THE UNIVERSITY OF CHICAGO
Graduate School of Business
Business 424-01, Spring Quarter 2000, Mr. Ruey S. Tsay

Mid-term Exam

Notes:

1. Open book and notes. The exam time is 80 minutes.
2. Write your answers in a bluebook. Mark the solution clearly.

1. Let $\boldsymbol{Z} = (Z_1, Z_2, Z_3)'$ be a 3-dimensional Gaussian random variable with mean $\boldsymbol{\mu} = (-3, 4, 2)'$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

Consider the following questions or statements. If the statement is true, briefly justify it such as citing a result from the textbook. If the statement is false, please explain why.

(a) $Z_1 + Z_3$ is independent of Z_2.

(b) $(Z_1, Z_3)'$ is independent of Z_2.

(c) Consider the simple linear regression model

$$Z_1 = \beta_0 + \beta_1 Z_3 + \epsilon,$$

where $E(\epsilon) = 0$ and ϵ is uncorrelated with Z_3. Find the values of β_0 and β_1.

(d) What is the conditional distribution of Z_1 given that $Z_3 = 3$?

(e) $Z_1 + Z_2$ is normally distributed with mean 1 and variance 3.

2. Consider the multivariate regression of Problem 7.25 on page 455 of the textbook. There are 17 observations, two dependent variables, and five predictors. All variables but z_1(GEN) and z_4(DIAP) were transformed by taking natural logarithm. The output of a multivariate regression is attached. Answer the following questions:

(a) Does gender (z_1) significantly affect the two response variables at the 5% significance level? You may use individual tests.

(b) What is the covariance between the two least squares estimates of the effects of gender?

(c) Instead of using the ordinary least squares method, one may use the maximum likelihood (ML) method. What is the ML estimate of $\boldsymbol{\Sigma}$, the covariance matrix of the error term?

(d) Describe a method that can be used to test the hypothesis that the predictor z_4(DIAP) can be removed from the model. [No numerical calculation is needed.]

3. A simple experiment collected two responses for two treatments. The data are as follows:

$$\text{Treatment 1:} \quad \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

$$\text{Treatment 2:} \quad \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

The sample means and covariance matrices are:

$$\text{Trt 1:} \quad \hat{\boldsymbol{x}}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \ \boldsymbol{S}_1 = \begin{bmatrix} 1 & -1.5 \\ -1.5 & 3 \end{bmatrix}.$$

$$\text{Trt 2:} \quad \hat{\boldsymbol{x}}_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \ \boldsymbol{S}_2 = \begin{bmatrix} 2 & -1.33 \\ -1.33 & 1.33 \end{bmatrix}.$$

(a) What is $\boldsymbol{S}_{pooled}$?

(b) Test $H_o : \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \mathbf{0}$ using a two sample approach with significance level $\alpha = 0.05$.

4. Consider a two-way multivariate analysis of variance (MANOVA) model

$$\boldsymbol{X}_{ij} = \boldsymbol{\mu} + \boldsymbol{\tau}_i + \boldsymbol{\beta}_j + \boldsymbol{e}_{ij}, \quad i = 1, \cdots, g; \quad j = 1, \cdots, b$$

where $\sum_{i=1}^{g} \boldsymbol{\tau}_i = \mathbf{0}$ and $\sum_{j=1}^{b} \boldsymbol{\beta}_j = \mathbf{0}$, $\boldsymbol{e}_{ij}$ are independent $N_p(\mathbf{0}, \boldsymbol{\Sigma})$ random vectors. Assume that $\boldsymbol{x}_{ij}$ is the single observation for $\boldsymbol{X}_{ij}$.

(a) Construct a MANOVA table for the model. You should give the formulas and degrees of freedom.

(b) What additional assumption is needed (as compared with the case of n observations for $\boldsymbol{X}_{ij}$, where $n > 1$) in order to make statistical inference?