triangle B)$.

6. Verify the inequaliy $P(A \triangle C) \leq P(A \triangle B) + P(B \triangle C)$

7. Let $A_1, A_2 \cdots \in \mathcal{F}$. $\mathcal{F}$ is a σ field. Show that $limsup A_n \in \mathcal{F}$, liminf $A_n \in \mathcal{F}$.

8. Show that liminf $A_n \subset$ limsup A_n.

$P(\Omega) = \infty$.

×

9. Let $\Omega = \{1,2,3,4,.....\}$ and let $\mathcal{F}$ be the family of all subsets of Ω. Can $P(\{i\}) = \frac{1}{i}, i=1,2,3,4,...$, be extended to a probability measure on $\mathcal{F}$?

10. If $P(A_n)=1$ for n=1,2,3,.....,then $P(\cap_{n=1}^{\infty} A_n)=1$.

1. Let $(\Omega,\mathcal{A})=([0,1],\mathcal{B}_{[0,1]})$, $\mathcal{A}_0 = \{\phi, [0,1]\}$ and $\mathcal{A}_1 = \sigma[\{\phi, [0, \frac{1}{3}], (\frac{1}{3}, \frac{2}{3}], (\frac{2}{3}, 1]\}]$.
Consider $X_i(\omega), i = 1, 2$ defined by
$X_1(\omega) = 2\omega$
$X_2(\omega)=(\frac{\pi}{2})\mathbf{1}_{[0,\frac{1}{5}]}(\omega)+3\mathbf{1}_{[\frac{3}{4},1]}(\omega)$
(a) Is X_1 $\mathcal{B}$-$\mathcal{A}_0$ measurable ?
(b) Find $X_2^{-1}(\mathcal{B})$. Is it a σ field?
(c)Describe the collection of random variables that are $\mathcal{A}_1$ measurable .

$\mathcal{B}$

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$P(\Omega) = \infty$.

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1. Let $(\Omega,\mathcal{A})=([0,1],\mathcal{B}_{[0,1]})$, $\mathcal{A}_0 = \{\phi, [0,1]\}$ and $\mathcal{A}_1 = \sigma[\{\phi, [0, \frac{1}{3}], (\frac{1}{3}, \frac{2}{3}], (\frac{2}{3}, 1]\}]$.
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1) $X : ([0,1], \mathcal{B}_{[0,1]}) \longrightarrow (\mathbb{R}, \mathcal{B})$

(a) $X_1(\omega) = 2\omega$

X_1 is $\mathcal{B}$-$\mathcal{A}_0$ measurable if $X_1^{-1}(\mathcal{B}) \subset \mathcal{A}_0$. {equivalenly $X_1^{-1}([a,\infty)) \subset \mathcal{A}$

Now
$$X_1^{-1}([a,\infty)) = \begin{cases} \phi & \text{if } a > 2 \\ [a/2, 1] & 0 < a \leq 2 \\ \Omega & a \leq 0 \end{cases}$$

since $X^{-1}(\omega) = \{\omega \in \Omega : X(\omega) \geq a\}$

But $[a/2, 1] \not\subset \mathcal{A}_0$

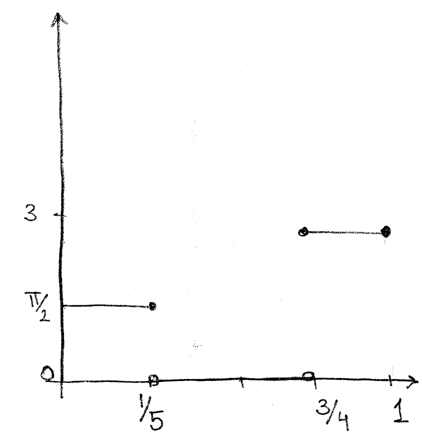
$\therefore$ X_1 is NOT $\mathcal{B}$-$\mathcal{A}_0$ measurable

(b) $X_2^{-1}(\mathcal{B}) = X_2^{-1}(\sigma(\mathcal{C})) = \sigma(X_2^{-1}(\mathcal{C}))$

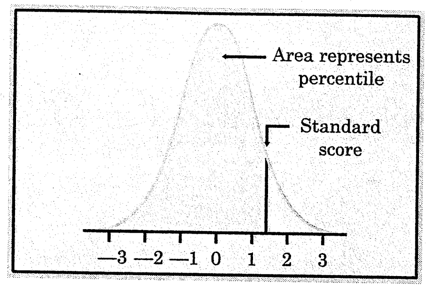
$= \sigma\{X^{-1}(c) : c \in \mathcal{C}\}$

$X_2^{-1}([a,\infty)) = \{\omega \in \Omega = [0,1] \mid X(\omega) \geq a\}$

$$X_2^{-1}([a,\infty)) = \begin{cases} \phi & \text{if } a > 3 \\ \left[\frac{3}{4}, 1\right] & \text{if } \frac{\pi}{2} < a \leq 3 \\ \left(\frac{1}{5}, \frac{3}{4}\right)^c & \text{if } 0 < a \leq \frac{\pi}{2} \\ \Omega & \text{if } a \leq 0 \end{cases}$$



$\therefore$ $X_2^{-1}(\mathcal{B}) = \sigma\left\{\Omega, \phi, \left[\frac{3}{4}, 1\right] \left(\frac{1}{5}, \frac{3}{4}\right)^c\right\}$



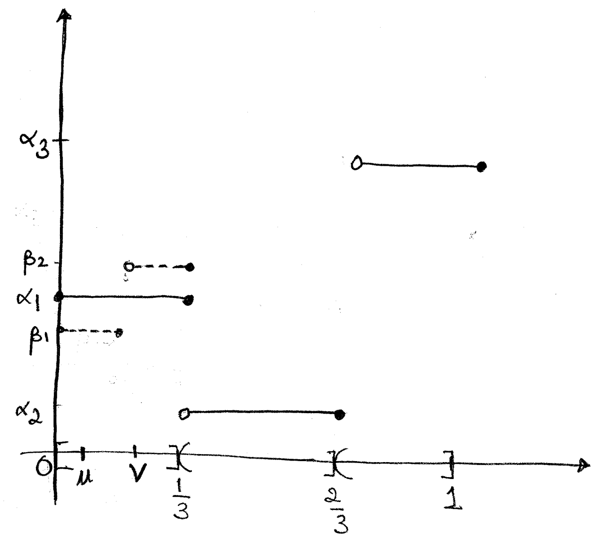
Percentiles of the normal distributions

Standard score	Percentile	Standard score	Percentile	Standard score	Percentile
−3.4	0.03	−1.1	13.57	1.2	88.49
−3.3	0.05	−1.0	15.87	1.3	90.32
−3.2	0.07	−0.9	18.41	1.4	91.92
−3.1	0.10	−0.8	21.19	1.5	93.32
−3.0	0.13	−0.7	24.20	1.6	94.52
−2.9	0.19	−0.6	27.42	1.7	95.54
−2.8	0.26	−0.5	30.85	1.8	96.41
−2.7	0.35	−0.4	34.46	1.9	97.13
−2.6	0.47	−0.3	38.21	2.0	97.73
−2.5	0.62	−0.2	42.07	2.1	98.21
−2.4	0.82	−0.1	46.02	2.2	98.61
−2.3	1.07	0.0	50.00	2.3	98.93
−2.2	1.39	0.1	53.98	2.4	99.18
−2.1	1.79	0.2	57.93	2.5	99.38
−2.0	2.27	0.3	61.79	2.6	99.53
−1.9	2.87	0.4	65.54	2.7	99.65
−1.8	3.59	0.5	69.15	2.8	99.74
−1.7	4.46	0.6	72.58	2.9	99.81
−1.6	5.48	0.7	75.80	3.0	99.87
−1.5	6.68	0.8	78.81	3.1	99.90
−1.4	8.08	0.9	81.59	3.2	99.93
−1.3	9.68	1.0	84.13	3.3	99.95
−1.2	11.51	1.1	86.43	3.4	99.97

(c) Describe a collection of sets which are $\mathcal{B}$-$\mathcal{A}_1$ measurable

$\mathcal{A}_1 = \sigma[\{\phi, [0, 1/3], (1/3, \frac{2}{3}], (\frac{2}{3}, 1]\}]$

Find X st $X^{-1}(\mathcal{B}) \subset \mathcal{A}_1$



let X be of the type

$$X(\omega) = \alpha_1 I_{[0,\frac{1}{3}]}(\omega) + \alpha_2 I_{(1/3, 2/3]}(\omega) + \alpha_3 I_{(2/3, 1]}(\omega)$$

$X^{-1}([a,\infty)) \subset \mathcal{B}$ always (easy to check). Also holds since $X(\cdot)$ is a simple function.

Claim : $X(\omega)$ must be constant over each of the intervals which generate $\mathcal{A}_1$, to be meas.

Let us suppose it was possible for $X(\omega)$ to be different for different values in $[0, 1/3]$ and still be measurable

ie let $u, v \in [0, 1/3]$ but $X(u) = \beta_1$ and $X(v) = \beta_2$ where $\beta_1 < \beta_2$

Since we have X is measurable. $X^{-1}((-\infty, x]) \subset \mathcal{A}_1$ for any $x \in \mathbb{R}$

$$u \in X^{-1}((-\infty, \beta_1]) \subset \mathcal{A}_1$$

$$v \in X^{-1}((-\infty, \beta_2]) \subset \mathcal{A}_1$$

Also $[0, 1/3] \in \mathcal{A}_0$ $\Rightarrow A_1 = [0, 1/3] \cap X^{-1}((-\infty, \beta_1]) \in \mathcal{A}_1$

$$A_2 = [0, 1/3] \cap X^{-1}((-\infty, \beta_2]) \in \mathcal{A}_1$$

A_1 and A_2 are disjoint sets, both $\subset [0, 1/3]$ and $A_1, A_2 \in \mathcal{A}_1$ & so are generated by some countable union, intersections etc of $[0, 1/3], (1/3, 2/3]$ $(2/3, 1]$ (since $\mathcal{A}_1$ is a σ-field). But this is not possible.

We have a contradiction and so our claim holds!

#2] <u>Shorter Proof</u>

"$\Longrightarrow$"

since $\mathcal{F}_{\mathcal{A}}$ is a σ-field by definition and $\mathcal{F}_{\mathcal{A}} = \mathcal{A}$

$\Rightarrow$ $\mathcal{A}$ is " a σ-field.

"$\Longleftarrow$" given $\mathcal{A}$ is a σ-field and $\mathcal{F}_{\mathcal{A}} = \bigcap\{\mathcal{F}_\alpha : \mathcal{F}_\alpha \text{ is } \sigma\text{-field } \& \ \mathcal{A} \subset \mathcal{F}\}$

Now $\mathcal{A} \subset \mathcal{A}$ and $\mathcal{A}$ is a σ-field $\therefore$ $\mathcal{F}_{\mathcal{A}} \subseteq \mathcal{A}$

Also for any $\mathcal{F}_\alpha \in \mathcal{F}_{\mathcal{A}}$ we have $\mathcal{A} \subset \mathcal{F}_\alpha$ $\forall \alpha$ satisfying

$\therefore$ $\mathcal{A} \subseteq \bigcap\{\mathcal{F}_\alpha : \mathcal{F}_\alpha \text{ is } \sigma\text{-field } \& \ \mathcal{A} \subset \mathcal{F}_\alpha\}$

ie $\mathcal{A} \subseteq \mathcal{F}_{\mathcal{A}}$

$\therefore$ $\mathcal{A} = \mathcal{F}_{\mathcal{A}}$

2] $\mathcal{F}_{\mathcal{A}} = \bigcap_{\alpha} \{ \mathcal{F}_\alpha : \mathcal{F}_\alpha \text{ is a } \sigma\text{-field } \& \ \mathcal{A} \subset \mathcal{F}_\alpha \}$

"$\Rightarrow$" Given $\mathcal{F}_{\mathcal{A}} = \mathcal{A} \rightarrow \mathcal{A} = \bigcap_{j \in \mathcal{J}_0} \{ \mathcal{F}_j : \mathcal{F}_j \text{ is a } \sigma\text{-field and } \mathcal{A} \subset \mathcal{F}_j \}$

(i) $\Omega \in \mathcal{F}_j \quad \forall \ j \in \mathcal{J}_0$ (since $\mathcal{F}_j$ is σ-field on Ω)

$\Rightarrow \Omega \in \bigcap_{j \in \mathcal{J}_0} \mathcal{F}_j$

$\Rightarrow \Omega \in \mathcal{A}$

(ii) $A \in \mathcal{A} \Rightarrow A \in \bigcap_{j \in \mathcal{J}_0} \{ \mathcal{F}_j : \mathcal{F}_j \text{ is } \sigma\text{-field and } \mathcal{A} \subset \mathcal{F}_j \}$

$\Rightarrow A \in \mathcal{F}_j \quad \forall j \in \mathcal{J}_0$

$\Rightarrow A^c \in \mathcal{F}_j \quad \forall j \in \mathcal{J}_0$

$\Rightarrow A^c \in \bigcap_{j \in \mathcal{J}_0} \{ \mathcal{F}_j \} \quad \Rightarrow \quad A^c \in \mathcal{A}$

(iii) $A_1, A_2 \ldots \in \mathcal{A} \Rightarrow A_1, A_2 \ldots A_n \ldots \in \mathcal{F}_j \quad \forall j \in \mathcal{J}_0$

$\Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}_j \quad \forall j \in \mathcal{J}_0$ {since $\mathcal{F}_j$ closed under countable unions

$\Rightarrow \left(\bigcup_{n=1}^{\infty} A_n \right) \in \bigcap_{j \in \mathcal{J}_0} \mathcal{F}_j$

$\therefore$ $\mathcal{A}$ is a σ-field

"$\Leftarrow$"

Given $\mathcal{A}$ is a σ-field and $\mathcal{F}_{\mathcal{A}} = \bigcap_{j \in \mathcal{J}_0} \{ \mathcal{F}_j : \mathcal{F}_j \text{ is } \sigma\text{-field } \& \ \mathcal{A} \subset \mathcal{F}_j \}$

Now $\mathcal{A}$ is a σ-field & $\mathcal{A} \subseteq \mathcal{A} \Rightarrow \mathcal{F}_{\mathcal{A}} \subseteq \mathcal{A}$ $\{ B = \bigcap_k A_k \Rightarrow B \subset A_k \ \forall k$

Also $\mathcal{A} \subset \mathcal{F}_j \ \forall \ j \in \mathcal{J}_0 \Rightarrow \mathcal{A} \subset \bigcap_{j \in \mathcal{J}_0} \mathcal{F}_j$

$\Rightarrow \mathcal{A} \subseteq \mathcal{F}_{\mathcal{A}}$

$\therefore \mathcal{A} = \mathcal{F}_{\mathcal{A}}$

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Yahoo! Driving Directions

Starting from: A E Tennessee St At N Monroe St, Tallahassee, FL 32301

Arriving at: B 2510 Killarney Way, Tallahassee, FL 32309-3163

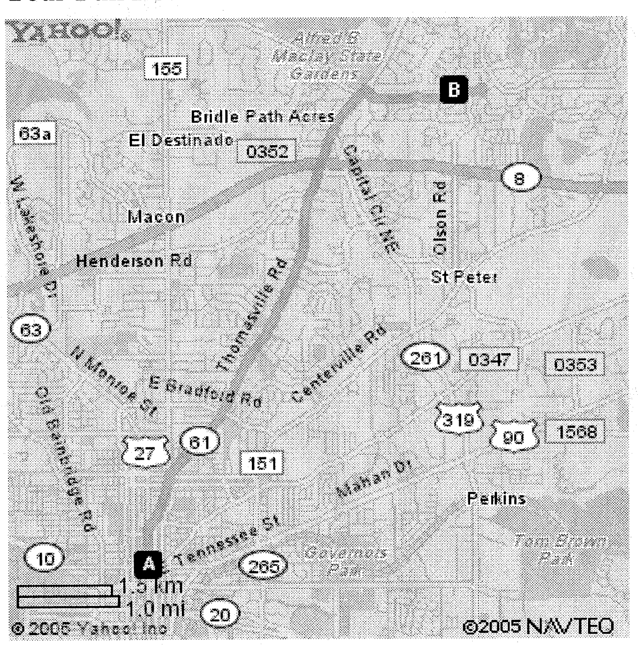
Distance: 6.8 miles **Approximate Travel Time:** 15 mins

Your Directions

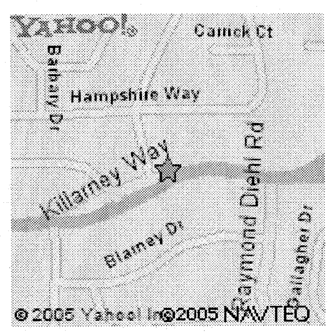
1.	Start on **N MONROE ST[US-27]** (at **E TENNESSEE ST & N MONROE ST** in **TALLAHASSEE)** going toward **E VIRGINIA ST** - go **0.4** mi
2.	Bear R on **THOMASVILLE RD[FL-61]** - go **4.9** mi
3.	Turn R on **KILLARNEY WAY** - go **1.2** mi
4.	Make a U-turn at **FOLEY DR** onto **KILLARNEY WAY** - go **0.3** mi
5.	Arrive at **2510 KILLARNEY WAY, TALLAHASSEE**

When using any driving directions or map, it's a good idea to do a reality check and make sure the road still exists, watch out for construction, and follow all traffic safety precautions. This is only to be used as an aid in planning.

Your Full Route



Your Destination



Address:
2510 Killarney Way
Tallahassee, FL 32309-3163

3). P_1 and P_2 are prob measures $\Rightarrow$ $P_1(\phi)=0$, $P_1(\Omega)=1$, $P_1(\sum_i A_i) = \sum_i P_1(A_i)$

& $P_2(\phi)=0$, $P_2(\Omega)=1$, $P_2(\sum_i A_i) = \sum_i P_2(A_i)$

(i) $P(\phi) = \alpha_1 P_1(\phi) + \alpha_2 P_2(\phi) = 0$ (where $\alpha_1, \alpha_2 \geq 0$ & $\alpha_1+\alpha_2=1$)

(ii) $P(\Omega) = \alpha_1 P_1(\Omega) + \alpha_2 P_2(\Omega) = \alpha_1+\alpha_2 = 1$

(iii) $P\left(\sum_{i=1}^{\infty} A_i\right) = \alpha_1 P_1\left(\sum_{i=1}^{\infty} A_i\right) + \alpha_2 P_2\left(\sum_{i=1}^{\infty} A_i\right)$

$= \alpha_1 \sum_{i=1}^{\infty} P_1(A_i) + \alpha_2 \sum_{i=1}^{\infty} P_2(A_i) = \sum_{i=1}^{\infty} [\alpha_1 P_1(A_i) + \alpha_2 P(A_i)]$

{since $\alpha_1 P_1(A_i)$ & $\alpha_2 P_2(A_i) \geq 0$}

$= \sum_{i=1}^{\infty} P(A_i)$

(iv) $P(A) = \alpha_1 P_1(A) + \alpha_2 P_2(A) \geq 0$ {since $P_1(A) \geq 0$ & $P_2(A) \geq 0$ $\forall A \subset \Omega$}

$\therefore$ P is also a prob measure

4) $\mathcal{F}$ is a σ-field on Ω

$A \subset \Omega$

$\tilde{\mathcal{F}} = \{A \cap B : B \in \mathcal{F}\}$. Show that $\tilde{\mathcal{F}}$ is a σ-field on A [ie our universal set = A]

(i) $A \subset \Omega$, also $\Omega \in \mathcal{F}$ since $\mathcal{F}$ is a σ-field

$\therefore$ $A \cap \Omega = A \in \tilde{\mathcal{F}}$ [ie the universal set $\in \tilde{\mathcal{F}}$]

(ii) let $C \in \tilde{\mathcal{F}}$ $\rightarrow$ $C = A \cap B$ where $A \subset \Omega$ and some $B \in \mathcal{F}$

$C^c = A \setminus C = A \setminus (A \cap B) = A \cap B^c$ {since universal set is A}

$C^c \in \tilde{\mathcal{F}}$ since $B^c \in \mathcal{F}$

(iii) let $C_1, C_2 \ldots \in \tilde{\mathcal{F}}$ $\Rightarrow$ $C_n = A \cap B_n$ for some $B_n \in \mathcal{F}$

$\bigcup_{n=1}^{\infty} C_n = \bigcup_{n=1}^{\infty} (A \cap B_n) = A \cap \left(\bigcup_{n=1}^{\infty} B_n\right) \in \tilde{\mathcal{F}}$ since $\left(\bigcup_{n=1}^{\infty} B_n\right) \in \mathcal{F}$

5(a) To show $\mu(\underline{\lim}\ A_n) \leq \underline{\lim}\ \mu(A_n)$

$$\mu(\underline{\lim}\ A_n) = \mu\left(\bigcup_{n=1}^{\infty} \bigcap_{m\geq n}^{\infty} A_m\right) \quad \{\text{def of lim inf}$$

$$= \mu\left(\bigcup_{n=1}^{\infty} B_n\right)$$

where we define $B_n = \bigcap_{m\geq n}^{\infty} A_m$

Notice $\{B_n\}_n$ is an $\uparrow$ seq

$$\Rightarrow \mu\left(\bigcup_{n=1}^{\infty} B_n\right) = \lim_{n\to\infty} \mu(B_n) \quad \{\text{Monotone prop of measures}$$

$$\therefore \mu(\underline{\lim}\ A_n) = \lim_{n\to\infty} \mu(B_n)$$

$$= \lim_{n\to\infty} \left[\mu\left(\bigcap_{m\geq n}^{\infty} A_m\right)\right]$$

Now $\bigcap_{m\geq n} A_m \subseteq A_m \quad \forall\ m \geq n$

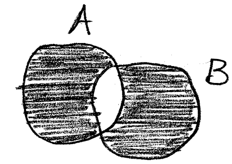
$$\therefore \mu\left(\bigcap_{m\geq n} A_m\right) \leq \mu(A_m) \quad \forall\ m \geq n$$

$$\therefore \mu\left(\bigcap_{m\geq n} A_m\right) \leq \inf_{m\geq n} \mu(A_m)$$

$$\therefore \lim_{n\to\infty} \left[\mu\left(\bigcap_{m\geq n}^{\infty} A_m\right)\right] \leq \lim_{n\to\infty} \left[\inf_{m\geq n} \mu(A_m)\right]$$

$$\mu(\underline{\lim}\ A_n) \leq \underline{\lim}\ \mu(A_m)$$

5] Show $|P(A) - P(B)| \leq P(A \triangle B)$



wlg assume $P(A) \geq P(B) \Rightarrow |P(A) - P(B)| = P(A) - P(B)$

If A and B are disjoint $\rightarrow P(A \triangle B) = P(A) + P(B) \geq P(A) - P(B)$

{since $P(A) \geq 0$ for any A

$$\therefore \quad |P(A) - P(B)| \leq P(A \triangle B)$$

If A and B are not disjoint $\Rightarrow P(A \triangle B) = P(AB^c) + P(BA^c)$

$$= P(A) + P(B) - 2P(A \cap B)$$

Now $(A \cap B) \subset B$

$$\therefore \quad P(A \cap B) \leq P(B)$$

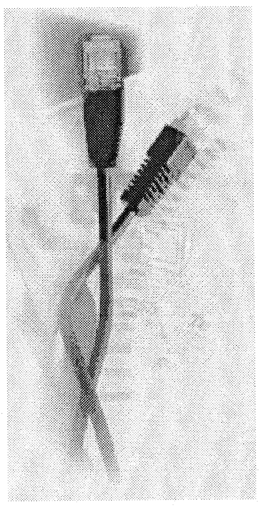
$$2P(A \cap B) \leq 2P(B)$$

$$\therefore \quad P(A) - 2P(A \cap B) \geq P(A) - 2P(B)$$

$$\therefore \quad P(A) + P(B) - 2P(A \cap B) \geq P(A) - P(B)$$

$$\therefore \quad P(A \triangle B) \geq |P(A) - P(B)|$$

actually worked backwards from and checked if end ineq holds



Article

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The analysis of putative environmental pollution gradients in spatially correlated epidemiological data

Andrew B. Lawson, Natalie W. Harrington

Abstract:

The analysis of putative sources of health hazard has recently received considerable attention. Most analyses, however, assume that the observations - such as counts in census tracts or exact locations of cases - are not spatially correlated. This assumption, if violated, could have serious consequences for inferences made from such data. This paper outlines a number of simple Monte Carlo test procedures for this situation.

The references of this article are secured to subscribers.

6] Verify $P(A \triangle C) \leq P(A \triangle B) + P(B \triangle C)$

$$P(A) + P(C) - 2P(AC) \overset{?}{\leq} P(A) + P(B) - 2P(AB) + P(B) + P(C) - 2P(BC)$$

$$-2P(AC) \overset{?}{\leq} -2P(AB) - 2P(BC) + 2P(B)$$

$$P(AB) + P(BC) - P(AC) \overset{?}{\leq} P(B)$$

$$[P(ABC^c) + P(ABC)] + [P(ABC) + P(A^cBC)] - [P(ABC) + P(AB^cC)] \overset{?}{\leq} [P(ABC^c) + P(A^cBC) + P(ABC) + P(A^cBC^c)]$$

$$-P(AB^cC) \overset{?}{\leq} P(A^cBC^c)$$

$$0 \overset{?}{\leq} P(A^cBC^c) + P(AB^cC)$$ which is true.

(ABC^c), (ABC), (BCA^c) are mutually disjoint & all are subsets of B

$$P(ABC^c) + P(ABC) + P(BCA^c) \leq P(B)$$

$$\Rightarrow P(ABC^c) + P(ABC) + P(BCA^c) \leq P(B) + P(AB^cC)$$

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Report A Problem

7) $A_1, A_2, \ldots \in \mathcal{F}$ $\quad$ $\mathcal{F}$ is a σ-field

$$\overline{\lim}\, A_n = \bigcap_{n=1}^{\infty} \bigcup_{m\geq n}^{\infty} A_m = \bigcap_{n=1}^{\infty} B_n \quad \text{where } B_n = \bigcup_{m\geq n}^{\infty} A_m$$

Now since $\mathcal{F}$ is a σ-field $\Rightarrow$ $B_n = \bigcup_{m\geq n}^{\infty} A_m \in \mathcal{F}$

Now $\bigcap_{n=1}^{\infty} B_n = \left(\bigcup_{n=1}^{\infty} B_n^c\right)^c$

$B_n^c \in \mathcal{F}$ and also $\bigcup_{n=1}^{\infty} B_n^c \in \mathcal{F} \Rightarrow \left(\bigcup_{n=1}^{\infty} B_n\right)^c \in \mathcal{F}$

$\therefore$ $\overline{\lim}\, A_n \in \mathcal{F}$

$$\underline{\lim}\, A_n = \bigcup_{n=1}^{\infty} \bigcap_{m\geq n}^{\infty} A_m = \bigcup_{n=1}^{\infty} B_n \quad \text{where } B_n = \bigcap_{m\geq n}^{\infty} A_m \in \mathcal{F}$$

Since $A_1 \ldots \in \mathcal{F} \Rightarrow A_1^c, A_2^c \ldots \in \mathcal{F}$

$\Rightarrow \bigcup_{m\geq n}^{\infty} A_m^c \in \mathcal{F} \Rightarrow \left(\bigcup_{m\geq n}^{\infty} A_m^c\right)^c \in \mathcal{F}$

Now $B_1, B_2 \ldots \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} B_n \in \mathcal{F}$

8] $\underline{\lim}\, A_n = \bigcup_{n=1}^{\infty} \bigcap_{m\geq n} A_m$ $\quad$ & $\quad$ $\overline{\lim}\, A_n = \bigcap_{n=1}^{\infty} \bigcup_{m\geq n} A_m$

$\omega \in \underline{\lim}\, A_n \Rightarrow \omega \in \bigcup_{n=1}^{\infty} \bigcap_{m\geq n} A_m \Rightarrow \exists\, n_0$ st $\omega \in \bigcap_{m\geq n_0} A_m$

$\Rightarrow \omega \in A_m \quad \forall\, m \geq n_0$

$\Rightarrow \omega \in$ all but finitely many A_m's

$\omega \in \overline{\lim}\, A_n \Rightarrow \omega \in \bigcap_{n=1}^{\infty} \bigcup_{m\geq n} A_m \Rightarrow \omega \in \left(\bigcup_{m\geq n} A_m\right) \quad \forall n$

$\Rightarrow$ given any arbitrary m, $\exists\, n_0$ st $\omega \in A_{n_0}$

$\Rightarrow \omega \in$ infinitely many A_m's

$\therefore$ $\underline{\lim}\, A_n \subset \overline{\lim}\, A_n$

3.3.1 SYSTEMATIC SAMPLING

Systematic sampling is a common type of sampling based on selecting every rth individual from a list or file after choosing a random number from 1 to r as a starting point. Going back to the example used for simple random sampling, choosing 300 from a population of 8,059, we now obtain a systematic sample by calculating the ratio $N/n = 8,059/300 = 26.9$ and selecting every twenty- sixth (or twenty-seventh) individual in the file beginning with a random start between 1 and 26 (or 27). Note that using $r = 26$ results in a sample size of 309 and $r = 27$ results in a sample size of 298. It is typical of systematic sampling procedures that they yield sample sizes approximately, rather than exactly, equal to the targeted sample size.

Systematic sampling is based on a fixed nonrandom rule and is not limited to selection from an actual file. Thus, selection of all those born on the (randomly chosen) fifth day of any month or of everyone whose social security number ends in (the randomly chosen digits) 12, 65, or 87 is similar to systematic sampling procedures yielding approximately 3 percent samples. Of course, the choice of the sampling scheme has to be relevant to the population being sampled. A population that is not completely covered by social security would be unsuitable for sampling by means of social security numbers. A primitive population that did not record and remember birthdays could not be sampled using date of birth. The reason for the popularity of systematic sampling is its simplicity and, in many instances, its superiority over simple sampling. The simplicity is obvious, the potential superiority less so. We shall consider this aspect in terms of an example. Suppose we want to systematically sample a chronological list of all hospital admissions for one year. If the sampling objective is to estimate the proportion of all hospital admissions during the year that are due to infectious diseases, it is quite probable that a systematic sample of admissions is preferable to a simple random sample. Consider that admissions for infectious diseases may have seasonal peaks and that by systematic sampling from a chronological list we are sure to obtain $(1/r)^{th}$ of the admissions in each season. In simple random sampling, however, some of the possible samples would include admissions only from a particular season and thereby improperly reflect the proportion of annual admissions for infectious disease. As in the foregoing example, if the list is ordered in a manner related to the study objective, systematic sampling is in some respects analogous to stratified sampling. If the list is in random order, systematic sampling is analogous to simple random sampling.

If however, the list is in cyclical order so that every rth element is in some way special, then systematic sampling can be disastrous. Investigators must be on guard with regard to this last situation, but it rarely occurs. An example commonly cited relates to choosing a sample of homes. In such a situation, it is possible that if the random start provides a corner house, every rth house thereafter might also

9] $\Omega = \{2, 3, 4, \ldots\}$ $\mathcal{F} = \{2^{\Omega}\}$

$P(\{i\}) = \frac{1}{i}$; $i = 2, 3 \ldots$

$$P(\Omega) = P\left(\bigcup_{i=2}^{\infty} \{i\}\right) = \sum_{i=2}^{\infty} P(\{i\}) = \sum_{i=2}^{\infty} \frac{1}{i} \neq 1$$

infact $P(\Omega) = \infty$

$\therefore$ P cannot be extended to a prob measure on $\mathcal{F}$.

10] $P(A_n) = 1$ for $n = 1, 2, 3 \ldots$

To show $P\left(\bigcap_{n=1}^{\infty} A_n\right) = 1$

We will show $P\left[\left(\bigcap_{n=1}^{\infty} A_n\right)^c\right] = 0 \rightarrow P\left(\bigcap_{n=1}^{\infty} A_n\right) = 1$

$$P\left[\left(\bigcap_{n=1}^{\infty} A_n\right)^c\right] = P\left(\bigcup_{n=1}^{\infty} A_n^c\right)$$

$$\leq \sum_{n=1}^{\infty} P(A_n^c) \quad \{\because \text{countable subadditivity}$$

Now $P(A_n) = 1 \rightarrow P(A_n^c) = 0$; $\forall\, n = 1, 2 \ldots$

$$\therefore P\left[\left(\bigcap_{n=1}^{\infty} A_n\right)^c\right] \leq 0 \quad \text{——} (*)$$

But $P(A) \geq 0$ for any $A \in \mathcal{A}$ $\{ P : (\Omega, \mathcal{A}) \rightarrow ([0,1], \mathcal{B}_{[0,1]})$

Since $A_1, A_2 \ldots \in \mathcal{A} \Rightarrow \left(\bigcap_{n=1}^{\infty} A_n\right) \in \mathcal{A} \Rightarrow \left(\bigcap_{n=1}^{\infty} A_n\right)^c \in \mathcal{A}$

$$\Rightarrow P\left[\left(\bigcap_{n=1}^{\infty} A_n\right)^c\right] \geq 0 \quad \text{——} (**)$$

$\therefore (*) \;\&\; (**) \Rightarrow$ $P\left[\left(\bigcap_{n=1}^{\infty} A_n\right)^c\right] = 0$

Data Generation and Simulation Procedures

This is a Monte Carlo study that generated sampling distributions of the school (second-level) fixed and random effects of HLM using specified parameter values. One application of Monte Carlo techniques is to generate sampling distributions under known population parameters and to investigate the properties of parameter estimates by observing their empirical (sampling) distribution (Hammersley & Handescomb, 1964).

This study adopted the general procedure of Busing's (1993) two-level sampling to generate the data, where school-level values were generated, then student-level values; and then one combined equation was used to generate the outcome values. The model that was used in this simulation study was described in Equation 3.4. The data were generated from known population parameters. These data represented the baseline model with no assumption violations as well as the systematic variations that represent different assumption violations and conditions specified in the study as shown in Table 3. 2.

To run the simulations, a large number of replications (100) was generated for each condition. The researcher generated response vectors using the true predictor variables and the true parameter values to generate level-1 and level-2 values from their respective distributions. Gammas (γ's) were fixed in the replications, while the second-level predictors and first-level (student) predictor values (*W's and X's*) were sampled, from pre-specified distributions, and were fixed for each level of collinearity and sample size factors. The second-level (school) residuals were sampled, from pre-specified distributions, and were fixed for each level of residual homoskedasticity assumption and school sizes to allow the comparison among the results.

The data obtained from each replication included fixed- and random-effect estimates and their standard errors. The statistics from each replication were saved on separate files and were compared with their true values. The analysis routine that is described in the following paragraphs was used to compute the required statistics to estimate the robustness of the estimates.

The sequences to generate the data that were completed and replicated for each condition are as follows: