## STA437/STA1005 Fall 2005 Assignment 1

- 1) 2.9.15 p52 Let **x** be  $N_3(\mu, \Sigma)$  with  $\mu' = [5, 10, 2]$  and  $\Sigma = \begin{bmatrix} 4 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ . a) What is the conditional distribution of  $x_2$  and  $x_3$  given  $x_1$ ?
- b) What is the conditional distribution of  $(x_1 + x_2, x_1 x_2)$  given  $x_3$ ?
- c) What is the distribution of  $A\mathbf{x}$ , where  $A = A = \begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$ ?

(5 marks)

2) 2.9.27 p55

Let **x** be 
$$N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$
 with  $\boldsymbol{\mu}' = [5, 10, 2]$  and  $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & 1 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 1 \end{bmatrix}$ .

Find the distribution of

- a)  $x_1 x_2$ ,
- b)  $x_1 + x_2 + x_3$
- c)  $(x_1, x_2)$ , the joint distribution of  $x_1, x_2$ .
- d)  $(x_1, x_2 | x_3)$ , the conditional distribution of  $x_1, x_2$  given  $x_3$ . (5 marks)
- 3) Let **x** be  $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  with  $\boldsymbol{\mu}' = [-3, 1, 4]$  and  $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ .

Which of the following variables are independent? Why?

a) X<sub>1</sub> and X<sub>2</sub>
b) X<sub>2</sub> and X<sub>3</sub>
c) (X<sub>1</sub>, X<sub>2</sub>) and X<sub>3</sub>
d) X<sub>2</sub> and X<sub>2</sub>-2.5 X<sub>1</sub>-X<sub>3</sub>

(5 marks)

4) The files, assign1dat1.txt, assign1dat2.txt and assign1dat3.txt (on course web page) contain generated multivariate data on three variables. I have also included a few outliers in these data sets. Use the methods we have discussed in lectures to identify these outlier. Test whether data are from a multivariate normal distribution. (Use data file assign1dat1.txt if your last name begins with A-E, assign1dat2.txt if your last name begins with F-N, and assign1dat3.txt if your last name begins with O-Z.)

(10 marks)