THE UNIVERSITY OF CHICAGO

Graduate School of Business

Business 424-01, Spring Quarter 2000, Mr. Ruey S. Tsay

Solutions to Midterm

1. By re-arranging the ordering of the components based on the covariance matrix, we have $\boldsymbol{Z}^* = (Z_1, Z_3, Z_2)'$ with mean $\boldsymbol{\mu} = (-3, 2, 4)'$ and covariance matrix

$$\boldsymbol{\Sigma}^* = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Using Result 4.5(b), $(Z_1, Z_3)'$ and Z_2 are independent.

   (a) True, because $Z_1 + Z_3$ normal and $\text{cov}(Z_1 + Z_3, Z_2) = 0$.

   (b) True, as stated above.

   (c) Taking expectation of the model, $E(Z_1) = \beta_0 + \beta_1 E(Z_3)$. Thus, $\beta_0 = -3 - 2\beta_1$. Next, $\text{cov}(Z_1, Z_3) = \text{cov}(\beta_0 + \beta_1 Z_3 + \epsilon, Z_3) = \beta_1 \text{ Var}(Z_3)$. Therefore, $\beta_1 = \text{cov}(Z_1, Z_3)/\text{Var}(Z_3) = -1/2 = -0.5$. Consequently, $\beta_0 = -3 + 0.5 \times 2 = -2$.

   (d) Use Result 4.6. $Z_1 | Z_3 = 3$ is normal with mean -3.5 and variance 0.5.

   (e) Yes, because Z_1 and Z_2 are independent.

2. Not covered by the midterm of 2002.

3. Answer:

   (a) $\boldsymbol{S}_{pooled} = \frac{n_1 - 1}{n_1 + n_2 - 2} S_1 + \frac{n_2 - 1}{n_1 + n_2 - 2} S_2$. Simple calculation shows

$$S_{pooled} = \begin{bmatrix} 1.6 & -1.398 \\ -1.398 & 1.998 \end{bmatrix}.$$

   (b) Use the Hotelling T^2 test of Result 6.2. Simple calculation gives $T^2 = 3.87$, which is distributed as $\frac{5 \times 2}{4} F_{2,4}$. The p value is 0.318. Thus, one cannot reject H_o at the 5% level.

4. Answer

   (a) See the two-way MANOVA table on page 310 with $n = 1$, i.e. no interaction.

   (b) There is no interaction between the two factors.