MAT4375 HW#4, 1999

Due to html limitations, ``X- bar'' will be noted X.
From the fourth edition of Johnson and Wichern

Problem 9.1

Here

$$L = \begin{bmatrix} .9 \\ .7 \\ .5 \end{bmatrix} \text{ and } LL' = \begin{bmatrix} .81 & .63 & .45 \\ .63 & .49 & .35 \\ .45 & .35 & .25 \end{bmatrix}.$$

Thus

$$L = \begin{bmatrix} .9 \\ .7 \\ .5 \end{bmatrix} \text{ and } \rho = LL'+\Psi = \begin{bmatrix} 1 & .63 & .45 \\ .63 & 1 & .35 \\ .45 & .35 & 1 \end{bmatrix} \text{ and } \Psi = \begin{bmatrix} .19 & 0 & 0 \\ 0 & .51 & 0 \\ 0 & 0 & .75 \end{bmatrix}.$$

Problem 9.2

(a) From the diagonal of LL' (see also line (9-6) on page 518), $h_1^2 = .81$, and the first factor explains 81% of the variance in the data. Similarly, $h_2^2 = .49$, $h_3^2 = .25$,

(b) Using (9-5) on page 517, $Corr(Z_i,F_1) = l_{i1}$. Here Z_1 carries the greatest ``weight'' in nameing F_1 since it has the strongest correlation with it among the original Z_i.

Problem 9.3

(a) For the m = 1 principal components solution

$$L = \sqrt{1.96}\begin{bmatrix} .625 \\ .593 \\ .507 \end{bmatrix}$$

and

$$LL' = 1.96\times\begin{bmatrix} .390625 & .371 & .317 \\ .371 & .351649 & .301 \\ .317 & .301 & .257049 \end{bmatrix} = \begin{bmatrix} .76563 & .72716 & .62132 \\ .72716 & .68932 & .58996 \\ .62132 & .58996 & .50382 \end{bmatrix}$$

Thus

$$\Psi \cong \begin{bmatrix} .23 & 0 & 0 \\ 0 & .31 & 0 \\ 0 & 0 & .50 \end{bmatrix} \text{ and Resid} \cong \begin{bmatrix} 0 & -.14 & -.17 \\ -.14 & 0 & -.24 \\ -.17 & -.24 & 0 \end{bmatrix}.$$

Here the m = 1 principal component solution is rather poor. The residual matrix is clearly non-zero, and the other elements poor approximations of the ones found in 9.1.

(b) The proportion of variance explained by the factor is $(1.96/(1+1+1))\times 100 = 65\%$.

Problem 9.4

Using Ψ from 9.1, we compute the reduced correlation matrix [(ρ)\tilde] = ρ-Ψ. We need the first principal component of this reduced correlation matrix, and then obtain the corresponding loading vector by (9-22) on page 529. Here $\lambda_1 = 1.55$ and

$$L_r^* = \begin{bmatrix} .89 \\ .69 \\ .50 \end{bmatrix}.$$

Result is close to that found in 9.1.

Problem 9.9

(a) Seems reasonable to me.

(b) I tried a -20^{o} rotation;

$$T = \begin{bmatrix} .94 & .34 \\ -.34 & .94 \end{bmatrix}$$

and obtained

$$[F_1, F_2]\,T = \begin{bmatrix} .59 & .24 \\ .49 & .11 \\ .51 & -.07 \\ -.09 & .75 \\ -.50 & .52 \\ -.25 & .08 \\ -.53 & .02 \\ -.46 & -.21 \\ -.51 & -.36 \end{bmatrix}$$

This shows F_1^{*} to be sweet vs non-sweet, and F_2^{*} to be rum and Marc versus the rest. The interpretation stays roughly the same.

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